FACTORISATION OF ALGEBRAIC EXPRESSIONS

7.

EXERCISE 7.1

Factorise the following :

1. $12x - 30$
2. $15x^2 - 5y$
3. $9m^2 + 81m^3$
4. $36x^2y^3 - 54x^3y^2$
5. $16m^2n - 20mn^2 + 2mn$
6. $-6 - 10t + 12t^2$
7. $z(z + 5) + 7(z + 5)$
8. $yz(p - 3) + xy(p - 3)$
9. $2(p - q)^2 - 3(p - q)$
10. $5a^2(t - p)^2 - 10(t - p)^2$
11. $a(p + q) - b(p + q)$
12. $(x + 2y)^2 - 4x - 8y$
13. $3p + 6q - 4(p + 2q)^2$
14. $ab(p - 1) - ac(p - 1)$
15. $z(x + y)^2 - t(x + y)$
16. $at + bt + bc + ac$
17. $a^3 - 9a^2 + a - 9$
18. $(a - b)(2b + 3) + (a - b)(2b - 3)$
19. $12ab - 2b^2 + 24ac - 4bc$
20. $18(2x - 3y)^2 - 6(3y - 2x)$
21. $14a^2b^2 - 49a^3b^2 - 7a^2b^3$
22. $a^2b + 3ab^2 - 6a^2b$
23. $12x^2y^3 - 60x^3y^2$
24. $ab - ac - db + dc$
25. $xy + yz + xz^2 + z^3$

Sol.

1. $12x - 30 = 6(2x - 5)$

2. $15x^2 - 5y = 5(3x^2 - y)$

3. $9m^2 + 81m^3 = 9m^2(1 + 9m)$

4. $36x^2y^3 - 54x^3y^2 = 18x^2y^2(2y - 3x)$

5. $16m^2n - 20mn^2 + 2mn = 2mn(8m - 10n + 1)$

6. $-6 - 10t + 12t^2 = -2(3 + 5t - 6t^2)$

7. $z(z + 5) + 7(z + 5) = (z + 5)(z + 7)$

8. $yz(p - 3) + xy(p - 3) = (p - 3)(yz + xy)$
$$= y(p - 3)(z + x).$$

9. $2(p-q)^2 - 3(p-q) = (p-q)[2(p-q) - 3]$

$$= (p-q)(2p-2q-3)$$

10. $5a^2(t-p)^2 - 10(t-p)^2 = 5(t-p)^2(a^2-2)$

11. $a(p+q) - b(p+q) = (p+q)(a-b)$

12. $(x+2y)^2 - 4x - 8y = (x+2y)^2 - 4(x+2y)$

$$= (x+2y)[x+2y - 4],$$

13. $3p + 6q - 4(p+2q)^2$

$$= 3(p+2q) - 4(p+2q)^2$$

$$= (p+2q)(3 - 4(p+2q)$$

$$= (p+2q)(3 - 4p - 8q)$$

14. $ab(p-1) - ac(p-1)$

$$= (p-1)(ab - ac)$$

$$= a(p-1)(b-c).$$

15. $z(x+y)^2 - t(x+y) = (x+y)[z(x+y)-t]$

$$= (x+y)(zx+zy-t)$$

16. $at + bt + bc + ac = t(a+b) + c(b+a)$

$$= (a+b)(t+c)$$

17. $a^3 - 9a^2 + a - 9 = a^2(a-9) + 1(a-9)$

$$= (a-9)(a^2+1)$$

18. $(a-b)(2b+3) + (a-b)(2b-3)$

$$= (a-b)(2b+3+2b-3)$$

$$= (a-b)(4b) = 4ab - 4b^2$$
$$= 4b(a-b)$$

19. $12ab - 2b^2 + 24ac - 4bc$

$$= 2b(6a-b) + 4c(6a-b)$$

$$= (6a-b)(2b+4c)$$

$$= 2(6a-b)(b+2c).$$

20. $18(2x-3y)^2 - 6(3y-2x)$

$$= 18(2x-3y)^2 + 6(2x-3y)$$

$$= 6(2x-3y)[3(2x-3y)+1]$$

$$= 6(2x-3y)(6x-9y+1)$$

21. $14a^2b^2 - 49a^3b^2 - 7a^2b^3$
$$= 7a^2b^2(2 - 7a - b)$$

22. $a^2b + 3ab^2 - 6a^2b = ab(a + 3b - 6a)$

23. $12x^2y^3 - 60x^3y^2 = 12x^2y^2(y - 5x)$

24. $ab - ac - db + dc = a(b-c) - d(b-c)$
$$= (b-c)(a-d)$$

25. $xy + yz + xz^2 + z^3$
$$= y(x+z) + z^2(x+z)$$
$$= (x+z)(y+z^2).$$

EXERCISE 7.2

Factorise the following :

1. $x^2 + 4xy + 4y^2$

2. $9 - 30z + 25z^2$

3. $4x^2 - 28x + 49$

4. $x^2 + 2 + \dfrac{1}{x^2}$

5. $x^2 - 2 + \dfrac{1}{x^2}$

6. $4x^2 + 12xy + 9y^2$

7. $81m^2 - 108mn + 36n^2$

8. $36a^4 + 24a^2b^3 + 4b^6$

9. $36a^2 + 60a + 25$

10. $9t^2 + 24pt + 16p^2$

11. $x^2y^2 + y^2z^2 + 2xy^2z$

12. $1 - 8n + 16n^2$

13. $1 - \dfrac{1}{3p} + \dfrac{1}{36p^2}$

14. $4m^4 + 4m^2 + 1$

15. $25z^6 - 10z^3 + 1$

16. $9m^2n^4 - 12mn^2a^2b + 4a^4b^2$

17. $225a^2b^2 - \dfrac{5}{a} + \dfrac{1}{36a^4b^2}$

18. $p^2q^2 - 6pqt + 9t^2$

19. $1 + 10t + 25t^2$

20. $z^4 - \dfrac{z^2y^2}{2} + \dfrac{y^2}{16}$

21. $\dfrac{p^2}{q^4} - 2 + \dfrac{q^4}{p^2}$

22. $x^4 + \dfrac{2}{x} + \dfrac{1}{x^6}$

23. $9x^4 + \dfrac{2}{3} + \dfrac{1}{81x^4}$

24. $25x^2 - 30xy + 9y^2$

25. $\dfrac{4}{81}y^2x^4 - \dfrac{2}{3}x^2y^2z^2 + \dfrac{9}{4}y^2z^4$

26. $16p^4 - 40p^2q^2 + 25q^4$

27. $81p^4 + 72p^2q^2 + 16q^4$

28. $16x^4y^2 - 48x^3y^3 + 36x^6y^4$

29. $\dfrac{1}{36}x^2 - \dfrac{3}{5}xz + \dfrac{81}{25}z^2$

30. $81r^4 - 108r^2p^3 + 36p^6$

Sol.

1. $x^2 + 4xy + 4y^2 = x^2 + 2 \times x \times 2y + (2y)^2$

$$= (x + 2y)^2$$

2. $9 - 30z + 25z^2 = 3^2 - 2 \times 3 \times 5z + (5z)^2$

$$= (3 - 5z)^2$$

3. $4x^2 - 28x + 49 = (2x)^2 - 2 \times 2x \times 7 + 7^2$

$$= (2x - 7)^2$$

4. $x^2 + 2 + \dfrac{1}{x^2} = x^2 + 2 \times x \times \dfrac{1}{x} + \left(\dfrac{1}{x}\right)^2$

$$= \left(x + \dfrac{1}{x}\right)^2$$

5. $x^2 - 2 + \frac{1}{x^2} = x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$$= \left(x - \frac{1}{x}\right)^2$$

6. $4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$$= (2x + 3y)^2$$

7. $81m^2 - 108mn + 36n^2 = (9m)^2 - 2 \times 9m \times 6n + (6n)^2$

$$= (9m - 6n)^2 = 9(3m - 2n)^2$$

8. $36a^4 + 24a^2 b^3 + 4b^6 = (6a^2)^2 + 2 \times 6a^2 \times 2b^3 + (2b^3)^2$

$$= (6a^2 + 2b^3)^2$$

$$= 4(3a^2 + b^3)^2$$

9. $36a^2 + 60a + 25 = (6a)^2 + 2 \times 6a \times 5 + (5)^2$

$$= (6a + 5)^2$$

10. $9t^2 + 24pt + 16p^2 = (3t)^2 + 2 \times 3t \times 4p + (4p)^2$

$$= (3t + 4p)^2$$

11. $x^2y^2 + y^2z^2 + 2xy^2z = y^2(x^2 + z^2 + 2xz)$

$$= y^2(x+z)^2$$

12. $1 - 8n + 16n^2 = (1)^2 - 2 \times 4n \times 1 + (4n)^2$

$$= (1 - 4n)^2$$

13. $1 - \frac{1}{3p} + \frac{1}{36p^2} = (1)^2 - 2 \times \frac{1}{6p} \times 1 + \left(\frac{1}{6p}\right)^2$

$$= \left(1 - \frac{1}{6p}\right)^2.$$

14. $4m^4 + 4m^2 + 1 = (2m^2)^2 + 2 \times 2m^2 \times 1 + (1)^2$

$$= (2m^2 + 1)^2$$

15. $25z^6 - 10z^3 + 1 = (5z^3)^2 - 2 \times 5z^3 \times 1 + (1)^2$

$$= (5z^3 - 1)^2$$

16. $9m^2n^4 - 12mn^2a^2b + 4a^4b^2$

$$= (3mn^2)^2 - 2 \times 3mn^2 \times 2a^2b + (2a^2b)^2$$

$$= (3mn^2 - 2a^2b)^2$$

17. $225a^2b^2 - \dfrac{5}{a} + \dfrac{1}{36a^4b^2}$

$$= (15ab)^2 - 2\times15ab\times\dfrac{1}{6a^2b} + \left(\dfrac{1}{6a^2b}\right)^2$$

$$= \left(15ab - \dfrac{1}{6a^2b}\right)^2$$

18. $p^2q^2 - 6pqt + 9t^2 = (pq)^2 - 2\times pq\times3t + (3t)^2$

$$= (pq - 3t)^2$$

19. $1 + 10t + 25t^2 = (1)^2 + 2\times1\times5t + (5t)^2$

$$= (1+5t)^2$$

20. $z^4 - \dfrac{z^2y^2}{2} + \dfrac{y^2}{16} = (z^2)^2 - 2\times z^2 y\left(\dfrac{y}{4}\right) + \left(\dfrac{y}{4}\right)^2$

$$= \left(z^2 - \dfrac{y}{4}\right)^2$$

21. $\dfrac{p^2}{q^4} - 2 + \dfrac{q^4}{p^2} = \left(\dfrac{p}{q^2}\right)^2 - 2\times\dfrac{p}{q^2}\times\dfrac{q^2}{p} + \left(\dfrac{q^2}{p}\right)^2$

$$= \left(\dfrac{p}{q^2} - \dfrac{q^2}{p}\right)^2$$

22. $x^4 + \dfrac{2}{x} + \dfrac{1}{x^6} = (x^2)^2 + 2 \times x^2 \times \dfrac{1}{x^3} + \left(\dfrac{1}{x^3}\right)^2$

$$= \left(x^2 + \dfrac{1}{x^3}\right)^2.$$

23. $9x^4 + \dfrac{2}{3} + \dfrac{1}{81x^4} = (3x^2)^2 + 2 \times 3x^2 \times \dfrac{1}{9x^2} + \left(\dfrac{1}{9x^2}\right)^2$

$$= \left(3x^2 + \dfrac{1}{9x^2}\right)^2$$

24. $25x^2 - 30xy + 9y^2 = (5x)^2 - 2 \times 5x \times 3y + (3y)^2$

$$= (5x - 3y)^2$$

25. $\dfrac{4}{81} y^2 x^4 - \dfrac{2}{3} x y^2 z^2 + \dfrac{9}{4} y^2 z^4$

$$= \left(\dfrac{2}{9} y x^2\right)^2 - 2 \times \dfrac{2}{9} y x^2 \times \dfrac{3}{2} y z^2 + \left(\dfrac{3}{2} y z^2\right)^2$$

$$= \left(\dfrac{2}{9} y x^2 - \dfrac{3}{2} y z^2\right)^2$$

$$= y^2 \left(\dfrac{2}{9} x^2 - \dfrac{3}{2} z^2\right)^2.$$

26. $16p^4 - 40p^2q^2 + 25q^4 = (4p^2)^2 - 2 \times 4p^2 \times 5q^2 + (5q^2)^2$

$$= (4p^2 - 5q^2)^2$$

27. $81p^4 + 72p^2q^2 + 16q^4 = (9p^2)^2 + 2 \times 9p^2 \times 4q^2 + (4q^2)^2$

$$= (9p^2 + 4q^2)^2$$

28. $16x^4y^2 - 48x^5y^3 + 36x^6y^4$

$$= 4x^4y^2 (4 - 12xy + 9x^2y^2)$$

$$= 4x^4y^2 [(2)^2 - 2 \times 2 \times 3xy + (3xy)^2]$$

$$= 4x^4y^2 (2 - 3xy)^2$$

29. $\frac{1}{36}x^2 - \frac{3}{5}xz + \frac{81}{25}z^2 = (\frac{1}{6}x)^2 - 2 \times \frac{1}{6}x \times \frac{9}{5}z + (\frac{9}{5}z)^2$

$$= (\frac{1}{6}x - \frac{9}{5}z)^2$$

30. $81t^4 - 108t^2p^3 + 36p^6 = (9t^2)^2 - 2 \times 9t^2 \times 6p^3 + (6p^3)^2$

$$= (9t^2 - 6p^3)^2 = 9(3t^2 - 2p^3)^2$$

EXERCISE 7.3

Factorise :

1. $9x^2 - 4y^2$
2. $81z^2 - 25$
3. $36 - 225y^2$
4. $4 - 25t^2$
5. $\dfrac{81}{4}a^2 - \dfrac{9}{25}b^2$
6. $\dfrac{18}{4}x^2 - \dfrac{50}{9}y^2$
7. $a^2b^2 - 4b^2c^2$
8. $12x^2z - 27y^2z$
9. $45ab^2 - 20ac^2$
10. $-27a^2b^2 + 12a^2c^2$
11. $-49a^3 + 64ab^2$
12. $-81x^3y^3 + 64xy$
13. $16\,x^4 - 1$
14. $16y^4 - 625$
15. $81z^4 - 256y^4$
16. $l^4 - 16m^4n^4$
17. $x^2 + 2xy + y^2 - 1$
18. $1 - y^2 - 2yz - z^2$ [HOTS]
19. $9 - y^2 + 2xy - x^2$ [HOTS]
20. $4a^2 + 12ab + 9b^2 - 25z^2$
21. $x^2 + 2 + \dfrac{1}{x^2} - 16y^2$
22. $y^6 - 6y^3 + 9 - z^4$
23. $16c^2 - 9a^2 - 6ab - b^2$ [HOTS]
24. $100 - x^2 - y^2 - 2xy$
25. $x^2 - y^2 - x - y$
26. $4x^2 - 6xy + 2x - 3y$
27. $81(x + y)^2 - 16(x - y)^2$
28. $4(2x + 3y)^2 - 9(2x - 3y)^2$
29. Evaluate using identity :
   (i) $36^2 - 40^2$
   (ii) $(18.4)^2 - (1.6)^2$
   (iii) $(608)^2 - (609)^2$
30. Factorise : $x^8 - y^8$ [Hint. $x^8 - y^8 = (x^4)^2 - (y^4)^2$] [HOTS]

Sol. 1. $9x^2 - 4y^2 = (3x)^2 - (2y)^2$
Using the identity $a^2 - b^2 = (a+b)(a-b)$ we get
$= (3x + 2y)(3x - 2y)$

2. $81z^2 - 25 = (9z)^2 - (5)^2 = (9z + 5)(9z - 5)$

3. $36 - 225y^2 = (6)^2 - (15y)^2 = (6 + 15y)(6 - 15y)$

4. $4 - 25t^2 = (2)^2 - (5t)^2 = (2 + 5t)(2 - 5t)$

5. $\dfrac{81}{4}a^2 - \dfrac{9}{25}b^2 = \left(\dfrac{9}{2}a\right)^2 - \left(\dfrac{3}{5}b\right)^2$

$= \left(\dfrac{9}{2}a + \dfrac{3}{5}b\right)\left(\dfrac{9}{2}a - \dfrac{3}{5}b\right)$

6. $\dfrac{18}{4}x^2 - \dfrac{50}{9}y^2 = 2\left[\dfrac{9}{4}x^2 - \dfrac{25}{9}y^2\right]$

$$= 2\left[\left(\dfrac{3}{2}x\right)^2 - \left(\dfrac{5}{3}y\right)^2\right]$$

$$= 2\left[\left(\dfrac{3}{2}x + \dfrac{5}{3}y\right)\left(\dfrac{3}{2}x - \dfrac{5}{3}y\right)\right].$$

7. $a^2b^2 - 4b^2c^2 = b^2(a^2 - 4c^2)$

$$= b^2\left[a^2 - (2c)^2\right]$$

$$= b^2(a+2c)(a-2c)$$

8. $12x^2z - 27y^2z = 3z(4x^2 - 9y^2)$

$$= 3z\left[(2x)^2 - (3y)^2\right]$$

$$= 3z(2x+3y)(2x-3y)$$

9. $45ab^2 - 20ac^2 = 5a(9b^2 - 4c^2)$

$$= 5a\left[(3b)^2 - (2c)^2\right]$$

$$= 5a(3b+2c)(3b-2c)$$

10. $-27a^2b^2 + 12a^2c^2 = -3a^2(9b^2 - 4c^2)$

$$= -3a^2\left[(3b)^2 - (2c)^2\right]$$

$$= -3a^2(3b+2c)(3b-2c)$$

11. $-49a^3 + 64ab^2 = -a(49a^2 - 64b^2)$

$$= -a[(7a)^2 - (8b)^2]$$

$$= -a(7a + 8b)(7a - 8b)$$

12. $-81x^3y^3 + 64xy = -xy(81x^2y^2 - 64)$

$$= -xy[(9xy)^2 - (8)^2]$$

$$= -xy(9xy + 8)(9xy - 8)$$

13. $16x^4 - 1 = (4x^2)^2 - (1)^2$

$$= (4x^2 + 1)(4x^2 - 1)$$

$$= (4x^2 + 1)[(2x)^2 - (1)^2]$$

$$= (4x^2 + 1)(2x + 1)(2x - 1)$$

14. $16y^4 - 625 = (4y^2)^2 - (25)^2$

$$= (4y^2 + 25)(4y^2 - 25)$$

$$= (4y^2 + 25)[(2y)^2 - (5)^2]$$

$$= (4y^2 + 25)(2y + 5)(2y - 5)$$

15. $81z^4 - 256y^4 = (9z^2)^2 - (16y^2)^2$

$$= (9z^2 + 16y^2)(9z^2 - 16y^2)$$
$$= (9z^2 + 16y^2)[(3z)^2 - (4y)^2]$$
$$= (9z^2 + 16y^2)(3z + 4y)(3z - 4y)$$

16. $t^4 - 16m^4n^4 = (t^2)^2 - (4m^2n^2)^2$

$$= (t^2 + 4m^2n^2)(t^2 - 4m^2n^2)$$
$$= (t^2 + 4m^2n^2)[(t)^2 - (2mn)^2]$$
$$= (t^2 + 4m^2n^2)(t + 2mn)(t - 2mn)$$

17. $x^2 + 2xy + y^2 - 1 = (x+y)^2 - (1)^2$

$$= (x + y + 1)(x + y - 1)$$

18. $1 - y^2 - 2yz - z^2 = 1 - (y^2 + 2yz + z^2)$

$$= (1)^2 - (y+z)^2$$
$$= (1 + y + z)(1 - y - z)$$

19. $9 - y^2 + 2xy - x^2 = 9 - (y^2 - 2xy + x^2)$

$$= (3)^2 - (y - x)^2 = (3 + y - x)(3 - y + x)$$

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20. $4a^2 + 12ab + 9b^2 - 25z^2 = (2a)^2 + 2 \times 2a \times 3b + (3b)^2 - (5z)^2$

$$= (2a+3b)^2 - (5z)^2$$

$$= (2a+3b+5z)(2a+3b-5z)$$

21. $x^2 + 2 + \frac{1}{x^2} - 16y^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 - (4y)^2$

$$= \left(x + \frac{1}{x}\right)^2 - (4y)^2$$

$$= \left(x + \frac{1}{x} + 4y\right)\left(x + \frac{1}{x} - 4y\right)$$

22. $y^6 - 6y^3 + 9 - z^4 = (y^3)^2 - 2 \times y^3 \times 3 + 3^2 - (z^2)^2$

$$= (y^3 - 3)^2 - (z^2)^2$$

$$= (y^3 - 3 + z^2)(y^3 - 3 - z^2)$$

23. $16c^2 - 9a^2 - 6ab - b^2 = (4c)^2 - (9a^2 + 6ab + b^2)$

$$= (4c)^2 - [(3a)^2 + 2 \times 3a \times b + b^2]$$

$$= (4c)^2 - (3a+b)^2$$

$$= (4c + 3a + b)(4c - 3a - b)$$

24. $100 - x^2 - y^2 - 2xy = (10)^2 - (x^2 + y^2 + 2xy)$

$$= (10)^2 - (x+y)^2$$

$$= (10 + x + y)(10 - x - y).$$

25. $x^2 - y^2 - x - y = (x+y)(x-y) - (x+y)$

$$= (x+y)(x-y-1)$$

26. $4x^2 - 6xy + 2x - 3y = 2x(2x-3y) + 1(2x-3y)$

$$= (2x-3y)(2x+1)$$

27. $81(x+y)^2 - 16(x-y)^2 = [9(x+y)]^2 - [4(x-y)]^2$

$$= [9(x+y) + 4(x-y)][9(x+y) - 4(x-y)]$$

$$= [9x + 9y + 4x - 4y][9x + 9y - 4x + 4y]$$

$$= (13x + 5y)(5x + 13y)$$

28. $4(2x+3y)^2 - 9(2x-3y)^2 = [2(2x+3y)]^2 - (3(2x-3y))^2$

$$= (4x + 6y + 6x - 9y)(4x + 6y - 6x + 9y)$$

$$= (10x - 3y)(-2x + 15y)$$

29. Using identity: $[a^2 - b^2 = (a+b)(a-b)]$

(i) $36^2 - 40^2 = (36+40)(36-40)$

$$= 76 \times -4 = -304$$

(ii) $(18.4)^2 - (1.6)^2 = (18.4 + 1.6)(18.4 - 1.6)$

$$= 20 \times 16.8$$

$$= 336$$

(iii) $(608)^2 - (609)^2 = (608 + 609)(608 - 609)$

$$= 1217 \times -1$$

$$= -1217$$

30. $x^8 - y^8 = (x^4)^2 - (y^4)^2$

$$= (x^4 + y^4)(x^4 - y^4)$$

$$= (x^4 + y^4)\left[(x^2)^2 - (y^2)^2\right]$$

$$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$

$$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y).$$

EXERCISE 7.4

Factorise :

1. $x^2 + 7x + 12$
2. $y^2 + 12y + 35$
3. $a^2 + 13a + 42$
4. $z^2 - 6z - 16$
5. $x^2 - 19x + 90$
6. $y^2 + 12y - 45$
7. $p^2 - 14p + 24$
8. $m^2 + 34mn - 72n^2$
9. $a^2 - 10ab - 75b^2$ [HOTS]
10. $p^2 - 22p + 121$
11. $x^2y^2 + 40xyz - 225z^2$ [HOTS]
12. $9 - 6y + y^2$
13. $a^2b^2 + abc - 156c^2$ [HOTS]
14. $t^2 - 9t - 36$
15. $y^2z^2 - xyz - 56x^2$ [HOTS]
16. $2\sqrt{2}x^2 + 11x + 6\sqrt{2}$ [HOTS]
17. $\sqrt{3}x^2 + 7x + 2\sqrt{3}$ [HOTS]

Sol. 1. $x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

$$= x(x + 4) + 3(x + 4)$$

$$= (x + 4)(x + 3)$$

2. $y^2 + 12y + 35 = y^2 + 7y + 5y + 35$
$$= y(y+7) + 5(y+7)$$
$$= (y+7)(y+5)$$

3. $a^2 + 13a + 42 = a^2 + 7a + 6a + 42$
$$= a(a+7) + 6(a+7)$$
$$= (a+7)(a+6)$$

4. $z^2 - 6z - 16 = z^2 - 8z + 2z - 16$
$$= z(z-8) + 2(z-8)$$
$$= (z-8)(z+2)$$

5. $x^2 - 19x + 90 = x^2 - 10x - 9x + 90$
$$= x(x-10) - 9(x-10)$$
$$= (x-10)(x-9)$$

6. $y^2 + 12y - 45 = y^2 + 15y - 3y - 45$
$$= y(y+15) - 3(y+15)$$
$$= (y+15)(y-3)$$

7. $p^2 - 14p + 24 = p^2 - 12p - 2p + 24$

$$= p(p-12) - 2(p-12)$$

$$= (p-12)(p-2)$$

8. $m^2 + 34mn - 72n^2 = m^2 + 36mn - 2mn - 72n^2$

$$= m(m+36n) - 2n(m+36n)$$

$$= (m+36n)(m-2n)$$

9. $a^2 - 10ab - 75b^2 = a^2 - 15ab + 5ab - 75b^2$

$$= a(a-15b) + 5b(a-15b)$$

$$= (a-15b)(a+5b)$$

10. $p^2 - 22p + 121 = p^2 - 11p - 11p + 121$

$$= p(p-11) - 11(p-11)$$

$$= (p-11)(p-11)$$

11. $x^2y^2 + 40xyz - 225z^2 = x^2y^2 + 45xyz - 5xyz - 225z^2$

$$= xy(xy+45z) - 5z(xy+45z)$$

$$= (xy+45z)(xy-5z)$$

12. $9 - 6y + y^2 = 9 - 3y - 3y + y^2$

$$= 3(3-y) - y(3-y)$$

$$= (3-y)(3-y).$$

13. $a^2b^2 + abc - 156c^2 = a^2b^2 + 13abc - 12abc - 156c^2$

$$= ab(ab + 13c) - 12c(ab + 13c)$$

$$= (ab + 13c)(ab - 12c)$$

14. $t^2 - 9t - 36 = t^2 - 12t + 3t - 36$

$$= t(t-12) + 3(t-12)$$

$$= (t-12)(t+3)$$

15. $y^2z^2 - xyz - 56x^2 = y^2z^2 - 8xyz + 7xyz - 56x^2$

$$= yz(yz - 8x) + 7x(yz - 8x)$$

$$= (yz - 8x)(yz + 7x)$$

16. $2\sqrt{2}\,x^2 + 11x + 6\sqrt{2} = 2\sqrt{2}x^2 + 8x + 3x + 6\sqrt{2}$

$$= 2\sqrt{2}x(x + 2\sqrt{2}) + 3(x + 2\sqrt{2})$$

$$= (x + 2\sqrt{2})(2\sqrt{2}x + 3)$$

17. $\sqrt{3}x^2 + 7x + 2\sqrt{3} = \sqrt{3}x^2 + 6x + x + 2\sqrt{3}$

$$= \sqrt{3}x(x + 2\sqrt{3}) + 1(x + 2\sqrt{3})$$

$$= (x + 2\sqrt{3})(\sqrt{3}x + 1)$$

EXERCISE 7.5

1. **Divide :**

   (i) $(7x^2 + 14x) + (x + 2)$

   (ii) $(10a - 35) + (2a - 7)$

   (iii) $3xyz\ (4x + 10) + (2x + 5)$

   (iv) $9a^2b^2\ (3x - 24) + 27ab\ (x - 2)$

   (v) $55ab\ (3a - 18)\ (2a - 6) + 22b\ (a - 6)$

   (vi) $40xyz\ (x + y)\ (y + z)\ (z + x) + 10x\ (y + z)\ (z + x)$

Sol. (i) $(7x^2 + 14x) \div (x+2) = \dfrac{7x\,(x+2)}{(x+2)} = 7x$

(ii) $(10a - 35) \div (2a - 7) = \dfrac{5\,(2a-7)}{(2a-7)} = 5.$

(iii) $3xyz\,(4x+10) \div (2x+5) = \dfrac{3xyz \times 2\,(2x+5)}{(2x+5)}$

$$= 6xyz$$

(iv) $9a^2b^2(3x-24) \div 27ab(x-2) = \dfrac{9a^2b^2(3x-24)}{27ab(x-2)}$

$$= \dfrac{9a^2b^2 \times 3(x-8)}{27ab(x-2)} = \dfrac{ab(x-8)}{(x-2)}$$

(v) $55ab\,(3a-18)(2a-6) \div 22b\,(a-6)$

$$= \dfrac{55ab \times 3(a-6)(2a-6)}{22b\,(a-6)}$$

$$= 15a\,(a-3).$$

(vi) $40xyz(x+y)(y+z)(z+x) \div 10x(y+z)(z+x)$

$$= \frac{40xyz(x+y)(y+z)(z+x)}{10x(y+z)(z+x)}$$

$$= 4yz(x+y)$$

2. Factorise the expressions and divide them as directed :

(i) $3(7x^2 + 105x) + 21(x + 5)$

(ii) $(a^2 - 2ab + b^2 - c^2) + (a - b - c)$

(iii) $3(x^4 - 256) + (x^2 + 16)$

(iv) $(x^4 + 2x^2y^2 + y^4) + (x^2 + y^2)$

(v) $(x^2 + 3x - 10) + (x + 5)$

(vi) $4ab(a^2 - 6a - 16) + 26(a - 8)$

(vii) $35(y^4 - 5y^3 - 24y^2) + 7y(y - 8)$

Sol. (i) $3(7x^2 + 105x) \div 21(x+5)$

$$= \frac{3 \times 7x(x+15)}{21(x+5)} = \frac{x(x+15)}{(x+5)}$$

(ii) $(a^2 - 2ab + b^2 - c^2) \div (a-b-c)$

$$= \frac{[(a-b)^2 - c^2]}{(a-b-c)} = \frac{(a-b+c)(a-b-c)}{(a-b-c)}$$

$$= (a-b+c)$$

(iii) $3(x^4 - 256) \div (x^2 + 16)$

$$= \frac{3[(x^2)^2 - (16)^2]}{(x^2+16)} = \frac{3(x^2+16)(x^2-16)}{(x^2+16)}$$

$$= 3(x^2 - 16).$$

(iv) $(x^4 + 2x^2y^2 + y^4) \div (x^2 + y^2)$

$$= \frac{(x^4 + 2x^2y^2 + y^4)}{(x^2 + y^2)} = \frac{(x^2 + y^2)^2}{(x^2 + y^2)} = (x^2 + y^2)$$

(v) $(x^2 + 3x - 10) \div (x + 5)$

$$= \frac{(x^2 + 3x - 10)}{(x + 5)} = \frac{(x^2 + 5x - 2x - 10)}{(x + 5)}$$

$$= \frac{x(x+5) - 2(x+5)}{(x+5)} = \frac{(x+5)(x-2)}{(x+5)} = (x-2)$$

(vi) $4ab(a^2 - 6a - 16) \div 26(a - 8)$

$$= \frac{4ab(a^2 - 6a - 16)}{26(a-8)} = \frac{4ab(a^2 - 8a + 2a - 16)}{26(a-8)}$$

$$= \frac{4ab(a-8)(a+2)}{26(a-8)} = \frac{2ab(a+2)}{13}.$$

(vii) $35(y^4 - 5y^3 - 24y^2) \div 7y(y-8)$

$$= \frac{35y^2(y^2 - 5y - 24)}{7y(y-8)}$$

$$= \frac{5y[y^2 - 8y + 3y - 24)}{(y-8)} = \frac{5y(y-8)(y+3)}{(y-8)}$$

$$= 5y(y+3).$$

3. Divide :

 (i) $(x^2 + 4x - 21)$ by $(x - 3)$ (ii) $(m^2 + 21m - 46)$ by $(m - 2)$

 (iii) $x^2 - 11x + 30$ by $x - 5$ (iv) $6x^2 - 31x + 35$ by $2x - 7$

 (v) $2x^2 + 11x + 5$ by $2x + 1$ (vi) $x^3 - 5x^2 - 2x + 24$ by $x - 4$

Sol. (i) $(x^2 + 4x - 21) \div (x - 3)$

$$= \frac{(x^2 + 7x - 3x - 21)}{(x-3)} = \frac{(x+7)(x-3)}{(x-3)}$$

$$= (x + 7)$$

(ii) $(m^2 + 21m - 46) \div (m - 2)$

$$= \frac{(m^2 + 23m - 2m - 46)}{(m-2)} = \frac{m(m+23) - 2(m+23)}{(m-2)}$$

$$= \frac{(m+23)(m-2)}{(m-2)} = (m + 23).$$

(iii) $(x^2 - 11x + 30) \div (x - 5)$

$$= \frac{(x^2 - 6x - 5x + 30)}{(x-5)} = \frac{(x-6)(x-5)}{(x-5)} = (x-6)$$

(iv) $(6x^2 - 31x + 35) \div (2x - 7)$

$$= \frac{(6x^2 - 21x - 10x + 35)}{(2x-7)} = \frac{[3x(2x-7) - 5(2x-7)]}{(2x-7)}$$

$$= \frac{(2x-7)(3x-5)}{(2x-7)} = (3x-5)$$

(v) $(2x^2 + 11x + 5) \div (2x + 1)$

$$= \frac{(2x^2 + 10x + x + 5)}{(2x + 1)} = \frac{2x(x + 5) + 1(x + 5)}{(2x + 1)}$$

$$= \frac{(2x + 1)(x + 5)}{(2x + 1)} = (x + 5)$$

(vi) $(x^3 - 5x^2 - 2x + 24) \div (x - 4)$

$$\Rightarrow \quad (x-4) \overline{\smash{\big)}\, x^3 - 5x^2 - 2x + 24} \, \big(\, x^2 - x - 6$$

$$\underline{x^3 - 4x^2}$$
$$+$$
$$-x^2 - 2x$$
$$\underline{-x^2 + 4x}$$
$$+$$
$$-6x + 24$$
$$\underline{-6x + 24}$$
$$+$$
$$0$$

Thus, Quotient $= x^2 - x - 6$

MISCELLANEOUS EXERCISE

Factorise :

1. $x^2 - xz + xy - yz$
2. $6ab - b^2 + 12ac - 2bc$
3. $(x - 2y)^2 + 4x - 8y$
4. $y^2 - xy(1 - x) - x^3$ [HOTS]
5. $(ax + by)^2 + (bx - ay)^2$
6. $ab^2 + (a - 1)b - 1$
7. $x^3 - 3x^2 + x - 3$
8. $ab(x^2 + y^2) - xy(a^2 + b^2)$
9. $x^2 - x(a + 2b) + 2ab$ [HOTS]
10. $16y^3 - 4y$
11. $3x^5 - 48x$
12. $(3x - 4y)^2 - 25z^2$
13. $(2x + 3y)^2 - 1$
14. $16c^2 - (5a + b)^2$
15. $100 - (x - 5)^2$

16. Evaluate :
   (i) $(405)^2 - (395)^2$
   (ii) $(17.8)^2 - (2.2)^2$

Sol. 1. $x^2 - xz + xy - yz = x(x-z) + y(x-z)$

$$= (x-z)(x+y)$$

2. $6ab - b^2 + 12ac - 2bc = b(6a - b) + 2c(6a - b)$

$$= (6a - b)(b + 2c)$$

3. $(x - 2y)^2 + 4x - 8y = (x - 2y)^2 + 4(x - 2y)$

$$= (x - 2y)(x - 2y + 4)$$

4. $y^2 - xy(1-x) - x^3 = y^2 - xy + x^2y - x^3$

$$= y(y - x) + x^2(y - x)$$

$$= (y - x)(y + x^2)$$

5. $(ax + by)^2 + (bx - ay)^2$

$$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$$

$$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$$

$$= (x^2 + y^2)(a^2 + b^2)$$

6. $ab^2 + (a-1)b - 1 = ab^2 + ab - b - 1$

$$= ab(b+1) - 1(b+1)$$
$$= (b+1)(ab-1)$$

7. $x^3 - 3x^2 + x - 3 = x^2(x-3) + 1(x-3)$

$$= (x-3)(x^2+1)$$

8. $ab(x^2+y^2) - xy(a^2+b^2)$

$$= abx^2 + aby^2 - xya^2 - xyb^2$$
$$= abx^2 - xya^2 + aby^2 - xyb^2$$
$$= ax(bx - ay) + by(ay - bx)$$
$$= ax(bx - ay) - by(bx - ay)$$
$$= (bx - ay)(ax - by).$$

9. $x^2 - x(a+2b) + 2ab$

$$= x^2 - ax - 2bx + 2ab$$
$$= x(x-a) - 2b(x-a)$$
$$= (x-a)(x-2b)$$

10. $16y^3 - 4y = 4y(4y^2 - 1)$

$$= 4y[(2y)^2 - (1)^2]$$
$$= 4y(2y+1)(2y-1)$$

11. $3x^5 - 48x = 3x(x^4 - 16)$

$$= 3x[(x^2)^2 - (4)^2]$$

$$= 3x(x^2 + 4)(x^2 - 4)$$

$$= 3x(x^2 + 4)(x^2 - 2^2)$$

$$= 3x(x^2 + 4)(x + 2)(x - 2)$$

12. $(3x - 4y)^2 - 25z^2 = (3x - 4y)^2 - (5z)^2$

$$= (3x - 4y + 5z)(3x - 4y - 5z)$$

13. $(2x + 3y)^2 - (1)^2 = (2x + 3y + 1)(2x + 3y - 1)$

14. $16c^2 - (5a + b)^2 = (4c)^2 - (5a + b)^2$

$$= (4c + 5a + b)(4c - 5a - b)$$

15. $100 - (x - 5)^2 = (10)^2 - (x - 5)^2$

$$= (10 + x - 5)(10 - x + 5)$$

$$= (x + 5)(15 - x)$$

16 (i) $(405)^2 - (395)^2 = (405 + 395)(405 - 395) = 800 \times 10 = 8000$

(ii) $(17.8)^2 - (2.2)^2 = (17.8 + 2.2)(17.8 - 2.2) = 20 \times 15.6$

$$= 312$$

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct option :

1. On factorising $ax + by + bx + ay$, we get
    (a) $(x + y)(a + b)$
    (b) $(x + y) + (a + b)$
    (c) $(x + a)(y + b)$
    (d) $(x + a) + (y + b)$

2. $ax + by - ay - bx$ is written as :
    (a) $(a - b)(x - y)$
    (b) $(a + b)(x - y)$
    (c) $(a + b)(x + y)$
    (d) $(a - b)(x + y)$

3. On factorising $3a^2bc + 9ab^2c + 21abc^2$, we get
    (a) $abc(a + 3b + 7c)$
    (b) $3abc(a + b + c)$
    (c) $3abc(a + 3b + c)$
    (d) $3abc(a + 3b + 7c)$

4. $x^3 - 144x$ is equivalent to :
    (a) $(x + 12)(x - 12)$
    (b) $x(x - 12)(x + 12)$
    (c) $x(x + 12)^2$
    (d) $x^2(x + 12)(x - 12)$

5. On factorising $(x - y)^2 + 4xy - z^2$, we get
    (a) $(x + y + z)(x + y - z)$
    (b) $(x - y - z)(x + y - z)$
    (c) $(x - y + z)(x + y - z)$
    (d) none of these

6. $1 + a + b + ab$ can be written as :
    (a) $a(b + 1)$
    (b) $b(a + 1)^2$
    (c) $(a + 1)(b + 1)$
    (d) $(a + 1)^2(b + 1)$

7. $2xy + 5x + 2y + 5$ can be written as :
    (a) $(x + 1)(y + 5)$
    (b) $(2x + 1)(2y + 5)$
    (c) $(x - 1)(2y + 5)$
    (d) $(x + 1)(2y + 5)$

8. On factorising $12ab - 8a - 6 + 9b$, we get :
    (a) $(4a + 3)(3b - 2)$
    (b) $(4a - 3)(3b + 2)$
    (c) $(4a - 3)(3b - 2)$
    (d) $(3a + 4)(2b - 3)$

9. $-36 + 4p^2$ is same as :
    (a) $(p + 3)(p - 3)$
    (b) $9(p + 4)(p - 4)$
    (c) $4(p + 3)(p - 3)$
    (d) $4p(p + 3)(p - 3)$

10. $x^2 + 4x - 12$ can be factorised as :
    (a) $(x - 2)(x + 6)$
    (b) $(x - 3)(x + 4)$
    (c) $(x + 2)(x - 6)$
    (d) $(x - 2)(x - 6)$

VALUE BASED QUESTIONS

Sameer and Balwant are very good friends. They study in the same class. Sameer always completes his homework, while Balwant takes a casual approach towards it. One day they got their homework as below :

Factorise : $3x^2 - 24x + 36$

Sameer's answer was $3(x - 2)(x - 6)$ and Balwant's answer was $(x - 2)(x + 6)$.

(a) Whose answer is correct? Verify it by factorising the given expression.

(b) Here, which value is depicted by Sameer?

(c) Which values are missing in Balwant?

Sol. (a) $\quad 3x^2 - 24x + 36 = 3(x^2 - 8x + 12)$
$$= 3(x^2 - 6x - 2x + 12)$$
$$= 3(x - 6)(x - 2)$$

So, Sameer's answer is correct.

MENTAL MATHS

Write T for true and F for false statements :

1. $10a^2b + 30ab$ can be expressed as $10ab(a + 3)$ T

2. On dividing $(a^2 - b^2)$ by $(a + b)$, we get $(a - b)$.   T

3. $xy + ab + ay + bx$ can be factorised as $(x + y)(a + b)$ F

4. $3(x + y) + z(x + y)$ can be factorised as $(x + y)(z + 3)$ T

5. On dividing $7a^2 + 14a$ by $7(a + 2)$, we get 1.   F

6. On dividing $a^2x^2 - 25$ by $(ax + 5)$, we get $(x - 5)$.   F

7. If we divide $(x^2 + 2 + \dfrac{1}{x^2})$ by $\left(x + \dfrac{1}{x}\right)$, we get $\left(x - \dfrac{1}{x}\right)$ as quotient.   F

8. $6x^4y^2 - 18x^2y^4$ can be factorised as $6x^2y^2(x^2 - 3y^2)$.   T

9. $x + 2xy + 2xy + 11$ can be expressed as $(x + 11)(x + 2y)$.   F

10. $x^2 + x - 2$ can be written as $(x - 1)^2$.   F