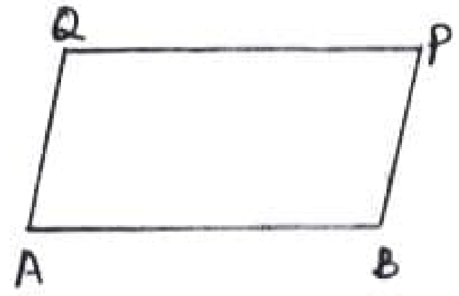
3. In a quadrilateral ABQP, AB = PQ and AB ∥ PQ. Show that ABQP is a parallelogram.

Solution:

Given that

AB = PQ and

AB ∥ PQ.



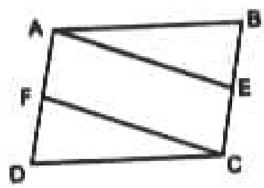
A quadrilateral is a parallelogram, if it has one pair of opposite sides parallel an equal.

Thus, ABPQ is a parallelogram.

4. In a parallelogram ABCD, E and F are the mid-points of BC and AD. Show that

(i) AFCE is a parallelogram

(ii) ΔABE ≅ ΔCDF. [HOTS]



Solution:

(i)

Given that, ABCD is a parallelogram.

⇒ AD = BC and AD ∥ BC

E and F are mid points of BC and AD respectively.

Thus,

$CE = AF \quad (\because AD = BC)$

$CE \parallel AF \quad (\because AD \parallel BC)$

We know that if one pair of opposite sides are parallel and equal, then the quadrilateral is a parallelogram.

Hence, AFCE is a parallelogram.

(ii)

In $\triangle ABE$ and $\triangle CDF$, we have:

$BE = FD$

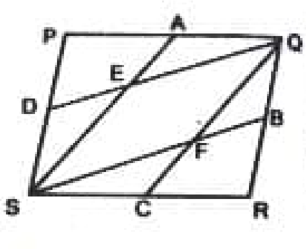
$CD = AB \quad (\because ABCD$ is a parallelogram$)$

and $AE = FC \quad (\because AFCE$ is a parallelogram$)$

$\therefore$ By SSS rule of congruence, $\triangle ABE \cong \triangle CDF$.

5. A, B, C, D are the mid-points of the sides PQ, QR, RS and SP of a parallelogram PQRS respectively. SA, SB, QC and QD have been joined to intersect at E and F. Show that EQFS is a parallelogram.

[HOTS]



Solution:

Given that, PQRS is a parallelogram.

$\Rightarrow$ $PQ = SR$ and $PQ \parallel SR$

A and C are mid points of PQ and SR respectively.

Thus, $AQ = SC$ and $AQ \parallel SC$

We know that, if one pair of opposite sides are parallel and equal, then the obtained quadrilateral is a parallelogram.

Hence, AQCS is a parallelogram.

Now in $\triangle APS$ and $\triangle CRQ$, we have:

$PS = RQ$ (opposite sides of a parallelogram)

$\angle SPA = \angle QRC$ (opposite angles of a ||gm)

$AP = CR$ (A and C are mid points of opposite sides of a ||gm)

$\therefore \triangle APS \cong \triangle CRQ$ (By SAS)

$\Rightarrow AE = CF$ (by CPCT) ---- (i)

Again,

$AS = CQ$ and $AS \parallel CQ$

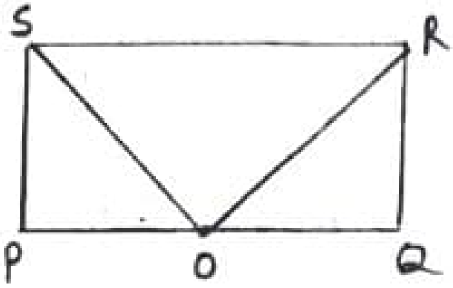
$AS - AE = CQ - CF$ from (i)

$\Rightarrow SE = QF$ and $SE \parallel QF$

Thus, EQFS is a parallelogram.

6. **In rectangle PQRS, O is the mid-point of PQ. Show that $\Delta POS \cong \Delta QOR$.**

Solution:



In $\triangle POS$ and $\triangle QOR$,

we have:

$SP = RQ$ (opposites sides of a rectangle)

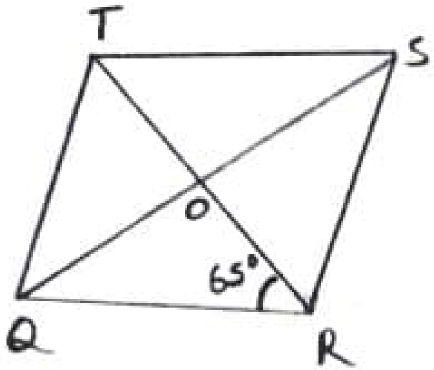
$\angle SPO = \angle RQO$ (each 90°)

$PO = OQ$ (O is the mid point of PQ)

$\therefore \Delta POS \cong \Delta QOR$.

7. **In a rhombus QRST, diagonals QS and RT intersect at O and $\angle ORQ = 65^\circ$. Find the angles of the rhombus.**

Solution:



Given that

$\angle ORQ = 65^\circ$

We know that diagonals of a rhombus bisect the interior angle.

$\therefore \angle QRS = 2 \times 65^\circ = 130^\circ$.

Again,

consecutive angles of a rhombus are supplementary.

$\therefore \angle QRS + \angle RST = 180^\circ$

$\Rightarrow 130^\circ + \angle RST = 180^\circ$

$\Rightarrow \angle RST = 180^\circ - 130^\circ = 50^\circ$

opposite angles of a rhombus are equal.

$\therefore \angle TQR = \angle RST = 50^\circ$

and $\angle STQ = \angle QRS = 130^\circ$

Thus, angles of a rhombus are 130°, 50°, 130° and 50°.

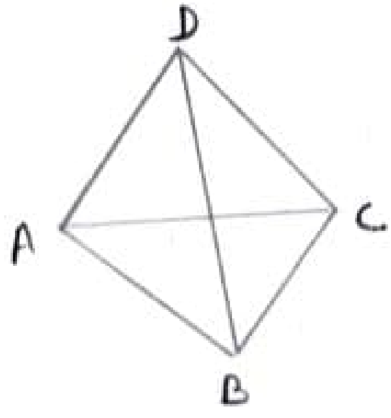
8. One of the diagonals of a rhombus is equal to the side of the rhombus, find the angles of the rhombus. [HOTS]

Solution:

Given that,

ABCD is a rhombus.

$\Rightarrow AC = BC$ --- (i)



$BC = AB$ ------ (ii)

From eq (i) and (ii)

$AC = BC = AB$

$\Rightarrow \triangle ABC$ is an equilateral triangle.

So, $\angle ABC = 60^\circ$

$\angle BCA = 60^\circ$ ------ (iii)

and $\angle CAB = 60^\circ$ ------ (iv)

Similarly, in $\triangle ADC$, $AD = DC$ (sides of a rhombus)

$AD = BC$

But $BC = AC$

$\therefore AD = AC$

$\therefore AD = DC = AC$

$\therefore DAC$ is an equilateral triangle.

$\Rightarrow \angle CAD = 60^\circ$ ------ (v)

$\angle ADC = 60^\circ$

$\angle DCA = 60^\circ$ ------ (vi)

From eq (iii) and (vi), we get:

$\angle BCA + \angle DCA = 60^\circ + 60^\circ = 120^\circ$

$\therefore \angle C = 120°$

From eq (iv) and (v)

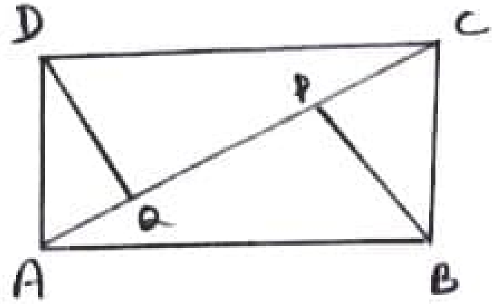
$\angle CAB + \angle CAD = 60° + 60° = 120°$

$\angle A = 120°$

Therefore, four angles of the rhombus are $120°$, $60°$, $120°$, $60°$.

9. In a rectangle ABCD, BP and DQ are perpendicular to the diagonal AC. Is BP = DQ? Justify.

Solution:



In $\triangle ABP$ and $\triangle CQD$,

we have:

$\angle APB = \angle DQC$ (each $90°$)

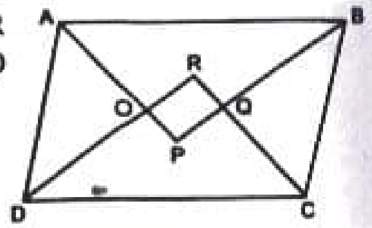
$AB = CD$ (opposite sides of rectangle)

$\angle BAP = \angle QCD$ (alternate angles)

$\therefore \triangle ABP \cong \triangle DCQ$ (By AAS)

$\Rightarrow BP = DQ$ (By CPCT)

10. AP and BP bisect ∠A and ∠B in a parallelogram ABCD. Also CR and DR bisect ∠C and ∠D intersecting BP and AP at Q and O respectively.
Find ∠AOD, ∠BQC, ∠APB and ∠CRD
Is OPQR a rectangle?



Solution:

To prove OPQR is a rectangle.

Proof: Since ABCD is a parallelogram.

Therefore $AB \parallel DC$

Now, $AB \parallel DC$ and transversal AD intersects them at A and D respectively. Therefore,

$\angle A + \angle D = 180^\circ$ [∵ Sum of consecutive interior angles is 180°]

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D = 90^\circ$

$\Rightarrow \angle DAO + \angle ADO = 90^\circ$ --- (i)

[∵ AO and DO are bisectors of ∠A and ∠D respectively]

But, in △ ADO, we have:

$\angle DAO + \angle ADO + \angle AOD = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle AOD = 180^\circ$

$\rightarrow \angle AOD = 90^\circ$

$\therefore \angle ROP = 90^\circ$

[$\because$ $\angle AOD$ and $\angle ROP$ are vertically opposite angles, $\therefore \angle AOD = \angle ROP$]

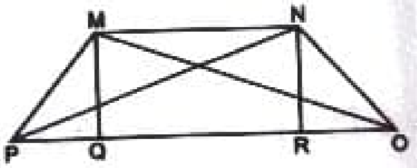
Similarly, we can prove that

$\angle ORQ = 90^\circ$, $\angle RQP = 90^\circ$ and

$\angle QPO = 90^\circ$.

Hence, OPRQ is a rectangle.

11. In an isosceles trapezium MNOP, MQ and NR are perpendiculars to PO.
Show that $\Delta PQM \equiv \Delta ORN$ and $\Delta PNR \equiv \Delta OMQ$



Solution:

In ΔPQM and ΔORN, we have:

$PM = ON$ (Sides of isosceles trapezium)

$QM = RN$ (Perpendiculars)

$\angle PQM = \angle ORN$ (each 90°)

$\therefore \Delta PQM \cong \Delta ORN$ (By RHS)

$\Rightarrow PQ = RO$ (CPCT)

Now,

In $\triangle PNR$ and $\triangle OMQ$

$NR = MQ$ (Equal perpendiculars)

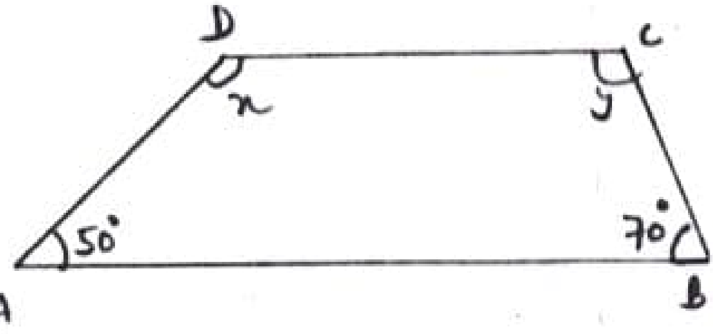
$\angle NRQ = \angle MQO$ (Each $90°$)

$RP = QO$ ($\because$ PQ and RO are equal and QR is common)

$\therefore \triangle PNR \cong \triangle OMQ$ (By SAS)

12. **In a trapezium, two adjacent angles are 50° and 70°. Find the remaining two angles.**

Solution:



The sum of two adjacent angles which one of the non-parallel sides is 180°.

$\therefore 50° + x = 180°$

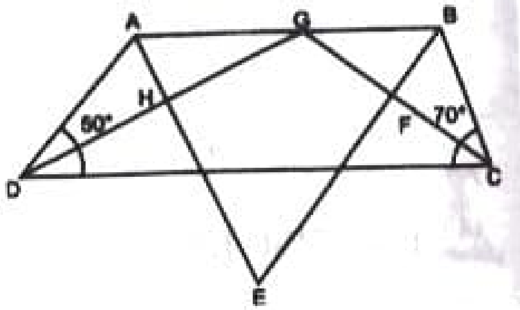
$\Rightarrow x = 180° - 50° = 130°$

and $70° + y = 180°$

$\Rightarrow y = 180° - 70° = 110°$

Hence, remaining two angles are 130° and 110°.

13. **The angle bisectors of a trapezium ABCD intersect at points E, F, G and H. If $\angle ADC = 50^\circ$, $\angle BCD = 70^\circ$, find the angles of EFGH.** [HOTS]



Solution:

Given that,

$\angle ADC = 50^\circ$

$\therefore \angle ADH = \frac{50^\circ}{2} = 25^\circ$

$\angle AHE = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\angle AHD = 180^\circ - (25^\circ + 65^\circ) = 90^\circ$

$\angle AHD$ and $\angle GHE$ are vertically opposite angles.

Thus $\angle GHE = 90^\circ$

Now,

$\angle BCD = 70^\circ$

$\Rightarrow \angle BCG = \frac{70^\circ}{2} = 35^\circ$

$\angle CBE = \dfrac{180^\circ - 70^\circ}{2} = 55^\circ$

$\angle BFC = 180^\circ - (35^\circ + 55^\circ) = 90^\circ$

$\angle BFC$ and $\angle GFE$ are vertically opposite angles.

$\therefore \angle GFE = 90^\circ$

$\angle AEB = 180^\circ - (65^\circ + 55^\circ) = 60^\circ$

we know that sum of all the angles in a quadrilateral $= 360^\circ$

$\therefore \angle HGH = 360^\circ - (90^\circ + 90^\circ + 60^\circ)$

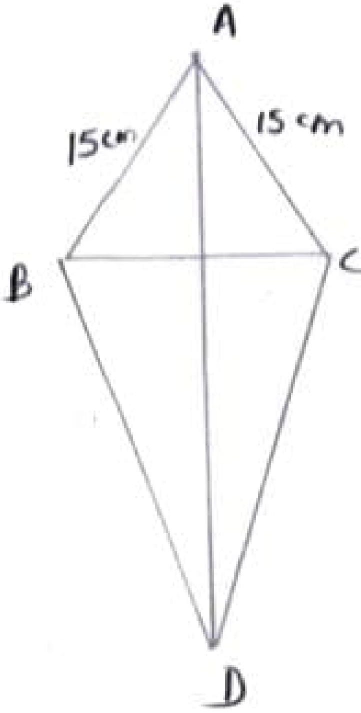
$= 120^\circ$

Thus, $\angle E = 60^\circ$, $\angle F = 90^\circ$, $G = 120^\circ$ and $\angle H = 90^\circ$.

14. Perimeter of a kite is 70 cm and its one side is 15 cm. Find the other three sides.

Solution: Adjacent sides of a kites are equal. Therefore $AC = AB = 15$ cm

Let the length of equal sides be x.



Perimeter of a kite

$= AB + AC + DB + DC$

$70 = 15 + 15 + x + x$

$70 = 30 + 2x$

$2x = 70 - 30 = 40$

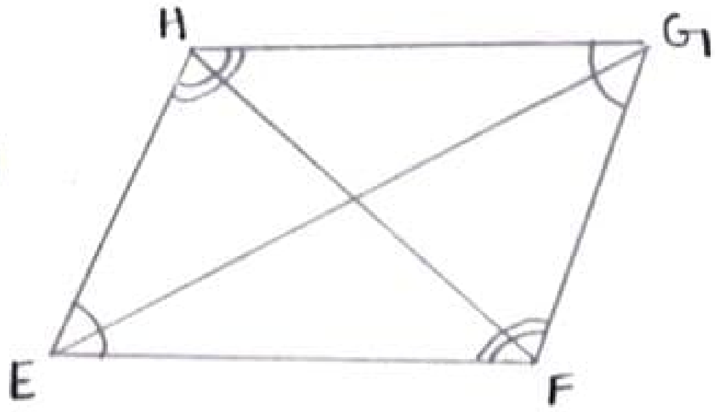
$x = \frac{40}{2} = 20$

$\therefore DB = DC = 20cm$

Thus, remaining sides of a kite are 15cm, 20 cm, 20 cm.

15. EFGH is a rhombus. Show that EG bisects $\angle E$ and $\angle G$ both.

Solution:



Given that EFGH is a rhombus.

$\Rightarrow EF = FG = GH = HE$

To prove:

EG bisects both $\angle E$ and $\angle G$.

i.e. $\angle FEG = \angle HEG$

and

$\angle HGE = \angle FGE$

Proof:

In $\triangle EHG$ and $\triangle EFG$

$EH = FG$ (Given)

$GH = EF$ (Given)

$EG = EG$ (common)

$\Rightarrow \triangle EHG \cong \triangle EFG$ (by SSS)

$\therefore \angle FEG = \angle HEG$ (by CPCT)

and

$\angle HGE = \angle FGE$ (by CPCT)

Thus, EG bisects $\angle E$ and $\angle G$.

16. In an isosceles $\triangle ABC$, $AP \perp BC$. On AP produced, a point D has been taken such that $PD > AP$. What is the quadrilateral ABDC called? If $PD = AP$, what will be the figure ABDC? [HOTS]

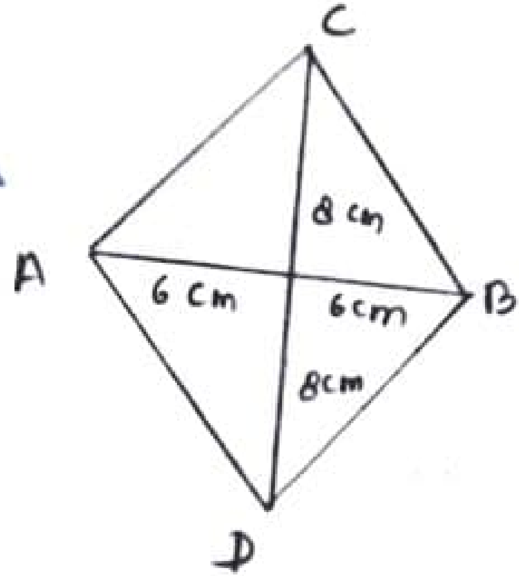
Solution:

when $PD > AP$, the obtained quadrilateral is called kite.

when, $PD = AP$, the obtained quadrilateral is called rhombus.

17. **In a quadrilateral, the diagonals are perpendicular bisectors of each other and their lengths are 12 cm and 16 cm. Find the lengths of the sides of the quadrilateral.**

Solution:



In a rhombus, diagonals are perpendicular bisectors of each other.

Thus,

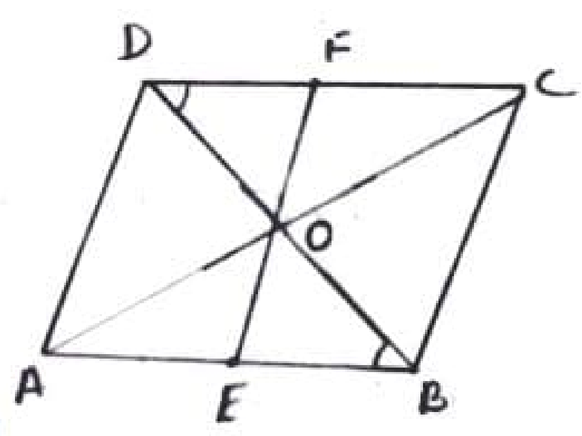
$AC = \sqrt{8^2 + 6^2} = 10\text{cm}$

All sides of a rhombus are equal.

Thus, the lengths of the sides of the quadrilateral is 10 cm each.

18. **In a parallelogram ABCD, E and F are the mid-points of AB and DC respectively. Show that BD bisects EF.**

Solution:



Construction:

Draw Diagonals BD and AC.

In llgm diagonals are bisect to each other.

In $\triangle DFO$ and $\triangle EBO$, we have:

$DF = BE$ (F and E are midpoints)

$\angle EBO = \angle FDO$ (alternate angles)

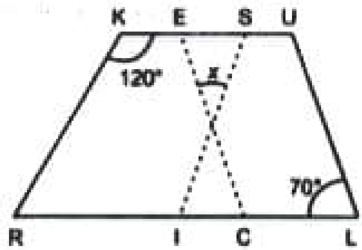
$OB = OD$ (Digonals bisects)

$\therefore \triangle DFO \cong \triangle EBO$ (By SAS)

Thus, $EO = OF$

This means, BD bisects EF.

19. In the figure, RISK and CLUE are parallelograms. Find the value of x. [HOTS]



Solution:

Adjacent angles of a parallelogram are supplementary.

In parallelogram, RISK,

$\angle RKS + \angle ISK = 180°$

$\Rightarrow 120^\circ + \angle ISK = 180^\circ$

$\angle ISK = 60^\circ$.

Also, opposite angles of a parallelogram are equal.

In parallelogram CLUE,

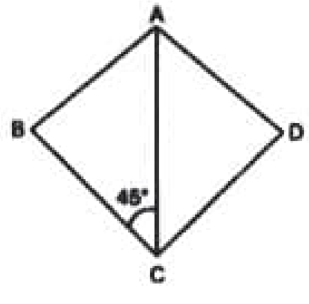
$\angle ULC = \angle CEU = 70^\circ$.

The sum of the measures of all the interior angle of a triangle is 180°.

$x + 60^\circ + 70^\circ = 180^\circ$

$\Rightarrow x = 50^\circ$.

20. In the figure, ABCD is a rhombus and $\angle ACB = 45^\circ$. Find $\angle ADC$. [HOTS]



Solution:

We know that each diagonal divides the rhombus in two congruent triangles

$\therefore \angle BCD = 2 \times 45^\circ = 90^\circ$.

Consecutive angles of a rhombus are supplementary.

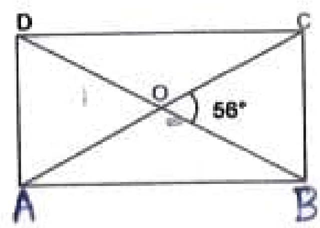
$$\angle C + \angle D = 180^\circ$$

$$90^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 90^\circ.$$

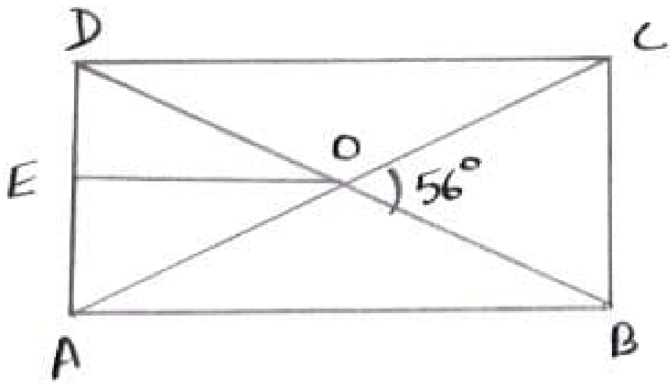
Thus, $\angle ADC = 90^\circ$.

21. In the figure, ABCD is a rectangle and $\angle BOC = 56^\circ$. Find $\angle ADO$.



[HOTS]

Solution:



Construction: Let E be the mid point of AD.

In rectangle, length of diagonals are equal and they bisect each other.

It means that $OA = OB = OC = OD$

In $\triangle OED$ and $\triangle OEA$, we have:

$OA = OD$ (by above)

$OE = OE$ (common)

$AE = ED$ (By construction)

So, $\triangle OED \cong \triangle OEA$ (By SSS)

Now

$\angle AOD = \angle BOC = 56^\circ$ (vertically opposite angles)

As $\angle EOD = \angle EOA$ (CPCT)

Also $\angle EOD + \angle EOA = 56^\circ$

$\therefore \angle EOD = 28^\circ$

Similarly,

$\angle OED = \angle OEA = 90^\circ$

Now in $\triangle OED$, $\angle OED + \angle EDO + \angle DOE = 180^\circ$

$\Rightarrow 90^\circ + \angle EDO + 28^\circ = 180^\circ$

$\Rightarrow \angle EDO = 62^\circ$

Thus, $\angle ADO = 62^\circ$.

In $\triangle OED$ and $\triangle OEA$, we have:

$OA = OD$ (by above)

$OE = OE$ (common)

$AE = ED$ (By construction)

So, $\triangle OED \cong \triangle OEA$ (By SSS)

Now

$\angle AOD = \angle BOC = 56°$ (vertically opposite angles)

As $\angle EOD = \angle EOA$ (CPCT)

Also $\angle EOD + \angle EOA = 56°$

$\therefore \angle EOD = 28°$

Similarly,

$\angle OED = \angle OEA = 90°$

Now in $\triangle OED$, $\angle OED + \angle EDO + \angle DOE = 180°$

$\Rightarrow 90° + \angle EDO + 28° = 180°$

$\Rightarrow \angle EDO = 62°$

Thus, $\angle ADO = 62°$.