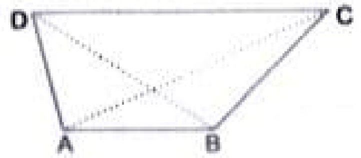
# Chapter-12

# Quadrilaterals

## WHAT WE HAVE LEARNT....

1. How many diagonals does a quadrilateral have?
2. In the quadrilateral ABCD, name the following :
   (a) Vertices (b) Angles
   (c) Pairs of opposite sides (d) Diagonals




3. Observe the following quadrilaterals and name them :

(a) (b) (c) (d) (e)

4. Name the quadrilateral which has just one pair of parallel sides.
5. Name the quadrilateral whose two equal diagonals are perpendicular to each other.
6. Say true or false :
   (a) Every rectangle is a parallelogram.
   (b) The diagonals of a parallelogram are equal.
   (c) The diagonals of a rhombus are equal and bisect each other at right angles.
   (d) Every parallelogram is a rhombus.
   (e) In a trapezium a pair of opposite sides are parallel.

Solution:

(1) A quadrilateral have 2 diagonals.

(2)

(a) Vertices: A, B, C and D

(b) Angles : $\angle A$, $\angle B$, $\angle C$ and $\angle D$

(c) Pairs of opposite sides :

- Side AB opposite to side CD
- Side AD opposite to side BC

(d) Diagonals: AC and BD

(3)

(a) Square

(b) Rectangle

(c) Rhombus

(d) Trapezium

(e) Parallelogram

(4) Trapezium is the quadrilateral which has just one pair of parallel sides.

(5) Rhombus.

(6) (a) True

(b) False

(c) False

(d) False

(e) True

# Exercise 12·1

## EXERCISE 12.1

1. How many diagonals does each of the following have?
   (i) A triangle (ii) A pentagon (iii) A heptagon

Solution:

(i) A triangle has 0 diagonals.

(ii) A pentagon has 5 diagonals.

(iii) A heptagon has 14 diagonals.

2. Find the sum of the angles of a polygon of :
   (i) 5 sides (ii) 6 sides (iii) 8 sides (iv) 16 sides

Solution:

We know that,

Sum of the angles of a polygon of n-sides

$= (2n-4)$ right angles

(i) 5 sides

Here $n=5$

$\therefore$ Sum of the angles $= (2\times5-4)\times90^\circ = 540^\circ$

(ii) 6 sides

Here $n=6$

$\therefore$ Sum of the angles $= (2\times6-4)\times90^\circ$
$= 720^\circ.$

(iii) 8 sides

Here $n=8$

$\therefore$ Sum of the angles $= (2\times8-4)\times90^\circ$
$= 1080^\circ$

(iv)

16 sides

Here $n=16$

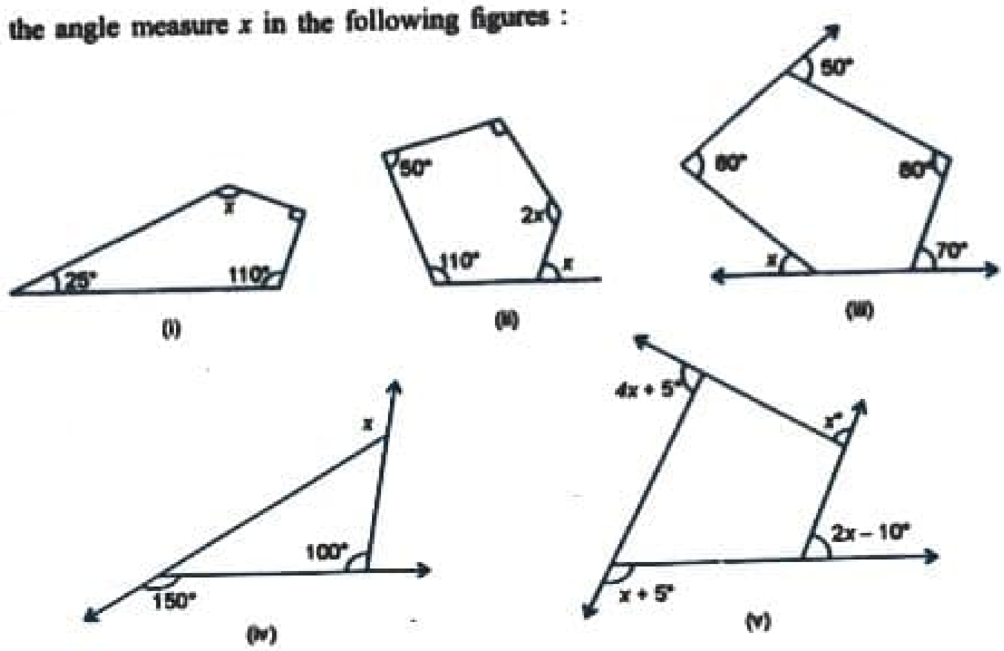
$\therefore$ Sum of the angles $= (2\times16-4)\times90^\circ$
$= 2520^\circ$

**3. All the angles of a rectangle are equal. Is it a regular polygon?**

Solution.

No, because in a rectangle all angles equal but sides are not equal. Hence, it is not a regular polygon.

4. **Find the angle measure $x$ in the following figures :**




Solution:

Given figure is quadrilateral.

(i) $x = 360^\circ - (25^\circ + 110^\circ + 90^\circ)$

$= 360^\circ - 225^\circ$

$= 135^\circ$

(ii) Given figure is pentagon.

$\therefore\ 110^\circ + 50^\circ + 90^\circ + 2x + 180^\circ - x = 540^\circ$

$\Rightarrow 250^\circ + 180^\circ + x = 540^\circ$

$\Rightarrow 430^\circ + x = 540^\circ$

$\Rightarrow x = 540^\circ - 430^\circ = 110^\circ$

(iii)

Given figure is pentagon.

$\therefore (180^\circ - 50^\circ) + 80^\circ + (180^\circ - x) + (180^\circ - 70^\circ) + 80^\circ$
$= 540^\circ$

$\Rightarrow 130^\circ + 80^\circ + 180^\circ - x + 110^\circ + 80^\circ = 540^\circ$

$\Rightarrow 580^\circ - x = 540^\circ$

$\Rightarrow x = 580^\circ - 540^\circ = 40^\circ$

(iv)

Given figure is triangle.

$\therefore (180^\circ - 150)^\circ + 100^\circ + (180^\circ - x) = 180^\circ$

$\Rightarrow 30^\circ + 100^\circ + \cancel{180^\circ} - x = \cancel{180}$

$\Rightarrow 130^\circ - x = 0$

$\Rightarrow x = 130^\circ$

(v) The given figure is quadrilateral.

$\therefore$ Sum of exterior angles of a polygon $= 360^\circ$

$\therefore x + 5^\circ + 2x - 10^\circ + x^\circ + 4x + 5^\circ = 360^\circ$

$\Rightarrow 8x + \cancel{5^\circ} - \cancel{10^\circ} + \cancel{5^\circ} = 360^\circ$

$\Rightarrow 8x = 360^\circ \Rightarrow x = \dfrac{360^\circ}{8} = 45^\circ$

5. Find the number of sides of a polygon, the sum of whose interior angle is :

(i) 720°  (ii) 1620°  (iii) 10 right angles

Solution: Let the polygon has n sides. Then.

(i)

$(2n-4)\times 90^\circ = 720^\circ$

$\Rightarrow (2n-4) = \frac{720^\circ}{90^\circ} = 8$

$\Rightarrow 2n-4 = 8$

$\Rightarrow 2n = 12 \Rightarrow n = \frac{12}{2} = 6$

(ii)

$(2n-4)\times 90^\circ = 1620^\circ$

$2n-4 = \frac{1620^\circ}{90^\circ} = 18$

$2n-4 = 18$

$2n = 18+4 = 22$

$n = \frac{22}{2} = 11$

(iii) $(2n-4)\times 90^\circ =$ 10 right angles

$\Rightarrow (2n-4)\times 90^\circ = 10\times 90^\circ$

$\Rightarrow (2n-4)\times 90^\circ = 900^\circ$

$\Rightarrow (2n-4) = \frac{900^\circ}{90^\circ} = 10$

$\Rightarrow 2n-4 = 10$

$\Rightarrow 2n = 10+4 = 14$

$\Rightarrow n = \frac{14}{2} = 7$

**6. Is it possible to have a polygon, the sum of whose interior angles is 750°?**

Solution:

No, because number of sides cannot be in fraction. It should be a whole number.

**7. Find the measures of each interior angle of a regular :**

**(i) pentagon (ii) hexagon (iii) decagon**

Solution:

(i) Sum of the interior angles of the pentagon

$= (2\times5-4) \times 90^\circ = 540^\circ$

$\therefore x+x+x+x+x = 540^\circ$

$\Rightarrow x = \frac{540^\circ}{5} = 108^\circ.$

(ii) Sum of the interior angles of the hexagon $= (2\times6-4)\times90^\circ = 720^\circ$

$\therefore\ x+x+x+x+x+x = 720^\circ$

$\Rightarrow\ 6x = 720^\circ$

$\Rightarrow\ x\ \dfrac{720^\circ}{6} = 120^\circ$

(iii)

Sum of the interior angles of the decagon $= (2\times10-4)\times90^\circ = 1440^\circ$

$\therefore\ (x+x+x+x\ \ldots\ 10\text{ times}) = 1440^\circ$

$\Rightarrow\ 10x = 1440^\circ$

$x = \dfrac{1440^\circ}{10} = 144^\circ$.

**8. Find the measures of each exterior angle of a regular polygon having :**

**(i) 8 sides (ii) 9 sides (iii) 12 sides**

Solution:

For a regular polygon of n-sides, each exterior angle $= \dfrac{360^\circ}{\ }$

(i) 8 sides

Here $n=8$

$\Rightarrow \therefore$ Exterior angle $= \frac{360^\circ}{8} = 45^\circ$

(ii) 9 sides

Here $n=9$

$\Rightarrow \therefore$ Exterior angle $= \frac{360^\circ}{9} = 40^\circ$

(iii) 12 sides

Here $n=12$

$\Rightarrow \therefore$ Exterior angle $= \frac{360^\circ}{12} = 30^\circ$

9. **Find the number of sides of a regular polygon each of whose exterior angles measures :** [HOTS]

(i) 45° (ii) 30° (iii) 20°

Solution:

We know that,

$$\text{number of sides}(n) = \frac{360^\circ}{\text{Measure of each exterior angle.}}$$

(i)

$$n = \frac{360^\circ}{45^\circ} = 8$$

(ii) $n = \frac{360°}{30°} = 12$

(iii) $n = \frac{360°}{20°} = 18$

10. How many sides does a regular polygon have, if each of its interior angles is :
(i) 144°? (ii) 160°?

Solution:

(i) Let the regular polygon has n sides.

Then, $n \times 144° = (2n-4) \times 90°$

$\Rightarrow 144°n = 180°n - 360°$

$\Rightarrow 36°n = 360°$

$\Rightarrow n = \frac{360°}{36°} = 10$

Hence the regular polygon has 10 sides.

(ii)

Let the regular polygon has n sides.

Then, $n \times 160° = (2n-4) \times 90°$

$\Rightarrow \quad 160^\circ n = 180^\circ n - 360^\circ$

$\Rightarrow \quad 20^\circ n = 360^\circ$

$\Rightarrow \quad n = \frac{360^\circ}{20^\circ} = 18$

Hence, the regular polygon has 18 sides.

11. Is it possible to have a regular polygon each of whose interior angles measures 145°? [HOTS]

Solution:

Let the regular polygon has n sides.

Then, $n \times 145^\circ = (2n-4) \times 90^\circ$

$\Rightarrow \quad 145^\circ n = 180^\circ n - 360^\circ$

$\Rightarrow \quad 35^\circ n = 360^\circ$

$\Rightarrow \quad n = \frac{360^\circ}{35^\circ} = 10.28$

Here, we see that, number of sides in decimal which is not possible. It should be a whole number.

So, the above regular polygon is not possible.

12. Is it possible to have a regular polygon with measure of each exterior angle as 25°? [HOTS]

Solution:

We know that for a regular polygon,

$$\text{number of sides} = \frac{360^\circ}{\text{each exterior angle}}$$

$$= \frac{360^\circ}{25^\circ} = 14.4$$

Here, number of sides of a regular polygon should be a whole number. It cannot be a decimal number.

Hence, it is not possible to have a regular polygon with measure of each exterior angle as 25°.

## EXERCISE 12.2

1. Find the angles of a quadrilateral whose all angles are equal.

Solution:

A quadrilateral has 4 angles. Given that all angles are equal.

$\therefore \quad x+x+x+x = 360^\circ$

$\Rightarrow \quad 4x = 360^\circ$ (Angle sum property)

$\Rightarrow \quad x = \frac{360^\circ}{4} = 90^\circ$

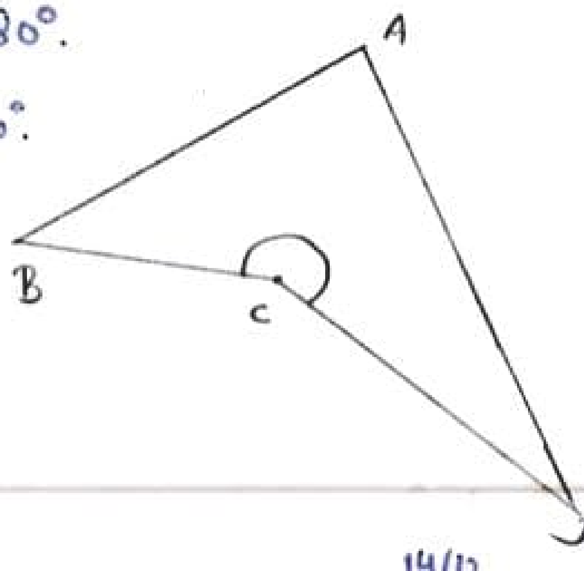
Hence, required angle is $90^\circ$.

2. How many angles of a concave quadrilateral are greater than $180^\circ$? Show by figure.

Solution:

In a concave quadrilateral only one angle will be greater than $180^\circ$.

Here, $\angle C$ is greater than $180^\circ$.



3. How many angles of a concave quadrilateral can be greater than 90°?

Solution:

Maximum two angles of a concave quadrilateral can be greater than 90°.

4. Is it possible that all the angles of a quadrilateral are acute? Explain why?

Solution:

No, because sum of 4 acute angles cannot be 360°.

5. Find the maximum number of angles in a quadrilateral that can be obtuse.

Solution:

In a quadrilateral, maximum 3 angles can be obtuse.

6. **In a quadrilateral PQRS, $\angle P = \angle Q$ and $\angle R = \angle S$. Also, $\angle P$ is double of $\angle R$. Find all the four angles.**

Solution:

Let the measure of $\angle R$ be $x$.

Then. $\angle P = 2$ times $\angle R = 2x$

Given that,

$$\angle P = \angle Q = 2x$$
$$\angle R = \angle S = x$$

Also, sum of four interior angles of a quadrilateral is $360°$.

$$\therefore \quad x + x + 2x + 2x = 360°$$

$$\Rightarrow \quad 6x = 360° \Rightarrow x = 60°$$

and $2x = 2 \times 60° = 120°$.

Thus, measure of 4 angles will be:

$120°$, $60°$, $120°$, $60°$.

7. Sum of two angles of a quadrilateral is 180°. What can you say about the other two angles?

Solution:

The sum of other two angles should be 180°. as the sum of the four interior angles of a quadrilateral is 360°.

8. One of the angles in a quadrilateral is 150°, remaining three angles are equal. Find the angles.

Solution:

We know that sum of the four interior angles of a quadrilateral is 360°.

Given that, one angle = 150°

∴ Sum of three equal angles = 360° − 150°
= 210°.

Thus, measure of each angle $= \frac{210^\circ}{3} = 70^\circ$.

Hence, measure of all three angles are 70°, 70° and 70° respectively.

9. Four angles of a quadrilateral are in the ratio 4 : 5 : 7 : 8. Find their measures.

Solution:

Let the angles have the measures $4x°$, $5x°$, $7x°$ and $8x°$.

Then,

$$4x° + 5x° + 7x° + 8x° = 360°$$

$$24x° = 360°$$

$$x° = \frac{360°}{24} = 15°$$

Thus,

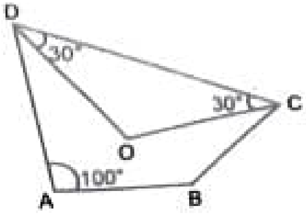
Measure of first angle $= 4x° = 4 \times 15° = 60°$

Measure of second angle $= 5x° = 5 \times 15° = 75°$

Measure of third angle $= 7x° = 7 \times 15° = 105°$

Measure of fourth angle $= 8x° = 8 \times 15° = 120°$

10. In the given figure, DO and CO are the bisectors of $\angle ADC$ and $\angle BCD$ respectively. If $\angle ADC = \angle BCD = 60^\circ$ and $\angle DAB = 100^\circ$, find the measures of $\angle DOC$ and $\angle ABC$. [HOTS]




Solution:

We know that, sum of three interior angles of a triangle DOC $= 180^\circ$

$$\therefore \angle DOC = 180^\circ - (\angle ODC + \angle OCD)$$
$$= 180^\circ - (30^\circ + 30^\circ)$$
$$= 180^\circ - 60^\circ$$
$$= 120^\circ$$

Also, sum of the four interior angles of a quadrilateral ABCD $= 360^\circ$.

$$\therefore \angle ADC + \angle BCD + \angle DAB + \angle ABC = 360^\circ$$
$$\Rightarrow 60^\circ + 60^\circ + 100^\circ + \angle ABC = 360^\circ$$
$$\Rightarrow 220^\circ + \angle ABC = 360^\circ$$
$$\Rightarrow \angle ABC = 360^\circ - 220^\circ$$
$$= 140^\circ.$$

Book-8

## EXERCISE 12.3

1. (i) In a quadrilateral, opposite sides are equal and one angle is five times the other adjacent angle. What is the figure called?

   (ii) In the above quadrilateral, if all the sides are equal, what will it be called?

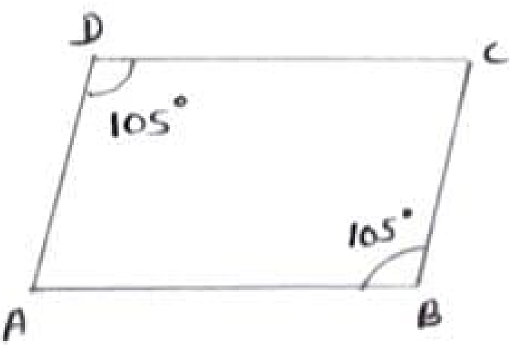
Solution:

(i) The figure is parallelogram.

(ii) The required figure is rhombus.

2. If one angle of a parallelogram is 105°, find the other three angles and the ratio between the four angles.

Solution:




From figure, in a Parallelogram opposite angles are equal.

Thus,

$\angle B = \angle D = 105^\circ$

$\angle A = \angle C = 180^\circ - 105^\circ$

$= 75^\circ$

Thus, the other three angles are $75^\circ$, $105^\circ$ and $75^\circ$.

Required Ratio = $\angle A : \angle B : C : \angle D = 5:7:5:7$