Chapter 1 - LINEAR SIMULTANEOUS EQUATIONS

1.1 Introduction

For an unknown variable x, the linear equation is represented by ax+b=0 where a≠0. In this equation 'a' and 'b' are real numbers and 'a' is the coefficient of x and 'b' is a constant. The solution for x is $-\frac{b}{a}$. For two unknown variables in linear equation can be represented as

$$a_1 x + b_1 y + c_1 = 0 \ldots\ldots\ldots\ldots\ldots(1)$$

where a_1 and b_1 are coefficients of x and y and c_1 is a constant and a_1 b_1 and c_1 are real numbers where the coefficients a_1 and b_1 are also not equal.

The geometric interpretation a system involving two variable x and y, each linear equation determines a line on the xy-plane. It has already been discussed in previous class. So the linear equation for equation1 is always determines a line.

In order to solve the values of x and y of linear equation (1), we need to take one more equation such as $a_2 x + b_2 y + c_2 = 0$ ……………(2) where a_2 and b_2 are coefficients of x and y and c_2 is a constant and a_2 b_2 and c_2 are real numbers where the coefficients a_2 and b_2 are not equal to zero at same time.

1.2 Geometrical Representation

The above given two linear equations $a_1 x + b_1 y + c_1 = 0$ ……………(1)

$a_2 x + b_2 y + c_2 = 0$ ……………(2) where $a_1^2+b_1^2 \neq 0$ and $a_2^2+b_2^2 \neq 0$ which means at the same time, a_1, b_1 and a_2, b_2 are not equal to 0. The graphical representation of equation 1 and 2 on xy-plane is a line representing as L_1 and L_2. In short

$L_1 : a_1 x + b_1 y + c_1 = 0$ ……………(1) and $L_2 : a_2 x + b_2 y + c_2 = 0$ ……………(2)

We get three types of lines when we represent both the equations on xy-plane

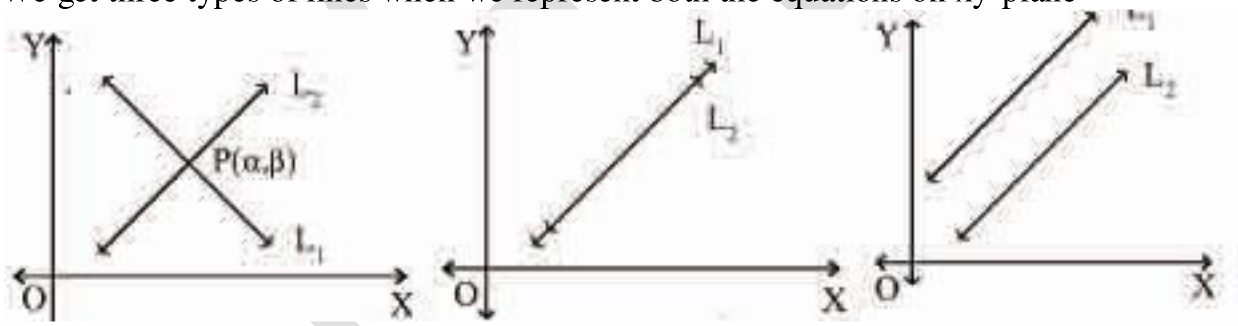


Fig 1 (i) — L_1 and L_2 intersect each other

Fig 1 (ii) — L_1 and L_2 are one and coincident

Fig 1 (iii) — L_1 and L_2 are parallel

In Fig 1(i), the straight lines L_1 and L_2 intersect each other at point P(α,β) where x=α and y=β. Hence there is only one and unique solution is obtained from it.

In Fig 1(ii) L_1 and L_2 are one and coincident i.e. they have infinite common points, as a result there are infinite common points.

In Fig 1(iii) L_1 and L_2 are parallel to each other i.e. they don't intersect at any point. Hence solution is not possible.

1.3 Solution of simultaneous equations by use of Graphs :

How to represent graphically a linear equation is discussed in previous class. Now we will solve the simultaneous linear equation through the following example :

Example 1 Solve the following given simultaneous equation

$x + 2y - 3 = 0$ .... (i) $\qquad$ $2x - y - 1 = 0$ .... (ii)

Solution : Put the values of x into y or vice versa

$x + 2y - 3 = 0 \Rightarrow y = \frac{1}{2}(3 - x)$ .... (i)

$2x - y - 1 = 0 \Rightarrow y = 2x - 1$ .... (ii)

In equation (i) – the value of y is derived from the values of x i.e 3 and 1given below in the table

x	3	1
y	0	1

∴ the co-ordinates of P_1 and P_2 is (3,0), (1,1) respectively

Similarly in equation (ii) – the value of y is derived from the values of x i.e 1 and 2given below in the table

x	1	2
y	1	3

∴ the co-ordinates of Q_1 and Q_2 is (1,1), (2,3) respectively.

The graphical representation of straight line L_1 whose co-ordinates are P_1 (3,0) and P_2(1,1), and for straight line L_2, the co-ordinates are Q_1(1,1) and Q_2(2,3) intersect at a point P on xy-plane.

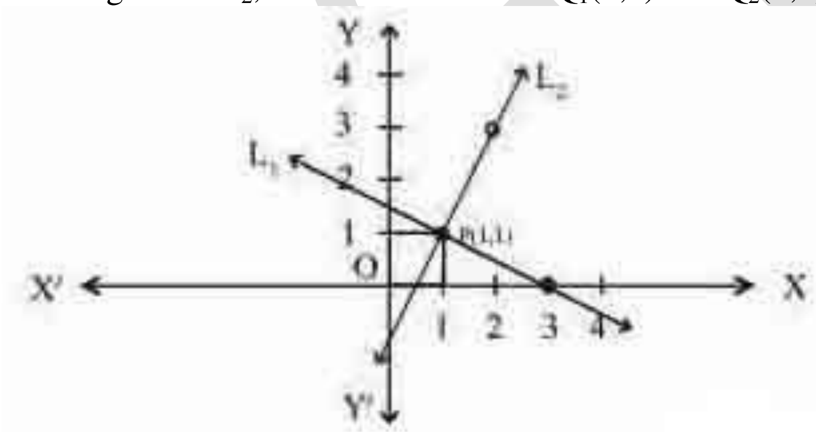


Fig 1.2

∴ the point of intersection P has the coordinates (1,1)

Example 2 : Solve the following equation by the use of graph

$x - 2y - 7 = 0$; $x + y + 2 = 0$

Solution : Put the values of x into y or vice versa

$x + 2y - 7 = 0 \Rightarrow y = \frac{1}{2}(7 - x)$ .... (i)

$x + y + 2 = 0 \Rightarrow y = -2 - x$ .... (ii)

In equation (i) – the value of y is derived from the values of x is given below in the table

x	-1	3
y	-4	-2

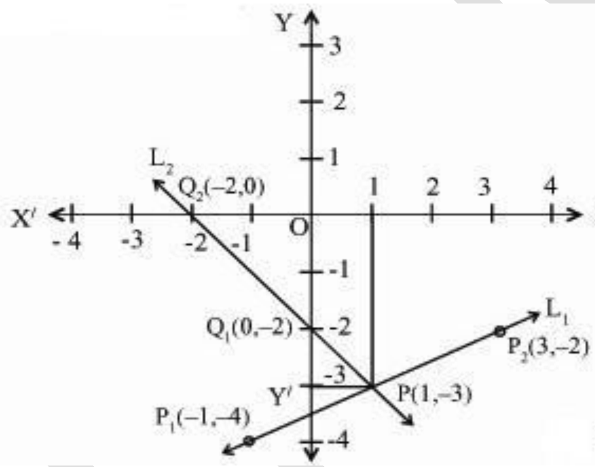
∴ the co-ordinates of P_1 and P_2 is (-1,-4), (3,-2) respectively

Similarly in equation (ii) – the value of y is derived from the values of x i.e 1 and 2given below in the table

x	0	-2
y	-2	0

∴ the co-ordinates of Q_1 and Q_2 is (0,-2), (-2,0) respectively.

The graphical representation of straight line L_1 whose co-ordinates are P_1 (-1,-4) and P_2(-(3,-2), and for straight line L_2, the co-ordinates are Q_1(0,-2) and Q_2(-2,0) intersect at a point P on xy-plane.



1.3

∴ the point of intersection P(1,-3) has the coordinates x=1, y=-3

Example 3 : Show graphically that the following systems of equations has infinitely many solutions

(a) x + y – 3 = 0 and 2x + 2y – 6 = 0, (b) x + y – 3 = 0 and x + y – 5 = 0

Solution : the equations for (a)

x + y – 3 = 0………. (i) and 2x + 2y – 6 = 0 …………(ii)

equation (ii) => 2x + 2y – 6 = 0 => x + y – 3 = 0

observed that above two equations are same. Every equation is determined by points (0,3) and (3,0). Therefore the graph drawn by the equation is straight line and every solution of one equation is a solution of the other. The system of equations has infinitely many solutions.

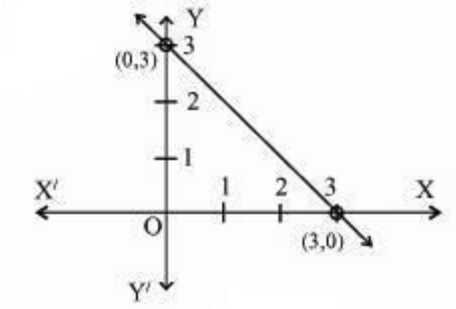


Fig 1.4

From fig 1.4 we obtained two points (0,3) and (3,0) from infinitely many solutions. Similarly we obtain two solutions (0,5) and (5,0) from 2nd equation (b). When we plot the points of 2nd equation we obtain the fig 1.5

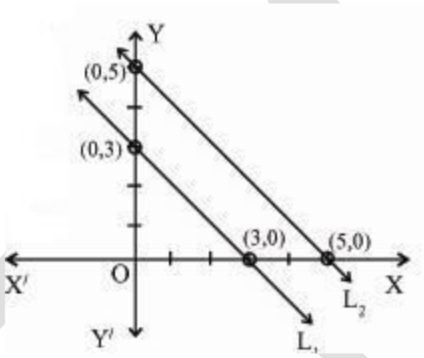


Fig 1.5

We find the lines represented by equation (b) are parallel. So, the we lines have no common point. Hence, the given system of equations has no solution.

1.4 Conditions of solvability of two linear simultaneous equations :

Consider the two linear simultaneous equations

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0.$$

Each equation gives a linear graph where a_1, b_1 is not equal to 0 and a_2, b_2 is also not equal 0 at one time.

We have seen that we obtained a unique solution from Example 1 and example 2.

From example 1 we obtained a_1=1, b_1=2 and c_1=-3 and a_2=2, b_2=-1 and c_2=-1

$\therefore \frac{a1}{a2} = \frac{1}{2}$ and $\frac{b1}{b2} = \frac{2}{-1} =$ -2 and => $\frac{a1}{a2} \neq \frac{b1}{b2}$

Similarly in example 2, $\frac{a1}{a2} = \frac{1}{1}$ and $\frac{b1}{b2} = \frac{-2}{1} =$ -2 and => $\frac{a1}{a2} \neq \frac{b1}{b2}$

It follows from this that the ratio of coefficients of unknown variables X and Y is not equal hence the equations are consistent with unique solution. They lines drawn, intersect each other at one point. Therefore the above two linear equations are **consistent and independent**.

From the Example 3(i), we have seen a_1=1, b_1=2 and c_1= -3 and a_2=2, b_2=-1 and c_2=-6

$\therefore \frac{a1}{a2} = \frac{1}{2}$ and $\frac{b1}{b2} = \frac{1}{2} =$ -2 and $\frac{c1}{c2} = \frac{-3}{-6} = \frac{1}{2}$

=> $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$ hence no unique solution can be obtained from the above two equations but they form infinitely many solutions i.e. lines represented by two equations are coincident. Hence they are **consistent and dependent**.

From the Example 3(ii), a_1=1, b_1=1 and c_1= -3
a_2=1, b_2=1 and c_2=-5

here $\frac{a1}{a2} = \frac{1}{1}$ and $\frac{b1}{b2} = \frac{1}{1}$ = 1 and $\frac{c1}{c2} = \frac{-3}{-5} = \frac{-3}{-5} = \frac{3}{5}$

=> $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, hence no solution can be obtained from the above two equations as the two lines of equation are parallel to each other. Therefore the equations are inconsistent.

Table shows the Example 1, Example 2 and Example 3

Difference between $a_1x+b_1y+c_1=0$, $a_2 x + b_2 y + c_2 = 0$ and the ratio of $\frac{a1}{a2}, \frac{b1}{b2}, \frac{c1}{c2}$	$L_1 : a_1 x + b_1 y + c_1 = 0$ $L_2 : a_2 x + b_2 y + c_2 = 0$	Defined as per the solutions obtained from the two equations
$\frac{a1}{a2} \neq \frac{b1}{b2}$	L_1 and L_2 intersect each other	Consistent and independent Obtain unique solution
$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$	L_1 and L_2 are infinite and coincident	Consistent and dependent Form infinitely many solutions
$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	L_1 and L_2 are parallel	Inconsistent No solution can be formed

Hence , unique solution (0,0) is formed when $a_1x+b_1y+c_1=0$, $a_2 x + b_2 y + c_2 = 0$ and when $\frac{a1}{a2} \neq \frac{b1}{b2}$ whereas infinite solutions are formed when $\frac{a1}{a2} = \frac{b1}{b2}$. Both the equations are consistent.

Example 4 :

(i)Determine the value of k for which unique solution is possible for the given system of equations 4x + ky + 8 = 0, 2x + 2y + 2 = 0.

(ii)Determine the value of k for which infinite solutions are possible for the given system of equations kx + 3y – (k–3) = 0 and 12x + ky – k = 0.

(i)Determine the value of k for which the given system of equations 5x–3y = 0 and 2x + ky = 0 has a infinite solutions.

Solution

(i) $a_1 = 4$, $b_1 = k$ and $c_1 = 8$, $a_2 = 2$, $b_2 = 2$ and $c_2 = 2$

$$\frac{a1}{a2} \neq \frac{b1}{b2} => \frac{4}{2} \neq \frac{k}{2} => k \neq 4$$

∴ k=4, so given system of equations has unique solution.

(ii) $a_1 = k$, $b_1 = 3$, $c_1 = -(k-3)$, $a_2 = 12$, $b_2 = k$, $c_2 = -k$

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2} => \frac{k}{12} = \frac{3}{k} = \frac{c-(k-3)}{-k}$$

k^2=12x3=36 ; k= $\pm$6 .... (i)

–3k = –k(k–3) => k^2 – 6k = 0 => k (k– 6) = 0 => k =0 or k = 6

From both the equations it is clear k=6 hence the give equations has infinite solutions. If k=-6, $\frac{a1}{a2} = \frac{b1}{b2} = \frac{1}{2} \neq \frac{c1}{c2} = \frac{3}{2}$ hence both the equations are inconsistent. Infinite solutions are possible if k $\neq\pm$ 6.

(iii) Here a1 = 5, b1 = –3, a2 = 2, b2 = k

Infinite solutions are possible provided $\frac{a1}{a2} = \frac{b1}{b2}$ => $\frac{5}{2} = \frac{-3}{k}$ => k = - $\frac{6}{5}$

Exercise 1(a)

1.Fill in the blanks choosing correct answer given in brackets

(i) Find the alternative solution for x+ y = 0 ----- [(4,5), (5,5), (–4, 4), (–4, 5)]

(ii) Find the alternative solution for x –2y = 0 ----- [(4,2), (–4,2), (4, –2), (–4, –2)]

(iii) Find the alternative solution for 2x + y+2 = 0 -----[(0,2), (2,0), (–2,0), (0, –2)]

(iv) if x – 4y +1 = 0, find x = ----- [4y – 1, 4y+1, –4y+1, –4y –1]

(v) if 2x – y + 2 = 0, find y = ----- [2x – 2, 2x+2, 2x – 2, –2x –2]

(vi) if x – 2y+ 3 = 0, find y = ----- [½(x+3), – ½(x –3), –½(–x+3), –½(x+3)]

2.In each of the following systems of equations determine whether the system has a (i) unique solution, (ii)infinitely many solutions or (iii) no solution.

(i) x+y+1 = 0, x –y +1 = 0,
(ii) x+y+1 = 0, 2x +2y +2 = 0
(iii) x+y+1 = 0, x +y +3 = 0,
(iv) 2x – y+3=0, – 4x+2y – 6=0
(v) 2x – y + 3 = 0, 2x + y – 3 = 0,
(vi) 2x – y + 3 = 0, –6x + 3y + 5 = 0

3. Find any three co-ordinates of the given systems of equations to show in the graph.

(i) x – y = 0
(ii) x + y = 0
(iii) x – 2y = 0
(iv) x + 2y – 4 = 0
(v) x – 2y – 4 = 0
(vi) 2x – y + 4 = 0

4. Find the answers for the following equations.

i)If the systems of equations, kx + my + 4 = 0 and 2x + y +1 = 0 are inconsistent Find k:m.
ii)If the solution for the equations 2x + 3y – 5=0 & 7x – 6y –1 = 0 is (1, β), find the value of β.
iii) For what value of t, (1,1) is the alternative solution of the equation 3x + ty – 6 = 0.
iv) For what value of t, (1,1) is the alternative solution of the equation tx – 2y – 10 = 0.
v) For what value of t, the system of equations tx + 2y = 0 and 3x + ty = 0 has infinite solutions.
vi) Prove that the system of equations 6x – 3y +10 = 0 and 2x– y+ 9 = 0 has no solution.
vii) Prove that the system of equations 2x + 5y = 17 and 5x + 3y = 14 are consistent and independent.
viii) Prove that the system of equations 3x–5y –10 = 0 and 6x – 10y = 20 has infinite solutions.

5. Find the solutions for the following system of equations using graph.

(i) $x + y - 4 = 0$ and $x - y = 0$, (ii) $x - y = 0$ and $x + y - 2 = 0$
(iii) $x + y = 0$ and $-x + y - 2 = 0$, (iv) $2x + y - 3 = 0$ and $x + y - 2 = 0$
(v) $3x + y + 2 = 0$ and $2x + y + 1 = 0$, (vi) $x + 2y + 3 = 0$ and $2x + y + 3 = 0$
(vii) $2x + y - 6 = 0$ and $2x - y + 2 = 0$, (viii) $x + y - 1 = 0$ and $2x + y - 8 = 0$
(ix) $3x + y - 11 = 0$ and $x - y - 1 = 0$, (x) $2x - 3y - 5 = 0$ and $-4x + 6y - 3 = 0$
(xi) $2x + y + 2 = 0$ and $4x - y - 8 = 0$, (xii) $3x + 4y - 7 = 0$ and $5x + 2y - 7 = 0$

6.

i) Using graph, prove that the system of equation $2x - 2y = 2$ and $4x - 4y - 8 = 0$ has no solution.
ii) Using graph, prove that the system of equation $2x - 3y = 1$ and $3x - 4y = 1$ has unique solution.
iii) Using graph, prove that the system of equation $9x + 3y + 12 = 0$ and $18x + 6y + 24= 0$ are unique and dependent.
iv) Using graph, find the y co-ordinates for system of equation $2x - y = 1$ and $x + 2y= 8$ where they intersect.

7. Find the value of k for the following system of equations has unique solutions.

(i) $x - 2y - 3 = 0, 3x + ky - 1 = 0$, (ii) $kx - y - 2 = 0, 6x + 2y - 3 = 0$
(iii) $kx + 3y + 8 = 0, 12x + 5y - 2 = 0$, (iv) $kx + 2y =5, 3x + y = 1$
(v) $x - ky = 2, 3x + 2y + 5 = 0$, (vi) $4x - ky = 5, 2x - 3y = 12$

8. Find the value of k for the following system of equations has infinite solutions.

(i) $7x - y - 5 = 0, 21x - 3y - k = 0$, (ii) $8x + 2y - 9 = 0, kx + 10y - 18 = 0$
(iii) $kx - 2y + 6 = 0, 4x - 3y + 9 = 0$, (iv) $2x + 3y = 5, 6x + ky = 15$
(v) $5x + 2y = k, 10x + 4y = 3$, (vi) $kx - 2y - 6 = 0, 4x + 3y + 9 = 0$

9. For what value of k, the following system of equations are inconsistent.

(i) $8x + 5y - 9 = 0, kx + 10y - 15 = 0$, (ii) $kx - 5y - 2 = 0, 6x + 2y - 7 = 0$
(iii) $kx + 2y - 3 = 0, 5x + 5y - 7 = 0$, (iv) $kx - y - 2 = 0, 6x - 2y - 3 = 0$
(v) $x + 2y - 5 = 0, 8x + ky -10 = 0$, (vi) $3x - 4y + 7\ 0, kx + 3y - 5 = 0$

1.2. Algebraic method of solving simultaneous linear equations in two variables.

Let the following system of equations are consistent and independent.

$a_1x + b_1y + c_1 = 0$ ....(1)
$a_2x + b_2y + c_2 = 0$ ....(2)

these two equations can be solved either by graphical method or algebraic method. Lets us first discuss in algebraic method.

I) METHOD OF SUBSTITUTION

In this method choose either of the two equations 1 or 2 and find the value of one variable, say y, in terms of the other i.e. x or variable x in terms of variable y. For example if b≠0 for equation 1

$b_1y = -c_2 - a_1x \Rightarrow y = \frac{1}{b1}(-c_1 - a_1x)$ ....(3)

if we put the value of $y = \frac{1}{b1}(-c_1 - a_1x)$ of equation 3 into equation 2, we obtain

$a_2x + \frac{b2}{b1}\{-c_1 - a_1x\} + c_2 = 0 \Rightarrow (a_2b_1 - a_1b_2)x + (c_2b_1 - c_1b_2) = 0$

$\Rightarrow x = -\dfrac{c2b1-c1b2}{a2b1-a1b2} \Rightarrow x = \dfrac{b1c2-b2c1}{a1b2-a2b1}$ ....(4)

If we substitute the value of x in either equation 1 or 2, we obtain

$$a_1\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right)+b_1y+c_1=0 \Rightarrow y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1} \quad .... (5)$$

$$x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\ y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1} \quad ....(6)$$

By substitution method we obtain the require solution for given system of equations.

Example 5

Solve : $5x + 2y + 2 = 0,\ 3x + 4y - 10 = 0$

$5x + 2y + 2 = 0$ and ....(i)

$3x + 4y - 10 = 0$ ....(ii)

On solving equation 1 and substituting the value of y, we obtain

$2y = -5x - 2 \Rightarrow y = \frac{1}{2}(-5x - 2)$ ....(iii)

From equation (ii) and (iii)

$$3x + \frac{4}{2}(-5x-2) - 10 = 0 \Rightarrow 6x + 4(-5x-2) - 20 = 0$$

$$6x - 20x - 8 - 20 = 0 \Rightarrow -14x - 28 = 0 \Rightarrow x = -2$$

From equation 1, x=-2, on substitution of the values of x, we get $5(-2) + 2y + 2 = 0$

$\Rightarrow 2y - 8 = 0 \Rightarrow y = 4$

$\therefore$ the solution is (-2, 4) ....**(Ans)**

II) METHOD OF ELIMINATION

As per this method we can eliminate either of the variable x or y. Let's eliminate x. Multiply the 2nd equation with co-efficient of x i.e a_1 and multiplying equation 1 with co-efficient x of equation 2.

$a_2 \times (1) \Rightarrow a_1a_2x + a_2b_1y + a_2c_1 = 0$ ....(7)

$a_1 \times (2) \Rightarrow a_1a_2x + a_1b_2y + a_1c_2 = 0$ ....(8)

the co-efficients of x in equation 7 and 8 is equal. If we subtract 8 from 7, we obtain

$(a_2b_1 - a_1b_2)\,y + (a_2c_1 - a_1c_2) = 0$

$\Rightarrow y = \dfrac{-(a2c1-a1c2)}{a2b1-a1b2} \Rightarrow y = \dfrac{c1a2-c2a1}{a1b2-a2b1}$

After substituting the value of y into equation 1 (or into equation 2), we obtain

$x = \dfrac{b1c2-b2c1}{a1b2-a2b1}$

if we consider α and β, then $\alpha = \dfrac{b1c2-b2c1}{a1b2-a2b1}$ and $\beta = \dfrac{c1a2-c2a1}{a1b2-a2b1}$

Example 6 :

Solve $2x + 3y - 8 = 0,\ 3x + y - 5 = 0$

Solution The given system of equation is

$2x + 3y - 8 = 0$ ....(i)

$3x + y - 5 = 0$ ....(ii)

$3 \times (i) \Rightarrow 6x + 9y - 24 = 0$ ....(iii)
$2 \times (ii) \Rightarrow 6x + 2y - 10 = 0$ ....(iv)
$\quad - \quad - \quad +$

$(iii) - (iv) \Rightarrow 7y - 14 = 0 \Rightarrow y = 2$

If we replace the value of y i.e. y=2 in equation (i), we obtain

$$2x + 6 - 8 = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$$

$\therefore$ the solution is (1, 2) ....**(Ans)**

III. CROSS MULTIPLICATION

From previous methods we obtained the solution

$$a_1x + b_1y + c_1 = 0 \qquad \text{.......................(1)}$$

$$a_2x + b_2y + c_2 = 0 \qquad \text{.......................(2)}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1},\ y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Now we can find

$$\left.\begin{array}{l} \dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{1}{a_1b_2 - a_2b_1} \\ \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1} \end{array}\right\} \qquad \text{.......................(3)}$$

In order to balance L.H.S of both equations of Equation 3, we can write

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \qquad \text{.......................(4)}$$

we should remember that $a_1b_2 - a_2b_1 \neq 0$ and $\frac{a1}{a2} \neq \frac{b1}{b2}$. The above process is known as cross multiplication. To remember easily we can use the following

$$\frac{x}{\begin{smallmatrix} b_1 & c_1 \\ b_2 & c_2 \end{smallmatrix}} = \frac{y}{\begin{smallmatrix} c_1 & a_1 \\ c_2 & a_2 \end{smallmatrix}} = \frac{1}{\begin{smallmatrix} a_1 & b_1 \\ a_2 & b_2 \end{smallmatrix}}$$

Note :

1. If $c_1 = c_2 = 0$ and $a_1b_2 - a_2b_1 \neq 0$, then, $a_1x + b_1y = 0$, $a_2x + b_2y = 0$, the solution will be (0,0). This type of equation is known as Homogenous Simultaneous Equation. If $a_1b_2 - a_2b_1 = 0$, then the equation will be one and coincident. The system of equation will be infinite.

2. In order to solve system of equation, first we should prove that $a_1b_2 - a_2b_1 \neq 0$.

Example 7

Solve : $2x - 3y - 1 = 0$, $4x + y - 9 = 0$

Solution Given systems of equations is
$2x - 3y - 1 = 0$
$4x + y - 9 = 0$
Here it is observed that solution is possible if $2 \times 1 - 4(-3) = 2 + 12 = 14 \neq 0$.
Using cross multiplication method

$$\frac{x}{\begin{array}{cc}-3 & -1\\ 1 & -9\end{array}} = \frac{y}{\begin{array}{cc}-1 & 2\\ -9 & 4\end{array}} = \frac{1}{\begin{array}{cc}2 & -3\\ 4 & 1\end{array}}$$

$$\Rightarrow \quad \frac{x}{(-3)(-9)-1(-1)} = \frac{y}{(-1)4-(-9)2} = \frac{1}{2\times1-4(-3)}$$

$$\Rightarrow \quad \frac{x}{27+1} = \frac{y}{-4+18} = \frac{1}{2+12} \Rightarrow \frac{x}{28} = \frac{y}{14} = \frac{1}{14} \quad \Rightarrow x = \frac{28}{14} = 2, \quad y = \frac{14}{14} = 1$$

∴ the solution is (2, 1) ……………………………….**(Ans)**

1.3 NON LINEAR SIMULTANEOUS EQUATION

Till now we have discussed about the solutions of linear simultaneous equation

$$a_r x + b_r y + c_r = 0, \; r = 1, 2 \; ....(1)$$

Many simultaneous equations are not linear, they are to be converted into linear equation using the various methods algebraic equations. But it is possible only for few equations only not for every equation.

Example 8

Solve $6x + 3y = 7xy$, $3x + 9y = 11xy$ $(x \neq 0, y \neq 0)$

The given system of equations is not linear. If we divided both sides with xy we can obtain $(\therefore x \neq 0, y \neq 0 \text{ and } xy \neq 0)$

$$\frac{6}{y} + \frac{3}{x} = 7, \quad \frac{3}{y} + \frac{9}{x} = 11$$

if we consider

$$\frac{1}{x} = \upsilon \qquad \frac{1}{y} = \nu$$

$3\upsilon + 6\nu - 7 = 0$ and $9\upsilon + 3\nu - 11 = 0$

(here $3x3 - 9 \text{ x } 6 = -45 \neq 0$)

Through cross multiplication

$$\frac{\upsilon}{\begin{array}{cc}6 & -7\\ 3 & -11\end{array}} = \frac{\nu}{\begin{array}{cc}-7 & 3\\ -11 & 9\end{array}} = \frac{1}{\begin{array}{cc}3 & 6\\ 9 & 3\end{array}}$$

$$\Rightarrow \quad \frac{\upsilon}{-66+21} = \frac{\nu}{-63+33} = \frac{1}{9-54} \quad \Rightarrow \frac{\upsilon}{-45} = \frac{\nu}{-30} = \frac{1}{-45}$$

$$\Rightarrow \quad \upsilon = \frac{-45}{-45} = 1, \; \nu = \frac{-30}{-45} = \frac{2}{3} \quad \Rightarrow \frac{1}{x} = 1, \; \frac{1}{y} = \frac{2}{3} \Rightarrow x = 1, \; y = \frac{3}{2}$$

∴ the solution is $(1, \frac{3}{2})$ ……………………………….**(Ans)**

Example 9

Solve :

$$: \frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}, \ \frac{7}{2x+3y} + \frac{4}{3x-2y} = 2$$

Solution :

$$\upsilon = \frac{1}{2x+3y} \qquad \nu = \frac{1}{3x-2y} \qquad \text{.......... (i)}$$

∴ the converted equation will $\frac{1}{2}u + \frac{12}{7}v = \frac{1}{2}$, $7u + 4v = 2$

$7\upsilon + 24\nu - 7 = 0$ ……(ii)

$7\upsilon + 4\nu - 2 = 0$ ……(iii)

(ii) – (iii) $\Rightarrow 20\nu - 5 = 0 \Rightarrow \nu = \frac{1}{4}$

∴ $3x - 2y = 4$ ……(iv)

In Eq(iii) $\nu = \frac{1}{4}$, we get $7\upsilon + 1 - 2 = 0 \Rightarrow \upsilon = \frac{1}{7}$

∴ $2x + 3y = 7$ ……(v)

$2(iv) - 3(v) \Rightarrow 2(3x - 2y) - 3(2x + 3y) = 8 - 21$

$\Rightarrow -13y = -13 \Rightarrow y = 1$

In Eq(iv) $y = 1$, we get $3x - 2 = 4 \Rightarrow x = 2$

∴ the solution is (2, 1) ……………………………….**(Ans)**

Important Note

Study the given picture

$$: A = \begin{pmatrix} 5 & 7 \\ 2 & 1 \end{pmatrix}$$

In the above picture, numbers are written in two rows and two columns and are put inside a bracket. These numbers assigned to a variable A. This arrangement of numbers in row and columns is known as 2x2 Matrix. We can also have 3x3 or 4x4 rows and columned matrix. As the number of row and columns are equal in the above matrix, it is known as square matrix. We also have long matrix. We study only the square matrix. Every square matrix has a determined number hence it is known as determinant of square matrix. If matrix A= $\begin{matrix} a & b \\ c & d \end{matrix}$

Its determinant is $|A| = \begin{matrix} a & b \\ c & d \end{matrix}$ = ad – cb. If take A= $\begin{matrix} 5 & 7 \\ 2 & 1 \end{matrix}$ we have I A I=5 x 1 – 7 x 2 = 5 –14 = –9

Similarly, we have

$$\begin{vmatrix} 1 & -4 \\ 0 & 3 \end{vmatrix} = 1 \times 3 - 0 \times (-4) = 3 - 0 = 3;$$

$$\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = 2 \times 3 - 1 \times (-1) = 6 + 1 = 7$$

We know that

$$a_1x + b_1y + c_1 = 0 \qquad \text{......................(1)}$$

$$a_2x + b_2y + c_2 = 0 \qquad \text{......................(2)}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \ y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Now you see the given pair of equations

$a1x + b1y + c1 = 0 \Rightarrow a1x + b1y = -c1$ and

$a2x + b2y + c2 = 0 \Rightarrow a2x + b2y = -c2$

Using a1, b1, –c1, a2, b2, –c2, we can have following determinants

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $\Delta_X = \begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}$ and $\Delta_Y = \begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}$

[if first column of Δ is replaced by a constant] [if 2^{nd} column of Δ is replaced by a constant]

where $\Delta = a_1b_2 - a_2b_1$, $\Delta_x = -c_1b_2 - b_1(-c_2)$, $\Delta_y = -a_1c_2 - a_2(-c_1)$

$= b_1c_2 - b_2c_1$ $\quad = c_1a_2 - c_2a_1$

Resultant of above determinants :

$x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$ where $\Delta \neq 0$ because both equations should be consistant

we can show in determinants form using cross multiplication method

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{b}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow \frac{x}{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}} = \frac{b}{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{\Delta_x} = \frac{y}{\Delta_y} = \frac{1}{\Delta} \Rightarrow x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}$$

It is known as Cramer's Rule as it is developed by well known mathematician Cramer

Example - **10 :**

Solve following system of equation using Cramer's Rule

$x + 2y = -1$ & $2x - 3y = 12$

Solution: Here $\Delta = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1 \times (-3) - 2 \times 2 = -3 - 4 = -7 \qquad \therefore \Delta \neq 0$

$$\Delta_x = \begin{vmatrix} -1 & 2 \\ 12 & -3 \end{vmatrix} = (-1) \times (-3) - 2 \times 12 = 3 - 24 = -21$$

$$\Delta_y = \begin{vmatrix} 1 & -1 \\ 2 & 12 \end{vmatrix} = 1 \times 12 - 2 \times (-1) = 12 + 2 = 14$$

$$\therefore x = \frac{\Delta_x}{\Delta} = \frac{-21}{-7} = 3, y = \frac{\Delta_y}{\Delta} = \frac{14}{-7} = -2$$

$\therefore$ Resultant Sol. : $(x, y) = (3, -2)$

Exercise - 1 (b)

1. Solve the system of equation using Substitution Method

(i) $x + y - 8 = 0,\ 2x - 3y - 1 = 0$ (ii) $3x + 2y - 5 = 0,\ x - 3y - 9 = 0$

(iii) $2x - 5y + 8 = 0,\ x - 4y + 7 = 0$ (iv) $11x + 15y + 23 = 0,\ 7x - 2y - 20 = 0$

(v) $ax + by - a + b = 0,\ bx - ay - a - b = 0$ (vi) $x + y - a = 0,\ ax + by - b^2 = 0$

2. Solve the system of equation using Elimination method

(i) $x - y - 3 = 0,\ 3x - 2y - 10 = 0$ (ii) $3x + 4y = 10,\ 2x - 2y = 2$

(iii) $3x - 5y - 4 = 0,\ 9x = 2y - 1$ (iv) $0.4x - 1.5y = 6.5,\ 0.3x + 0.2y = 0.9$

(v) $\sqrt{2}x + \sqrt{3}y = 0,\ \sqrt{5}x + \sqrt{2}y = 0$ (vi) $ax + by = 0,\ x + y - c = 0\ (a+b \neq 0)$

3. Solve the give system of equations using cross multiplication method

(i) $x + 2y + 1 = 0,\ 2x - 3y - 12 = 0$ (ii) $2x + 5y = 1,\ 2x + 3y = 3$

(iii) $x + 6y + 1 = 0,\ 2x + 3y + 8 = 0$ (iv) $\frac{x}{a} + \frac{y}{b} = a + b,\ \frac{x}{a^2} + \frac{y}{b^2} = 2$

(v) $x + 6y + 1 = 0,\ 2x + 3y + 8 = 0$ (vi) $4x - 9y = 0,\ 3x + 2y - 35 = 0$

4. Solve following system of equations

(i) $\frac{2}{x} + \frac{3}{y} = 17,\ \frac{1}{x} + \frac{1}{y} = 7$ $(x \neq 0,\ y \neq 0)$

(ii) $\frac{5}{x} + 6y = 13,\ \frac{3}{x} + 20y = 35$ $(x \neq 0)$

(iii) $2x - \frac{3}{y} = 9,\ 3x + \frac{7}{y} = 2$ $(y \neq 0)$

(iv) $4x + 6y = 3xy,\ 8x + 9y = 5xy$ $(x \neq 0,\ y \neq 0)$

(v) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$
$(a+b)x + (a+b)y = a^2 + b^2$

(vi) $\frac{x}{a} + \frac{y}{b} = 2,\ ax - by = a^2 - b^2$

(vii) $\frac{5}{x+y} - \frac{2}{x-y} + 1 = 0$
$\frac{15}{x+y} + \frac{7}{x-y} - 10 = 0$

(viii) $\frac{xy}{x+y} = \frac{6}{5},\ \frac{xy}{y-x} = 6$ $(x + y \neq 0,\ x - y \neq 0)$

(ix) $6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$

(x) $\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

(xi) $\frac{x+y}{2} - \frac{x-y}{3} = 8,\ \frac{x+y}{3} + \frac{x-y}{4} = 11$ (xii) $\frac{x}{a} = \frac{y}{b},\ ax + by = a^2 + b^2$

5. Find the values of following equations

(i) $\begin{vmatrix} 2 & 5 \\ 6 & 0 \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$ (iii) $\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix}$ (iv) $\begin{vmatrix} \frac{1}{2} & 1 \\ \frac{3}{4} & \frac{1}{5} \end{vmatrix}$

6. Solve the following equation using Cramer's Rule

(i) $2x + 3y = 5,\ 3x + y = 4$ (ii) $x + y = 3,\ 2x + 3y = 8$

(iii) $x - y = 0,\ 2x + y = 3$ (iv) $2x - y = 3,\ x - 3y = -1$

Applications to word problems

In this section, we shall learn about some applications of simultaneous linear equations is solving problems related to our day-today life. There is wide variety of such problems which are generally called 'word problems'. In solving such problems, we may use the following algorithm.

1. Read the problem carefully and identify the unknown quantities. Give these quantities a variable name like x,y,v,w etc.
2. Identify the variables to be determined.
3. Read the problem carefully and formulate the equations in terms of the variables to be determined.
4. Solve the equations obtained in step III using any one of the algebraic methods learnt earlier.

Example 11

If twice the father's age in years is added to the son' age, the sum is 105 years. But fathers age is added to twice of his son's, the sum is 75 years. Find the ages of father and son.

Solution : Suppose father's age (in years) be x and that of son's be y. Then,

$$2x + y = 105$$
$$x + 2y = 75$$

This system of equations may be written as

$$2x + y - 105 = 0$$
$$x + 2y - 75 = 0$$

By cross-multiplications, we get

$$\frac{x}{x(-75) - 2x(-105)} = \frac{y}{-105x1 - (-75)x2} = \frac{1}{2x2 - 1x1}$$

$$\Rightarrow \frac{x}{135} = \frac{y}{45} = \frac{1}{3} \Rightarrow x = \frac{135}{3} = 45 \quad y = \frac{45}{3} = 15$$

$\therefore$ Father's age = 45 years and Son's age = 15 years

Example 12

The sum of digits of a two digit number is 12. The number is obtained by reversing the order of digits of the given number exceeds the given number by 18. Find the given number.

Solution : Suppose the numbers are x and y

As per questions the order of numbers in tens place x and units place is y. Then the original number will be 10x + y and when the order is reversed the formed will be 10y+x

$x + y = 12$ ....(i)

$(10y+x) - (10x + y) = 18 \Rightarrow 9y - 9x - 18 \Rightarrow y - x = 2$ ..... (ii)

On adding (i) and (ii) we get $2y = 14 \Rightarrow y = 7$

Putting the values of y in equation 1we get $x + 7 = 12 \Rightarrow x = 5$

Hence original number = 57 ……………………………………(Ans)

On reversing the number = 75 which is 18 more than original number.

Example 13

A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fractions becomes $\frac{1}{2}$. What is the fraction.

Solution : Let the fraction be $\frac{x}{y}$.

Then according to the given conditions, we have

$$\frac{x+1}{y+1} = \frac{4}{5} \text{ and } \frac{x-5}{y-5} = \frac{1}{2}$$

=> $5x + 5 = 4y + 4$ and $2x - 10 = y - 5$

=> $5x - 4y + 1 = 0$ .... (i)

$2x - y - 5 = 0$ .... (ii)

Equation (i) => $5x - 4y + 1 = 0$ .... (iii)

Equation (ii) x 4 => $8x - 4y - 20 = 0$ .... (iv)

On subtraction of Equation (iv) from Equation (iii) we get, $-3x + 21 = 0$ => $x = 7$

On putting the value of x in equation (i) we get the, $5 \times 7 - 4y + 1 = 0$ => $4y = 36$ => $y = 9$

∴ the given fraction is $\frac{7}{9}$ .... (Ans)

Example 14

8 men and 12 women can finish a piece of work in 10 days while 6 men and 8 women can finish it in 14 days. Find the time taken by one woman to finish the work.

Solution : Suppose that one man alone can finish the work in x days and one woman alone can finish it in y days. Then

One man's one day's work = $\frac{1}{x}$

One woman's one day's work = $\frac{1}{y}$

∴ 8 man's one day's work = $\frac{8}{x}$

12 woman's one day's work = $\frac{12}{y}$

Since 8 men and 12 women can finish the work in 10 days

As per the question, system of equations formed is

$$\frac{8}{x}+\frac{12}{y}=\frac{1}{10}, \quad \frac{6}{x}+\frac{8}{y}=\frac{1}{14}$$

$$\frac{1}{x} = \upsilon \qquad \frac{1}{y} = v$$

$$80\upsilon + 120v - 1 = 0 \qquad 84\upsilon + 112v - 1 = 0$$

To solve the equation, use cross-multiplication method

$$\frac{\upsilon}{120(-1)-112(-1)} = \frac{v}{84(-1)-80(-1)} = \frac{1}{80\times112-120\times84}$$

$$\Rightarrow \frac{\upsilon}{-8} = \frac{v}{-4} = \frac{1}{-1120} \Rightarrow \upsilon = \frac{8}{1120} = \frac{1}{140} \qquad v = \frac{4}{1120} = \frac{1}{280}$$

$$\Rightarrow x = 140 \quad y = 280$$

$\therefore$ Woman completes her work in 280 days

Example 15

The sum of two number is 15 and the sum of their reciprocal is $\frac{3}{10}$. Find the two numbers.

Solution : Suppose the two numbers are x and y and their reciprocal is $\frac{1}{x}$ and $\frac{1}{y}$.

Hence as per the question the equation is

$$x + y = 15 \ldots\ldots\ldots\ldots (i) \qquad \frac{1}{x}+\frac{1}{y}=\frac{3}{10} \ldots\ldots\ldots\ldots (ii)$$

$$\Rightarrow \frac{x+y}{xy} = \frac{3}{10} \Rightarrow \frac{15}{xy} = \frac{3}{10} \quad \text{From equation (i) } x+y = 15$$

$$\Rightarrow xy = \frac{15\times10}{3} = 50$$

But $x - y = \pm\sqrt{(x+y)^2 - 4xy} = \pm\sqrt{15^2 - 4\times50} = \pm\sqrt{25} = \pm5$

$$\therefore x - y = 5 \ldots\ldots\ldots\ldots\ldots\ldots (iii) \qquad x - y = -5 \ldots\ldots\ldots\ldots\ldots (iv)$$

On solving equation (i) and (ii), we get x = 10, y = 5
Or solving equation (i) and (iv), we get x = 5, y = 10
Hence the two numbers are 10 and 5 …………………………… (Ans)

Exercise 1 (c)

1. Sum of two numbers is 137 and their difference is 43. Find the two numbers.

2. The lengths of the three sides of an equilateral triangle is x + 4cm., 4x-y cm and y+2 cm. Find the their lengths.

3. In a rectangle ABCD, if AB = 3x + y cm., BC = 3x + 2 cm., CD = 3y– 2x cm. and DA = y + 3 cm. find its area.

4. A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

5. The sum of two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

6. The sum of two numbers is 4 times of their difference. If the sum of the two numbers is 8, find the numbers.

7. The sum of two-digit number is 10; and the number formed by reversing the order of digits is one less than the twice of the original number. Find the number.

8. In a two-digit number, we obtain 2 when two times of the 2^{nd} number is subtracted from three times of the 1^{st} number. When we add 7 to the 2^{nd} number, the resultant is two times of the 1^{st} digit. Find the numbers.

9. A fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

10. A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get $\frac{18}{11}$, but if the numerator is increased by 8 and the denominator is doubled, we get $\frac{2}{5}$. Find the fraction.

11. 5 pens and 6 pencils cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

12. A father is three times as old as his son. In 12 years time, he will be twice as old as his son. Find the present ages.

13. The area of the rectangle is reduced by 9 cm^2 if its length is reduced by 5 cm and the breadth is increased by 3 cm. If we increase the length by 3 cm and breadth by 2 cm, the area is increased by 67 cm^2. Find the length and breadth of the rectangle.

14. Two men and three women together finish a piece of work in 5 days. Four men and nine women together finish the same work in 2 days. Find the time taken by one man to finish work and that taken by one man alone.

15. A and B can together can do a piece of work in 8 days. A left the work after working together for 3 days and the rest of the work is finished by B in 15 days. Find the time take by each when work alone.

16. The incomes of A and B are in the ratio of 8:7 and their expenditures in the ratio of 19:16. If each saves Rs 1250, find their incomes.

17. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

18. If in a rectangle, the length is increased by 2 m and breadth is reduced by 2 m, the area is reduced by 28 m^2. If however the length is reduced by 1 m and the breadth increased by 2m, the area increases by 33 m^2. Find the area of the rectangle.

19. Show 50 as sum of two numbers such that the sum of their reciprocals is $\frac{1}{12}$.

20. One third of the sum of the numerator and denominator of a fraction is 4 less than the denominator. If 1 is added to the denominator, the fraction reduces to $\frac{1}{4}$. Find the fraction.

CHAPTER 2 : QUADRATIC EQUATION

2.1 Introduction

P (x) = a x^2 + b x + c (a ≠ 0) is a quadratic polynomial where a, b are co-efficients of variables x^2 and x respectively and c is a constant. $ax^2 + bx + c = 0$, (a≠0) is known as quadratic equation.

We have learnt in our earlier classes about the linear equation i.e. ax + b = 0, (a ≠ 0). A quadratic equation can have at most two real roots where as linear equation has one.

Note :

In a equation have n number of roots, then the equation look $anx^2+an-1x^{n-1}+...+a1x+a0=0$, (an ≠ 0). It is known as **Fundamental Theorem of Algebra**.

In class IX you have leant about quadratic equation $ax^2 + bx + c = 0$, (a≠0).

Suppose $x^2 - 5x + 6 = 0$

=> $x^2 - 3x - 2x + 6 = 0$ => $x (x - 3) - 2 (x - 3) = 0$

=> $(x - 2) (x - 3) = 0$ => x = 2 or x = 3

Hence the roots are 2 and 3

for x= α, the value of quadratic polynomial $ax^2 + bx + c$ is zero. Hence α is known as zero of a polynomial. For an example 3 is 0 for a polynomial $3x^2 - 5x + 6$ as for x=3, the value of $ax^2 + bx + c=0$. In quadratic equation, 0 of an equation mean it has one root.

Every quadratic polynomial is related to a quadratic equation. Example $ax^2 + bx + c$ is related to $ax^2 + bx + c=0$, (a≠0).

2.2 Solution by completing the squares

Take the quadratic equation $ax^2 + bx + c=0$, a≠0

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ (Divide both the sides of the equation by a)

$\Rightarrow x^2 + 2.x.\frac{b}{2a} = -\frac{c}{a}$ ($\frac{c}{a}$ is brought to RHS of the equation)

$\Rightarrow x^2 + 2.x.\frac{b}{2a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$ ($\left(\frac{b}{2a}\right)^2$ is added to both sides)

$$\Rightarrow \left\{x^2 + 2.x.\frac{b}{2a} + \left(\frac{b}{2a}\right)^2\right\} = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2-4ac}{4a^2} \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \left(\pm\frac{\sqrt{(b^2-4ac)}}{2a}\right)^2$$

(Note that both the sides of the equation are changed into square roots

$$\Rightarrow x + \frac{b}{2a} = \pm\frac{\sqrt{(b^2-4ac)}}{2a} \Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{(b^2-4ac)}}{2a} = \frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$$

$$\Rightarrow x = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{or} \quad x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Hence $\alpha = \frac{-b+\sqrt{(b^2-4ac)}}{2a}$; $\beta = \frac{-b-\sqrt{(b^2-4ac)}}{2a}$

Alternate Method

$ax^2 + bx + c = 0 \ (a \neq 0)$

$\Rightarrow ax^2 + bx = -c$ (c has been brought to RHS)

$\Rightarrow 4a\,(ax^2 + bx) = -4ac$ (both sides of equation is multiplied by 4)

$\Rightarrow 4a^2x^2 + 4abx = -4\,ac$

$\Rightarrow (2ax)^2 + 2.\,2ax.\,b = -4ac$

$\Rightarrow (2ax)^2 + 2\,.\,2ax.\,b + b^2 = b^2 - 4ac$ (b^2 is add to both sides of equation)

$$\Rightarrow (2ax+b)^2 = \left(\pm\sqrt{(b^2-4ac)}\right)^2 \quad \text{(both sides are converted into root)}$$

$$\Rightarrow 2ax + b = \pm\sqrt{(b^2-4ac)} \Rightarrow 2ax = -b \pm\sqrt{(b^2-4ac)}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{(b^2-4ac)}}{2a} \Rightarrow x = \frac{-b + \sqrt{(b^2-4ac)}}{2a} \text{ or } x = \frac{-b - \sqrt{(b^2-4ac)}}{2a}$$

If quadratic equation is α β

$$\alpha = \frac{-b + \sqrt{(b^2-4ac)}}{2a} \quad \text{ଏବଂ} \quad \beta = \frac{-b - \sqrt{(b^2-4ac)}}{2a} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Hence it is known as Quadratic Formula

Example – 1

Solve the quadratic equation $6x^2 + 11x + 3=0$ by the method of completing the square.

Solution : we have a=6, b=11 and c=3

Multiply both sides of equation by 4a = 24, we get

$24 (6x2 + 11x) = (– 3) \times 24$

$\Rightarrow 144 x2 + 264x = – 72 \Rightarrow (12x)^2 + 2 (12x) \times 11 = – 72$

$\Rightarrow (12x)^2 + 2 (12x) \times 11 + (11)^2 = (– 72) + (11)^2$

$\Rightarrow (12x + 11)^2 = – 72 + 121 = 49 = (\pm 7)^2$

$\Rightarrow 12x + 11 = \pm 7 \Rightarrow 12x = – 11 \pm 7$

$\Rightarrow 12x = – 11 + 7$ or $– 11 – 7$

$\Rightarrow 12x = – 4$ or $– 18$

$\Rightarrow x = - \frac{4}{12} = -\frac{1}{3}$ or $-\frac{18}{12} = -\frac{3}{2}$

$\therefore$ roots of the equation are $-\frac{1}{3}$ and $-\frac{3}{2}$

Alternate method

The equation is $6x^2 + 11x + 3=0$

$\Rightarrow x^2 + \frac{11}{6}x + \frac{1}{2} = 0$ (dividing throughout by 6)

$$\Rightarrow x^2 + 2.x.\frac{11}{12} + \left(\frac{11}{12}\right)^2 = \left(\frac{11}{12}\right)^2 - \frac{1}{2}$$

$$\Rightarrow \left\{x^2 + 2.x.\frac{11}{12} + \left(\frac{11}{12}\right)^2\right\} = \frac{121}{144} - \frac{1}{2} = \frac{49}{144}$$

$$\Rightarrow \left(x + \frac{11}{12}\right)^2 = \left(\pm\frac{7}{12}\right)^2 \Rightarrow x + \frac{11}{12} = \pm\frac{7}{12}$$

$$\Rightarrow x = \frac{-11}{12} \pm \frac{7}{12} \Rightarrow x = \frac{-11}{12} + \frac{7}{12} \qquad \frac{-11}{12} - \frac{7}{12}$$

$$\Rightarrow x = -\frac{4}{12} = -\frac{1}{3} \qquad \frac{-18}{12} = \frac{-3}{2}$$

$\therefore$ roots of the equation are $-\frac{1}{3}$ and $-\frac{3}{2}$

Example 2

Using quadratic formula, solve the equation $x^2 + 2x – 63 = 0$ in terms of α and β.

Solution : we have a=1, b=2, and c = -63

$$\alpha = \frac{-b + \sqrt{(b^2-4ac)}}{2a} = \frac{-2 + \sqrt{(2^2 - 4 \times 1 \times (-63)}}{2 \times 1} = \frac{-2 + \sqrt{(4+252)}}{2} = \frac{-2+16}{2} = 7$$

$$\beta = \frac{-b - \sqrt{(b^2-4ac)}}{2a} = \frac{-2 - \sqrt{\{2^2 - 4 \times 1 \times (-63)\}}}{2 \times 1} = \frac{-2 - \sqrt{(4+252)}}{2} = \frac{-2-16}{2} = \frac{-18}{2} = -9$$

Hence $\alpha = 7$ and $\beta = -9$ …………………. (ans)

Discriminant

In a quadratic equation $ax^2 + bx + c=0$, $b^2 - 4ac$ is known as discriminant and it is denoted by 'D'. Hence $D = b^2 - 4ac$. When we consider the quadratic equation $ax^2 + bx + c=0$, where variable a, b, and c are whole numbers and $a\neq0$. If we consider the root of the equation as D then

$$\alpha = \frac{-b+\sqrt{(b^2-4ac)}}{2a} = \frac{-b+\sqrt{D}}{2a}$$

$$\beta = \frac{-b-\sqrt{(b^2-4ac)}}{2a} = \frac{-b-\sqrt{D}}{2a}$$

Nature of roots

Depending on determinants of quadratic equation, roots are obtained

i. If $D=b^2-4ac>0$, the α and β are real and they distinct i.e $\alpha\neq\beta$.
ii. If $D= b^2-4ac=0$, the α and β are real and they are one and coincident i.e $\alpha=\beta$
iii. If $D= b^2-4ac<0$, the α and β are not real roots of the given quadratic equation.

From the above discussion it is clear that roots are real and either distinct or coincident only when $D\geq0$.

Value of D	**Nature of roots**	**Roots are**
1. $D>0$	Roots are real and distinct	$\frac{-b+\sqrt{D}}{2a}$, $\frac{-b-\sqrt{D}}{2a}$
(i) Whole number	Roots are rational and not equal	
(ii) Not a whole number	Roots are irrational and not equal	
2. $D = 0$	Real (rational) and equal	$\frac{-b}{2a}$
3. $D<0$	Not a real number	

Example 3

Determine the nature of roots of quadratic equation $x^2 - 2x - 8=0$

Solution : here a = 1, b = – 2 and c = – 8

Determiner $D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-8) = 4 + 32 = 36$

Since the $D>0$, hence the roots are real and distinct. .... (Ans)

Note : since 36 is a whole number, roots are rational and not equal.

2.5 Relation between roots and coefficients

Suppose α and β are toots of a quadratic equation, $(a\neq0)$, where in

$$\alpha = \frac{-b+\sqrt{(b^2-4ac)}}{2a} \text{ and } \beta = \frac{-b-\sqrt{(b^2-4ac)}}{2a}$$

(a and b are coefficients of x2 and x wheras c is a constant

(I) SUM OF THE ROOTS

$$\alpha+\beta = \frac{-b+\sqrt{(b^2-4ac)}}{2a} + \frac{-b-\sqrt{(b^2-4ac)}}{2a}$$

$$= \frac{-b+\sqrt{(b^2-4ac)}-b-\sqrt{(b^2-4ac)}}{2a} = \frac{-2b}{2a} = \frac{-b}{a} = -\frac{\text{coefficient of x}}{\text{coefficient of x2}}$$

$$\alpha+\beta = \frac{-b}{a} \text{ i.e } \boxed{\text{sum of roots } = \frac{-b}{a}}$$

(II) product of roots

$$: \alpha\beta = \left[\frac{-b+\sqrt{(b^2-4ac)}}{2a}\right]\left[\frac{-b-\sqrt{(b^2-4ac)}}{2a}\right]$$

$$= \frac{(-b)^2-\left(\sqrt{(b^2-4ac)}\right)^2}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a} = \frac{\text{constant}}{\text{coefficient of x2}}$$

$$\alpha\beta = \frac{c}{a} \text{ i.e. } \boxed{\text{product of root } = \frac{c}{a}}$$

Example 4

If α and β are roots of a quadratic equation $25x^2 + 30x + 7=0$, $(a\neq0)$, find the value of α+β and αβ.

Solution : a=25, b=30 and c=7

$$\alpha+\beta = -\frac{b}{a} = -\frac{30}{25} = -\frac{6}{5} \text{ and } \alpha\beta = \frac{c}{a} = \frac{7}{25} \qquad \text{.................... (Ans)}$$

2.6 Some known results

Suppose $ax^2 + bx + c=0$, $a\neq0$ is a quadratic equation and its roots are α and β

$$\therefore \alpha+\beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

$$\text{(I) } \alpha-\beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\sqrt{\left\{\left(-\frac{b}{a}\right)^2 - 4\frac{c}{a}\right\}} = \pm\frac{\sqrt{(b^2-4ac)}}{a}$$

$$\text{(II) } \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2-2ac}{a^2}$$

$$\text{(III) } \alpha^2-\beta^2 = (\alpha+\beta)(\alpha-\beta)$$

$$= \left(-\frac{b}{a}\right)\frac{\sqrt{(b^2-4ac)}}{a} = \frac{-b\sqrt{(b^2-4ac)}}{a^2}$$

$$\text{(IV) } \alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$= \left(-\frac{b}{a}\right)^3 - 3\frac{c}{a}\left(-\frac{b}{a}\right) = \frac{-b^3+3abc}{a^3} = \frac{-b(b^2-3ac)}{a^3}$$

$$\text{(V) } \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-b}{a}\right)^2 - 2\frac{c}{a}}{\frac{c}{a}} = \frac{b^2-2ac}{ca}$$

Example 5

If α and β are roots of a quadratic equation $2x^2 - 6x + 3 = 0$, prove that

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+3\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+2\alpha\beta = 13$$

Solution : a=2, b=-6, c=3

$$\therefore \alpha+\beta = -\frac{b}{a} = \frac{-(-6)}{2} = 3 \text{ and } \alpha\beta = \frac{c}{a} = \frac{3}{2}$$

$$\text{Now } \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{3}{\left(\frac{3}{2}\right)} = \frac{3\text{x}2}{3} = 2 \text{ and } \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$= \frac{(3)^2 - 2\text{x}\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)} = \frac{(9-3)\text{x}2}{3} = \frac{12}{3} = 4$$

$$\therefore \text{LHS} = \frac{\alpha}{\beta}+\frac{\beta}{\alpha}+3\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+2\alpha\beta = 4 + (3\text{ x}2) + 2\text{ x}\left(\frac{3}{2}\right)$$

= 4 + 6 + 3 = 13 = RHS .... (Ans)

2.7 Formation of a quadratic equation :

If α and β are roots of a quadratic equation $2x^2 - 6x + 3 = 0$, prove that $\alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now $ax^2 + bx + c=0 \Rightarrow x^2 + \frac{b}{a} + \frac{c}{a} = 0$ (dividing both sides with a)

⇨ $x^2 - (-\frac{b}{a})x + \frac{c}{a} = 0 \Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$

Hence $x^2 -$ (sum of roots)x + product of roots = 0

Note : It is possible to formulate a quadratic equation provided you know the roots.

Example 6 : The sum of the roots of a quadratic equation is -5 and their product is 3. Find the equation.

Solution : Here $\alpha+\beta = -5$ and $\alpha\beta = 3$

Required equation = $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

⇨ $x^2 - (-5x) + 3 = 0 \Rightarrow x^2 + 5x + 3 = 0$

.................................(Ans)

Example 7 : If the product of roots of the following quadratic equation $ax^2 - 4x + (4a + 1) = 0$ is 2, find the value of a.

Solution : Suppose the roots are α and β.

Here $\alpha\beta = \frac{(4a+1)}{a} = 2 \Rightarrow 4a + 1 = 2a \Rightarrow 4a - 2a = -1 \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$ .... (Ans)

Example 8 : If sum of the roots and its products are equal for a quadratic equation $ax^2+4x+6a=0$, $a\neq0$, find the value of a.

Solution : here $\alpha+\beta = \frac{Coefficient\ of\ x}{coeffincient\ of\ x2} = -\frac{4}{a}$, $\alpha\beta = \frac{variable\ without\ x}{coeffincient\ of\ x} = \frac{6a}{a} = 6$

As per question : $\alpha+\beta = \alpha\beta \Rightarrow -\frac{4}{a} = 6 \Rightarrow a = -\frac{4}{6} = -\frac{2}{3}$ ....(Ans)

Exercise 2 (a)

1. Correct the following statement of the given quadratic equations
   - i) $x^2-4x+4=0$, the roots are real and distinct.
   - ii) $x^2-5x+6=0$, 2 is the discriminant.
   - iii) $ax^2+bx+c=0$, the sum of the roots is $\frac{c}{a}$
   - iv) $ax^2+bx+c=0$, the product of the roots is $\frac{b}{a}$
   - v) 1 and -1 is the roots of quatdratic equation $x^2+1=0$
   - vi) $x^2=0$, roots are not equal.
   - vii) $3x^2-2x-1=0$, the sum of roots is $-\frac{3}{2}$.
   - viii) $3x^2-2x-1=0$, the product of roots is $\frac{1}{3}$.

2. **Give short answer for the following questions**
   - i) If 3 and -5 is the roots of a quadratic equation, find the equation.
   - ii) If product of the roots of a quadratic equation, $mx^2-2x+(2m-1)=0$, find the value of m.
   - iii) If a root of a quadratic equation, $x^2-px+2=0$ is 2, find the value of p.
   - iv) If the roots of a quadratic equation, $4x^2+2x+c=0$ is unique and distinct, find the value of c.
   - v) If a root of a quadratic equation, $5x^2+2x+k=0$ is -2, find the value of k.
   - vi) If the roots of a quadratic equation, $2x^2+kx+3=0$ is real and equal, find the value of k.

3. **Choose the correct option**
   - i) Which of the following is correct quadratic equation of x
     (a) $x^2-x-12=0$ (b) $x^2+\frac{1}{x2}=3$ (c) $x+\frac{1}{x}=x^2$ (d) $x(x-1)(x+5)=0$
   - ii) Find the nature of roots for the equation $7x^2+9x+2=0$.
     (a) Real and distinct (b) real and unique (c) not real (d) none of these
   - iii) Which of the following equation have root -6 and 8
     (a) $(x+6)(x+8) = 0$ (b) $(x+6)(x-8) = 0$ (c) $(x-6)(x+8) = 0$ (d) $(x-6)(x-8) = 0$
   - iv) If the roots of the equation $3x^2+2\sqrt{2}x-5=0$ is α and β, find the product of αβ.
     (a) 3 (2) $2\sqrt{5}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{-5}{3}$
   - v) If α and β are roots of the equation $4x^2-2x+\frac{1}{4}=0$, find the value of α+β.
     (a) $\frac{1}{16}$ (b) 4 (c) $\frac{1}{2}$ (d)-8
   - vi) If α and β are roots of the equation $4x^2+3x+7=0$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.
   - vii) The sum of the roots and its product of a quadratic equation is 4 and $\frac{5}{2}$ respectively, find the equation
     (a) $2x^2+8x+5=0$ (b) $2x^2-8x+5=0$ (c) $2x^2+8x-5=0$ (d) $2x^2-8x-5=0$

4. **Solve the given quadratic equations by the method completing the square.**
   (i) $x^2 + x - 6 = 0$
   (ii) $2x^2 - 9x + 4 = 0$
   (iii) $14x^2 + x - 3 = 0$
   (iv) $3x^2 - 32x + 12 = 0$
   (v) $x^2 + 2px - 3qx - 6pq = 0$
   (vi) $\sqrt{3}x^2 + 10x + 8\sqrt{3} = 0$
   (vii) $25x^2 + 30x + 7 = 0$
   (viii) $3a^2x^2 + 8abx + 4b^2 = 0$ $(a\neq0)$
   (ix) $x^2 + ax + b = 0$
   (x) $x^2 + bx = a^2 - ab$

5. **Solve the following equation using the quadratic formula.**
   (i) $4x^2 - 11x + 6 = 0$
   (ii) $(2x - 1)(x - 2) = 0$
   (iii) $x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$
   (iv) $a(x^2+1) = x(a^2 + 1)$, $a\neq0$
   (v) $6x^2 + 11x + 3 = 0$
   (vi) $2x^2 + 41x - 115 = 0$
   (vii) $12x^2 + x - 6 = 0$
   (viii) $(6x + 5)(x - 2) = 0$
   (ix) $15x^2 - x - 28 = 0$
   (x) $(x + 5)(x - 5) = 39$

6. If one root of a quadratic equation $4x^2 - 13x + k = 0$ is 12 time more than the other, find the value of k.

7. If one root of a quadratic equation $x^2 - 5x + p = 0$ is 4 more than the other, find the value of p.
8. If α and β are roots of the equation $2x^2-5x+3=0$, find the value of $\alpha^2\beta+\alpha\beta^2$.
9. If α and β are roots of the equation $2x^2-6x+3=0$, find the value of $(\alpha+1)(\beta+1)$.
10. If the difference between the roots of the quadratic equation $2x^2-(p+1)x+p-1=0$ and it product is equal, find the value of p.
11. If α and β are roots of the equation $5x^2-3x-3=0$, prove that $\alpha^3+\beta^3=\frac{117}{125}$.
12. If α and β are roots of the equation $5x^2+17x+6=0$, find the value of $\frac{1}{\alpha 2} + \frac{1}{\beta 2}$.
13. If α and β are roots of the equation $x^2-8x+16=0$, find the value of $\frac{\alpha\beta}{\alpha+\beta}$ in the form of p.
14. Find the value of m, if the roots of the quadratic equation $x^2-2(5+2m)x+3(7+10m)=0$ is unique.

15. (i) If a=b=c=0 prove that the roots of given equation are real and unique $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$, .
 (ii) If a+b+c=0 and a, b, c ϵ Q, prove that the roots of given equation are rational. $(b+c-a)x^2 + (c+a-b)x + (a+b+c) = 0$
16. The sum of the roots of the quadratic equation is 3 and sum of its squares are 29, find the equation.

17. If α and β are roots of the equation $2x^2-4x+2=0$, prove that $\frac{\alpha}{\beta} + \frac{\alpha}{\beta} + 4(\frac{1}{\alpha}+\frac{1}{\beta})+2\alpha\beta=12$
18. (i) If one root of the quadratic equation $ax^2-bx+c=0$ is 4 times more than the other, prove $4b^225ac$.
 (ii) If one root of the quadratic equation $x^2-px+q=0$ is 2 times more than the other, prove $4p^2=9q$.

19. (i) If the roots of the quadratic equation $41x^2-2(5a+4b)x+(a^2+b^2)=0$, prove that $\frac{a}{b}=\frac{5}{4}$
 (ii) If sum of roots of a quadratic equation $x^2-px+q=0$ is equal to sum its squares, prove $2q=p(p+1)$
 (iii) If one root of a equation $x^2-px+q=0$ is square of other, find $p^3+q^2+q=3pq$.

20. If the roots of the equation $a(b-c)x^2+b(c-a)x + c(a-b) =0$ is equal prove $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

2.8 Equations reducible to quadratic form

Few equations which are not quadratic i.e. not in the form of quadratic equation $ax^2-bx+c=0$. We have to convert the unknown variable in such a way to form quadratic equation. Some examples are given below.

Example 9 : Find the roots of the following quadratic equation $4x^4-21x^2+20=0$

In the above equation power of the variable x is 4, hence it is not a quadratic equation.

We can write $x^2 = y$ hence the equation can be written as

$4x^4-21y+20=0$ .... (i)

Now it become a quadratic equation

Using quadratic equation, we first solve the equation

In equation (i) a=4, b=-21 and c=20

Determinant (D) = $b^2-4ac = (-21)^2 - 4 \times 4 \times 20 = 441 - 320 = 121$

$\therefore y = \frac{-b\pm\sqrt{D}}{2a} = \frac{-(-21)\pm\sqrt{121}}{2x4} = \frac{21\pm11}{8} = \frac{21+11}{8}$ or $\frac{21-11}{8} = 4$ or $\frac{5}{4}$

$y = \frac{5}{4} \Rightarrow x^2 = \frac{5}{4} \Rightarrow x=\pm \frac{\sqrt{5}}{4}$

Hence required roots are 2, -2, $\frac{1}{2}\sqrt{5}$, -$\frac{1}{2}\sqrt{5}$, or ±2,, $\pm\frac{1}{2}\sqrt{5}$ ....(Ans)

Example 10

Solve : $4\left(x-\frac{1}{x}\right)^2+8(x+\frac{1}{x})-29=0$

Solution : $\left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4$

$\therefore$ Given equation $\Rightarrow 4\left(x-\frac{1}{x}\right)^2+8(x+\frac{1}{x})-29=0$

$$\Rightarrow 4\left\{\left(x+\frac{1}{x}\right)^2-4\right\}+8\left(x+\frac{1}{x}\right)-29=0$$

$$\Rightarrow 4\left(x+\frac{1}{x}\right)^2+8\left(x+\frac{1}{x}\right)-45=0 \quad ....(i)$$

Now $x+\frac{1}{x}=y$ hence the equation can be $4y^2+8y-45=0$

now $a=4, b=8$ ଓ $c=-45$ । Now to find the value of determinant D,

$$(D)=b^2-4ac=(8)^2-4\times4\times(-45)=64+720=784=(28)^2$$

$\therefore$ Root is rational and not equal ($\because$ D is a square intiger

$\therefore\ y=\frac{-b\pm\sqrt{D}}{2a}=\frac{-8\pm\sqrt{784}}{2\times4}=\frac{-8\pm28}{8}=\frac{-8+28}{8}$ or $\frac{-8-28}{8}=\frac{5}{2}$ or $\frac{-9}{2}$

ଯଦି $x+\frac{1}{x}=\frac{5}{2}$ ହୁଏ, ତେବେ $2x^2-5x+2=0\Rightarrow x=\frac{-(-5)\pm\sqrt{25-16}}{4}=\frac{5\pm3}{4}$

$$=\frac{5+3}{4} \qquad \frac{5-3}{4}=2 \qquad \frac{1}{2}$$

Hence $x+\frac{1}{x}=\frac{-9}{2}$ but $2x^2+9x+2=0\Rightarrow x=\frac{-9\pm\sqrt{81-16}}{4}=\frac{-9\pm\sqrt{65}}{4}$

(Roots are irrational and not equal)

$\therefore$ The roots of the equation : $2, \frac{1}{2}, \frac{-9+\sqrt{65}}{4}, \frac{-9-\sqrt{65}}{4}$ (Ans)

Example 11

Solve : $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$

Solution : Let $\sqrt{\frac{x}{1-x}}=y$

The given equation will be $y+\frac{1}{y}=\frac{13}{6}\Rightarrow 6y^2-13y+6=0$

$$\therefore\ y=\frac{-(-13)\pm\sqrt{(-13)^2-4\times6\times6}}{2\times6}=\frac{13\pm\sqrt{169-144}}{12}=\frac{13\pm\sqrt{25}}{12}=\frac{13\pm5}{12}$$

$$\therefore\ y=\frac{18}{12}\qquad\frac{8}{12}\ \Rightarrow y=\frac{3}{2}\qquad y=\frac{2}{3}$$

Now $y=\frac{3}{2}\Rightarrow\sqrt{\frac{x}{1-x}}=\frac{3}{2}\Rightarrow\frac{x}{1-x}=\frac{9}{4}\Rightarrow 4x=9-9x\ \Rightarrow 13x=9\ \Rightarrow x=\frac{9}{13}$

For $y=\frac{2}{3}\ \Rightarrow\sqrt{\frac{x}{1-x}}=\frac{2}{3}\Rightarrow\frac{x}{1-x}=\frac{4}{9}$

$\Rightarrow 9x=4-4x\ \Rightarrow 13x=4\ \Rightarrow x=\frac{4}{13}$ ।

$\therefore$ The resultant roots $\frac{9}{13}$ ও $\frac{4}{13}$ । ....(Ans)

Example 12 :

Solve : $x(x+5)(x+7)(x+12)+150=0$

Solution : Given equation $x(x+5)(x+7)(x+12)+150=0$

$\Rightarrow \{x(x+12)\}\{(x+5)(x+7)\}+150=0$

$\Rightarrow (x^2+12x)(x^2+12x+35)+150=0$

$\Rightarrow y(y+35)+150=0$ Here $x^2+12x=y$

$\Rightarrow y^2+35y+150=0$

$\Rightarrow y=\frac{-35\pm\sqrt{(35)^2-4\times1\times150}}{2\times1}$ Using quadratic equation

$=\frac{-35\pm\sqrt{1225-600}}{2}=\frac{-35\pm\sqrt{625}}{2}=\frac{-35\pm25}{2}=-5$ or -30

$y=-30\ \Rightarrow x^2+12x=-30\ \Rightarrow x^2+12x+30=0$

$\Rightarrow x=\frac{-12\pm\sqrt{144-120}}{2}=\frac{-12\pm\sqrt{24}}{2}=\frac{-12\pm2\sqrt{6}}{2}=-6\pm\sqrt{6}$

again $y=-5\ \Rightarrow x^2+12x=-5\ \Rightarrow x^2+12x+5=0$

$\Rightarrow x=\frac{-12\pm\sqrt{144-20}}{2}=\frac{-12\pm\sqrt{124}}{2}=\frac{-12\pm2\sqrt{31}}{2}=-6\pm\sqrt{31}$

$\therefore$ The roots of given equation $-6+\sqrt{6},\ -6-\sqrt{6},\ -6+\sqrt{31},\ -6-\sqrt{31}$

$-6\pm\sqrt{6},\ -6\pm\sqrt{31}$ । ....(Ans)

Application of quadratic equation

In this section, we will discuss some simple problems on practical applications of quadratic equation. in this type of problems we first formulate quadratic equation whose solution is a solution of the given problem. sometimes it may happen that, out of the roots of the quadratic equation only one has a meaning for the problem. Any root of the quadratic equation, which does not satisfy the condition of the problem will be rejected.

In order to solve this type of problems, we may use the following algorithm.

- Translate the word problems into symbolic language and formulate the quadratic equation.
- Solve the quadratic equation formed in Step 1
- Translate the solution into verbal language and reject the solution which does not have a meaning for the problem.

Example 13 : A sum of a number and its positive square root is 90. Find the number.

Solution : Let the number be x^2.

$\therefore$ x is the positive square root of x^2

As per question $x^2+x=90 \Rightarrow x^2+x-90=0 \Rightarrow x^2+10x-9x-90=0$

$\Rightarrow x(x+10)-9(x+10)=0$

$\Rightarrow (x+10)(x-9)=0 \Rightarrow x=-10$ or $x=9$

As it is positive square root, and the value of x = -10 is negative, hence equation is not possible whereas x=9

∴ the number is $x^2=9^2=81$ .... (Ans)

Alternate solution

Let the number be x

∴ The positive square root is $\sqrt{x}$

As per the question $x+\sqrt{x}=90$ => $\sqrt{x}=90-x$

- ⇨ $x = (90-x)^2$ =>$x = 8100 – 180x + x^2$ =>$x^2-181x + 8100 =0$
- ⇨ $x^2-100x-81x+8100=0$ => $(x-100)(x-81)=0$
- ⇨ x-100=0 or x-81=0 => x=100 or x=81

x=100, hence the equation is not possible but x=81, the equation can be possible.

∴ the number is 81 .... (Ans)

Example 14 : The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the numbers.

Solution : let the two numbers be x and (15-x)

As per the question : $\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$ => $\frac{15-x+x}{x(15-x)}=\frac{3}{10}$ => $\frac{15}{15x-x2}=\frac{3}{10}$

- ⇨ $150 = 45x – 3x^2$ => $3x^2 – 45x + 150 = 0$
- ⇨ $x^2 – 15x + 50 = 0$

$= x^2 – 10x – 5x + 50 = 0$ => $(x-10)(x-5) = 0$ => x = 10 or x = 5

Hence the two numbers are 10 and 5 .... (Ans)

Example 15 : The speed of a boat in still water is 11 Km/hr. It can go 12 km upstream and return downstream to the original point in 2 hrs and 45 min. Find the speed of boat per hour.

Solution : let the speed of the stream be x km/hr Then,

Speed of the boat in upstream (11-x) km/hr

Speed of the boat in downstream (11+x) km/hr

∴ time taken by boat to go 12 km upstream $= \frac{12}{11-x}$

time taken by boat to go 12 km downstream $= \frac{12}{11+x}$

As per question $\frac{12}{11-x}+\frac{12}{11+x} = 2\frac{3}{4}$ (45 min = $\frac{45}{60}=\frac{3}{4}$)

- ⇨ $\frac{12(11-x)+12(11+x)}{121-x2}=\frac{11}{4}$ => $\frac{264}{121-x2}=\frac{11}{4}$ => $\frac{24}{121-x2}=\frac{1}{4}$

- ⇨ $121-x^2 = 96$ => $x^2 = 25$ => x =5

∴ Hence the speed of the stream 5 km/hr

Exercise - 2(b)

1. Answer the following question

i) The sum of a number and its reciprocal is 2. Form the quadratic equation taking the number as x.

ii) The product of two consecutive whole numbers is 20. Form a quadratic equation taking one number as y.

iii) The sum of two numbers is 18 and their product is 72. Form a quadratic equation taking one number as x.

iv) Find the number whose square is equal to its number.

v) The sum of the first n cardinal numbers S=$\frac{n(n+1)}{2}$. If s=120, form a quadratic equation to find the value of x

vi) Convert $\sqrt{x} + x = 6$ into a quadratic equation.

vii) Convert $\sqrt{x+9}$ + 3 = x into a quadratic equation.

viii) Convert x-2$\sqrt{2}$ - 6 = x into a quadratic equation.

2. Solve the following word problems.

i) A positive number is 12 more than its square, find the number.

ii) Sum of a number and its reciprocal is $\frac{41}{20}$, find the number.

iii) If sum of the reciprocals of two consecutive whole numbers is $\frac{11}{30}$, form the equation and find the number.

iv) Find the numbers if the sum of reciprocal of two consecutive whole numbers is $\frac{23}{132}$.

v) Divide 51 into two parts so that the product of two parts is 378. Find the numbers.

3. A two digit number is 4 times the product of its digits. The number in units place is two more than the digit in tens place. Find the number.

4. In a family, the age of α is product of β and δ. If β is elder than δ by 1 year and the age of α is 42 years, find the age of β after 5 years.

5. Square of $\frac{1}{8}$th of total number of monkeys living in a forest are playfull and rest 12 monkeys sit on the top. Find the total number of monkeys

6. The area of a triangle is 30 cm^2. If altitude (height) of the triangle exceeds the length of its base by 7 cm., find the length of the base.

7. The length of the sides forming right angle of a right angled triangle are 5x amd (3x-1). If the area of the triangle is 60 cm^2. Find the length of its sides.

8. The number of diagonals of n numbered polygon is $\frac{1}{2}n(n-3)$. If the polygon has 54 diagonals, find the number of sides of the polygon.

9. The sum of areas of two squares is 468 m^2 and the difference between the perimeters is 24 m. Find the squares of the two squares.

10. If a man increases his walking speed by 1 km/hr, he takes 10 min less to cover a distance of 2 km., find the speed of the man.

11. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.

12. Rs. 250 is equally distributed among certain number of students. Had there been 25 more students, each would have got Rs. 0.50 less. Find the total number of students.

13. The length of a rectangle exceeds its width by 8 cm and the area of the rectangle is 240 cm^2. Find the perimeter (dimensions) of the rectangle.

14. A passenger train takes 2 hours less for a journey of 300 km if its speed increased by 5 km/hr from its usual speed. Find the usual speed of the train.

15. A 25m long and 16m broad rectangular field is surrounded by a path of equal breadth. If the area of the path is 230 m^2, find the breadth of the path.

16. Some students planned a picnic. The budget for food was Rs. 480. But 8 of these failed to go and thus the cost of food for each member increased by Rs 10. How many students attended the picnic?

17. Solve the following

(i) $(x+1)(x+2)(x+3)(x+4) = 120$

(ii) $5\sqrt{\frac{3}{x}} + 7\sqrt{\frac{x}{3}} = 22\frac{2}{3}$

(iii) $3x + \frac{5}{16x} - 2 = 0$

(iv) $\left(\frac{2x+1}{x+1}\right)^4 - 6\left(\frac{2x+1}{x+1}\right)^2 + 8 = 0$

(v) $(3x^2 - 8)^2 - 23(3x^2 - 8) + 76 = 0$

(vi) $5(5^x + 5^{-x}) = 26$

(vii) $(x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = 0$

(viii) $x^{-4} - 5x^{-2} + 4 = 0$

(ix) $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0$

(x) $\frac{3}{\sqrt{2x}} - \frac{\sqrt{2x}}{5} = 5\frac{9}{10}$

(xi) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$ $(x \neq 0, x \neq -1)$

(xii) $x(2x+1)(x-2)(2x-3) = 63$

(xiii) $\frac{x-3}{x+3} - \frac{x+3}{x-3} = 6\frac{6}{7}$ $(x \neq -3, 3)$

(xiv) $3\left(x^2 + \frac{1}{x^2}\right) + 4\left(x - \frac{1}{x}\right) - 6 = 0$

(xv) $\left(\frac{x+1}{x-1}\right)^2 - \left(\frac{x+1}{x-1}\right) - 3 = 0$

(xvi) $\sqrt{2x+9} + x = 13$

(xvii) $\sqrt{2x + \sqrt{2x+4}} = 4$

■■■

CHAPTER 3 : ARITHMETIC PROGRESSION

An arrangement of numbers in a definite order according to some rule is known as Sequence.

For example 2, 4, 6, 8..............; 1, 3, 5, 7 ....;

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ .... ; 2, 6, 18, 54...............

Each number of a sequence is known as term. Hence depending up on two to three terms, we can get rest of the terms of a sequence. Generally it is written as $t_1, t_2, t_3, t_4, \ldots t_n$. Hence, $t_1, t_2, t_3, t_4, \ldots$ etc Here, the subscripts denote the positions of the terms i.e. term1, term2 and so on. In general, the number at the nth place is called nth term of the sequence and is denoted by t_n. This nth term is also called the general term of the sequence.

If $t_{n+1} = t_{n+2} = \ldots = 0$ then the sequence is $t_1, t_2, t_3, t_4, \ldots t_n$ and it is a infinite term. Hence most of the sequences are infinite. As per certain rules the sequence is classified is called progression. There are three types of progressions :

1. Arithmetic progression
2. Geometric progression
3. Harmonic progression

3.2 Arithmetic Progression (A.P.) :

Arithmetic progression is also known as A.P. In this section, we shall discuss a particular type of sequences in which each term, except the first, progress in a definite manner. The difference between the terms is known as common difference, is denoted by d.

Hence, for A.P. - $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots = t_n - t_n - 1 = d$

3.2.1 nth Term of Arithmetic Progression

The first term of any Arithmetic progression is **a** and common difference is **d**, then the sequence is as follows :

$$t_1 = a$$
$$t_2 = a + d = a + (2 - 1)\,d$$
$$t_3 = a + 2d = a + (3 - 1)\,d$$
$$t_4 = a + 3d = a + (4 - 1)\,d$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$t_n = a + (n - 1)\,d$$

hence AP can generally represented as a, a + d, a + 2d, a + 3d, ..., a + (n –1)d

$\therefore$ for nth term = $\boxed{t_n = a + (n - 1)d}$ Note : in AP a is first term and d is the difference.

Example 1 : Each number given below is a AP

(i) –18, –16, –14, – 12.......

If the first term a = –18, common difference d = – 16 – (–18)= –14–(–16) = –12 – (–14)=2

(ii) –11, 0, 11, 22, 33, 44.............

If the first term a = –11, common difference d = 0 – (–11) = 11 –0 = 22 –11 = 11

(iii) $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \ldots\ldots\ldots\ldots$

If the first term $a = \frac{1}{3}$, common difference $d = \frac{2}{3} - \frac{1}{3} = 1 - \frac{2}{3} = \frac{4}{3} - 1 = \frac{1}{3}$

For nth position $\mathbf{t_n}$ for above equation

(i) $t_n = -18 + (n - 1)\,2 = -18 + 2n - 2 = 2n - 20$ ($\because t_n = a + (n - 1)d$)

(ii) $t_n = -11 + (n - 1)11 = -11 + 11n - 11 = 11n - 22$

(iii) $t_n = \frac{1}{3} + (n - 1)\,\frac{1}{3} = \frac{1}{3} + \frac{1}{3}n - \frac{1}{3} = \frac{1}{3}n$

As per above equations, we can find nth position i.e. t_n by finding the values of a and n.

Tenth term of AP of the 1st equation = $t_{10} = -18 + (10 - 1)2 = -18 + 18 = 0$

3.2.2 Addition of n numbered term of A.P.

The formula of first n numbered term addition is found by famous German Mathematician Gauss in his childhood. He was asked by his teacher to add numbers from 1 to 100. His teacher thought that Gauss will take lot of time to save and keep quite in classroom. He took very less time to solve it. He used the method which is given below :

Addition of 1 to 100 numbers i.e S_{100}

$$S_{100} = 1 + 2 + 3 + ... + 98 + 99 + 100$$
$$S_{100} = 100 + 99 + 98 + ... + 3 + 2 + 1$$

On addition $2S_{100} = 101 + 101 + 101 + ... + 101 + 101 + 101$

$$2S_{100} = 101 \text{ x } 100 \Rightarrow S_{100} = \frac{101x100}{2} = 5050$$

Now we will add a, a + d, a + 2d, a + 3d, upto n terms.

The nth position $t_n = a + (n - 1)\, d = l$, prior to l is $l-d$ and prior to it is $l-2d$ etc.

Hence till nth position

$\therefore S_n = a + (a + d) + ... + (l - d) + l$

$S_n = l + (l - d) + ... + (a + d) + a$ (terms are written reverse order (backward count))

On addition $2S_n = (a + l) + (a + l) + ...$ nth position

$\therefore 2S_n = n\,(a + l) \qquad \therefore S_n = \frac{n}{2}(a + l)$

$\therefore$ Formula to find sum of n terms of AP $\boxed{S_n = \frac{n}{2}(a + l)}$

i.e $\boxed{S_n = \frac{n}{2}\text{(First term + last term)}}$

If we consider the formula $l = a + (n - 1)\, d$

$$S_n = \frac{n}{2}\{a + a + (n - 1)\, d\} \Rightarrow S_n = \frac{n}{2}\{\, 2a + (n - 1)\, d\}$$

$\therefore$ Alternative formula to find the sum of n terms $\boxed{S_n = \frac{n}{2}\{\, 2a + (n - 1)\, d\}}$

Note 1: Sum of first n numbered terms $S_n = \frac{n}{2}(a + l)$

Note 2: If first term is a and common difference d=0, each a,a,a,a…………. till n, i.e S_n = a+a+a+.... n, the AP will be $S_n = na$.

Note 3: In an Arithmetic Progression

(i) If we add same number to all the terms
(ii) If we subtract same number from all the terms
(iii) If we multiply every term with same number except 0
(iv) If we divide every term with same number except 0

then the resultants of every term of sequence will remain same in arithmetic progression.

Prove : Suppose the first term of AP is a and the common difference is d

Then the AP is a, a+d, a + 2d, ..., a+(n–1)d,

to prove (i) of note 3 -> k is added to every term then the sequence will be

(a +k), (a+k) + d, (a+k) + 2d, ..., (a+k) + (n–1)d

In the same way (ii) and (iii) can be proved.

Example 2 :

(a) Find the sum of the numbers from 15 to 85 in reverse order method.
(b) If 4 is the first term and common difference is 3, find
 (i) Write AP
 (ii) Find 33rd term i.e. t_{33}
 (iii) Find sum of first 40 terms (s_{40}) of AP

Solution : the total number of numbers present in between the numbers 15 and 85 are

(85-17)+1 = 71

The sum of these 71 numbers = S_{71}.

S_{71} = 15 + 16 + 17 + 18 +.... + 83 + 84 + 85

S_{71} = 85 + 84 + 83 + 82 +.... + 17 + 16 + 15 (in reverse order)

$2S_{71}$ = 100 + 100 + 100 + 100 ..... + 100 + 100 + 100

$\therefore 2S_{71} = 100 \times 71$

$\Rightarrow S_{71} = \frac{100x71}{2} = = 50 \times 71 = 3550$ ……….. (Ans)

Using formula $S_n = \frac{n}{2}(a + l)$

$S_{71} = \frac{71}{2}(15 + 85) = 50 \times 71 = 3550$

(here, first term is a=15 and last term l=85)

(b) (i) A. P. = 4, 7, 10, 13, 17, ... [$\because$ a = 4 and d = 3]

(ii) $t_{33} = 4 + (33 - 1) \times 3 = 100$ [$\because t_n = a + (n-1)d$]

(iv) Sum of first 40 terms of AP (S_{40}) = $S_n = \frac{40}{2}\{ 2x4 + (40 - 1) 3\} = 20(8+117)$

$\Rightarrow S_{40} = 20 \times 125$ ($\because S_n = \frac{n}{2}\{ 2a + (n - 1) d\}$)

$\Rightarrow S_{40} = 2500$ ………………..(Ans)

Example 3 : In a AP, if $t_4 = 11$, $t_{10} = 16$, find t_{21} and sum of first 40 numbers.

Solution : Suppose first term = a and common difference = d

Given $t_4 = 11 \Rightarrow a + (4 - 1) d = 11 \Rightarrow a + 3d = 11$ .... (1)

$t_{10} = 16 \Rightarrow a + (10 - 1) d = 16 \Rightarrow a + 9d = 16$ .... (2)

from eqn (1) and (2) => $(a + 9d) - (a + 3d) = 16 - 11 \Rightarrow 6d = 5 \Rightarrow d = \frac{5}{6}$

Now putting the value of d in eqn 1 and 2, we have

Eqn 1 = $a + 3 \times \frac{5}{6} = 11 \Rightarrow a = 11 - \frac{5}{2} = \frac{17}{2}$

$\therefore t_{21} = a + (21 - 1) d \Rightarrow \frac{17}{2} + (20) \times \frac{5}{6} \Rightarrow \frac{151}{6} = 25\frac{1}{6}$ …… (Ans)

Using formula $S_n = \frac{n}{2}\{ 2a + (n - 1) d\}$

$S_{40} = \frac{40}{2}\{ 2 \times \frac{17}{2} + (40 - 1) \frac{5}{6} \}$

$S_{40} = 20 \{ 17 + \frac{65}{2} \} = 340 + 650 = 990$ ……..(Ans)

Example 4 : Find S_{50} in a sequence of 2, 4, 6, 8, ...

Solution : in the above question $t_2 - t_1 = 4 - 2 = 2$, $t_3 - t_2 = 6 - 4 = 2$, $t_4 - t_3 = 8 - 6 = 2$ ..etc $\therefore$ sequence is AP, where a = 2 and common difference d=2

$\therefore S_n = \frac{50}{2}\{ 2x2 + (50 - 1) 2\} = 2550$ ($\because S_n = \frac{n}{2}\{ 2a + (n - 1) d\}$) ……(Ans)

Example : 5

How many terms of the series 27 + 24 + 21+ ...... be taken so that their sum is 132. Explain the double answer.

Solution : First term a=27 and common difference = -3 (d = 24 – 27 = 21 – 24 = – 3)

Suppose the term number is n = 132 $\therefore S_n = 132$

=> $S_n = \frac{n}{2}\{ 2a + (n - 1) d\}$ = 132 => $S_n = \frac{n}{2}\{ 2x27 + (n - 1) -3\} = 132$

=> $\frac{n}{2}(57 - 3n) = 132 \Rightarrow n(57 - 3n) = 264 \Rightarrow -3n^2 + 57n - 264 = 0$

=> $n^2 - 19n + 88 = 0 \Rightarrow (n - 11)(n - 8) = 0$

=> n=11 or n=8

$\therefore$ The sum of 11 term of AP or sum of 8 terms of AP is 132

Hence, it can be represented as

$t_9 = 27 + (9 - 1)(-3) = 3$, $t_{10} = t_9 + d = 3 + (-3) = 0$

$t_{11} = t_{10} + d = 0 + (-3) = -3$

$\Rightarrow t_9 + t_{10} + t_{11} = 3 + 0 + (-3) = 0$

$\Rightarrow S_{11} = S_8 + t_9 + t_{10} + t_{11} = S_8 + 0 = S_8$

Hence if we add 8 or 11 term of AP, the total will be 132.

Example 6 : If $t_n = 2n + 3$ is the sequence, find S_n.

Solution : Replace n with 1 from both sides.

$t_1 = 2 \times 1 + 3 = 5 \Rightarrow a = 5$

similarly, n will be replace by 2 and 3 and so on

$t_2 = 2 \times 2 + 3 = 7$ and $t_3 = 2 \times 3 + 3 = 9$

$t_3 - t_2 = 9 - 7 = 2$ and $t_2 - t_1 = 7 - 5 = 2 \therefore t_3 - t_2 = t_2 - t_1 = 2$

From the above solution, the common difference of the terms is d=2

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 5 + (n-1) \times 2]$

$= \frac{n}{2}(10+2n-2) = \frac{n}{2}(2n+8) = n(n+4) = n^2 + 4n$ ………… (Ans)

Note : take a number assign it to n and we can find the sum of n term i.e. if n=30, we can find S_{30}.

$\therefore S_{30} = 30^2 + 4 \times 30 = 900 + 120 = 1020$

Example 7 : If Sn, the sum of first n terms of an AP, is given by $S_n = 3n + 4n^2$, find t_7.

Solution : given - $S_n = 3n + 4n^2$

If sum of (n-1) term is Sn – 1 (if we replace n with n-1)

$S_n - 1 = 3(n-1) + 4(n-1)^2 = 3n - 3 + 4n2 - 8n + 4 = -5n + 4n^2 + 1$

$S_n = S_n - 1 + t_n \Rightarrow 3n + 4n^2 = -5n + 4n^2 + 1 + tn$

$\Rightarrow t_n = 8n - 1$ .... (i)

$\therefore t_7 = 8 \times 7 - 1 = 55$ [if we write n = 7 in equation (i)] ………………. (Ans)

Example 8 : Prove that, if the numbers like a^2 , b^2, c^2 present in AP, then $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ can present in AP.

Solution : As per Note 3, we add ab + bc + ca to above numbers a^2 , b^2, c^2

$\therefore a^2+ab + bc + ca, b^2 + ab + bc + ca, c^2 + ab + bc + ca$ can present in A.P

$\Rightarrow a(a+b) + c(a+b), b(a+b) + c(a+b), c(b+c)+a(b+c)$

$\Rightarrow (a+b)(c+a), (a+b)(b+c), (b+c)(c+a)$

(on dividing each term with $(a + b)(b + c)(c + a)$)

$\Rightarrow \frac{(a+b)(c+a)}{(a+b)(b+c)(c+a)}, \frac{(a+b)(b+c)}{(a+b)(b+c)(c+a)}, \frac{(b+c)(c+a)}{(a+b)(b+c)(c+a)}$

$\therefore$ terms $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ present in A.P. ……………. Proved

Example 9 : Show that $t_{m+n} + t_{m-n} = 2tm$ is of an A.P

Solution : Suppose the first term of AP is a and common difference is d

$\therefore t_{m+n} = a + (m+n-1)d$ and $t_{m-n} = a + (m-n-1)d$

$t_{m+n} + t_{m-n} = (a+a) + (m+n-1 + m - n - 1)d = 2a + (2m - 2)d$

$= 2\{a+(m-1)d\} = 2t_m$

$\therefore t_{m+n} + t_{m-n} = 2t_m$ …………….Proved

Exercise 1 (a)

Section (i)

1. **Choose the correct answer from the given options**

(i) In a sequence of 1, 2, 3, 4, ... find t_8 = ... [(a) 6 (b) 7 (c) 8 (d) 9]

(ii) In a sequence of 2, 4, 6, 8, ... find t_7 = ... [(a) 12 (b) 14 (c) 16 (d) 18]

(iii) In a sequence of -5, -3, -1, 1, ... find t_{11} = ... [(a) 13 (b) 15 (c) 17 (d) 19]

(iv) In a sequence of 3, 6, 9, ... find common difference d = ... [(a) 3 (b) 4 (c) 5 (d) 6]

(v) A sequence -4, -2, 0, 2, ... is A.P. find common difference d = ...[(a) -2 (b) -3 (c) 2 (d) 3]

(vi) In a sequence 10.2,10.4, 10.6, 10.8, ... , find t_5 = ... [(a) 11.0 (b) 11.2 (c) 11.4 (d) 11.6]

(vii) If 2.5, 2.9, 3.3, 3.7, ... A.P., find common difference d = ...[(a) 1.5 (b) 1.4 (c) 0.5 (d) 0.4]
(viii) If 3, x, 9, ... is A.P., find x = ... [(a) 4 (b) 5 (c) 6 (d) 7]
(ix) If 1.01, 1.51, 2.01, 2.51, ..is A.P, find common difference d=… [(a)1 (b) 0.5 (c) 1.5 (d) 1.05]
(x) If 5, 0, -5, -10, ... A.P., find common difference d = ... [(a) –5 (b) 5 (c) –10 (d) 10]

2. **Find out which of the following sequences are arithmetic progressions.**

(i) 1, 4, 7, 10, 15, 16, 19, 22
(ii) 1, 8, 15, 22, 29, 36, 43, 50
(iii) 1, 6, 11, 15, 22, 28, 34, 40
(iv) 1, 4, 7, 9, 11, 14, 17, 20
(v) –5, –3, –1, 0, 2, 4, 6, 8
(vi) a, a + d, a + 2d, a + 3d, a + 4d, a + 5d, a + 6d, a + 7d
(vii) 0.6, 0.8, 1.0, 1.5, 1.7, 1.8, 1.9, 2.0
(viii) –7, –4, –1, 2, 5, 8, 11, 14

3. **Find the common difference from above equations which are arithmetic progressions.**

4. **Write first 4 terms of A.P, when a=5 and common difference is as follows :**

(i) d = 5 (ii) d = 4 (iii) d = 2 (iv) d = –2 (v) d = –3

5. **nth term of A.P is given below as t_n. Find t5, t8 and t10 in each of the following terms :**

(i) $t_n = \frac{n+1}{2}$
(ii) $t_n = -10 + 2n$
(iii) $t_n = 10n + 5$
(iv) $t_n = 4n - 6$

6. **Find A.P for the following (1st , 2nd and 3rd Terms are important) where**

(i) 1st term a=4, and common difference d=3
(ii) 1st term a=4, and common difference d=3
(iii) 1st term a=7, and common difference d=-4
(iv) 1st term a=10, and common difference d=5
(v) 1st term $a=\frac{1}{2}$, and common difference $d=\frac{3}{2}$
(vi) 1st term $a=\frac{1}{2}$, and common difference d=-1

7. **Write True or False**

(a) The sequence 1, 2, 3, 4....... is an A.P.
(b) The sequence 1, –1, 1, –1,......... is an A.P.
(c) The sequence 2, 1, –1, –2 is an A.P.
(d) If a sequence $t_n = n - 1$, it is an A.P.
(e) If a sequence $S_n = \frac{n(n-1)}{2}$, it is an A.P
(f) If the ratio of three angles of a triangle is 2 : 3 : 4, it makes a A.P.
(g) The three sides of a right angle triangle make an A.P.
(h) Odd numbers cannot form A.P.
(i) All natural numbers divisible by 5 make an A.P.
(j) If 5, x, 9 are in A.P. then x=6.

Section (ii)

8.
(a) Find S_{30} in the sequence 1 + 2 + 3 + ...
(b) Find S_{10} in the sequence 1 + 3 + 5 +………
(c) Find S_{15} in the sequence 2 + 4 + 6 + ...
(d) Find S_{30} in the sequence 1 – 2 + 3 – 4 +…….
(e) Find S_{41} in the sequence 1 – 2 + 3 – 4 + ...
(f) Find S_{17} in the sequence 1+1 +2 + 2 + 3 + 3 ...
(g) Find S_{39} in the sequence 1 + 2 + 3 + 2 + 3 + 4 + 3 + 4 + 5 ...
(h) Find S_{21} in the sequence – 7 – 10 – 13 - …………....
(i) Find S_{15} in the sequence 10 + 6 + 2 + ……...
(j) Find S_{25} in the sequence 20 + 9 – 2 + ...
(k) Find S_n in the sequence n+(n– 1) + (n – 2) + ...
(l) Find S_{20} in the sequence $5 + 4\frac{1}{3} + 3\frac{2}{3}$ ……….

9.
(a) If a = 3, d = 4, n = 10, find S_n.
(b) If a = – 5, d = – 3, find S_{17}.
(c) If $t_n = 2n - 1$, find first 5 terms.
(d) If $t_n = 3n + 2$, find S_{61}.

(e) If $t_n = 3n - 5$, find S_{50}

(f) If $t_n = 2 - 3n$, find S_n.

(g) If $S_n = n2$, find t_{15}.

(h) In an A. P., if $a = 3$, $d = 4$, $S_n = 903$, find n.

(i) In an A. P., if $d = 2$, $S_{15} = 285$, find a.

(j) In an A. P., if $t_{15} = 30$, $t_{20} = 50$, find S_{17}.

10. (i)Using 'Reverse Order Method', find the sum of
    - (a) Natural numbers from 1 to 105
    - (b) Natural numbers between 25 to 93
    - (c) Natural numbers between 111 to 222

    (ii) In a sequence, 1, 2, 3, ..., find the following
    - (a) S_{20}
    - (b) S_{50}

    (iii) Find the sum of natural number between 32 to 85.

    (iv) Find the sum of positive even numbers which are less than 100.

    (v) Find the sum of positive odd numbers which are less than 150.

Section (iii)

11. Find the total terms, $t_{n,}$ of an A.P whose sum is 72, if its first term is 17 and common difference is 2. Give reason for getting two results.

12. (i) In a sequence, if the sum of three terms of an AP is 18 and their product is 192, find terms. (Note : take the numbers in a order of $a - d$, a, $a + d$)
    (ii) In a sequence, if the sum of first term and last term is 16 and the product of middle terms is 63, find the numbers.
    (Note: take the numbers as $a - 5d$, $a - 3d$, $a - d$, $a+d$, $a + 3d$ and $a + 5d$)

13. The sum of three terms of an AP is 21 and sum of their squares is 155, find the sequence.

14. If the lengths of the sides of a right angle triangle are in A.P, prove that they are in a ratio of 3 : 4 : 5.

15. Find the sum of all positive integers less than100 which are divisible by 5.

16. Find the sum of all positive integers less than 200 which are not divisible by 3.
    (Note : find 1+2+....+199 and 3+6+....+198, subtract 2nd sequence from 1st)

17. Divide 15 in 3 parts in such a manner that they will be in A.P and their product will be 120.

18. The sum of 3 terms of an A.P is 15 and the product of their square of first and last term is 58. Find the numbers.

19. The sum of the extremes of the A.P having 4 terms is 8 and product of middle two terms is 15, find the numbers. (Note – take the numbers as $a - 3d$, $a - d$, $a + d$ and $a + 3d$)

20. The sums of n terms of three sequences of an A.P is S_1, S_2 and S_3 respectively. The first term of each sequence is 1 and common difference is 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

21. The value of the first p, q, r terms of an A.P is a, b, c respectively.
    Show that $a (q - r) + b (r - p) + c (p - q) = 0$

22. If a, b, c are three terms of A.P, prove that the given 3 terms of each are in arithmetic progression.
    (i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$
    (ii) $b + c$, $c + a$, $a + b$

(iii) b + c–a, c + a– b, a + b– c (iv) $\frac{1}{a}\left[\frac{1}{b}+\frac{1}{c}\right], \frac{1}{b}\left[\frac{1}{c}+\frac{1}{a}\right], \frac{1}{c}\left[\frac{1}{a}+\frac{1}{b}\right]$

(iv) $a^2(b+c), b^2(c+a), c^2(a+b)$

23. (i) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the three terms of A.P and a + b + c ≠ 0, prove that $\frac{b+c}{a}$, $\frac{c+a}{b}$, $\frac{a+b}{c}$ is also the terms of A.P

(ii) If the sequence $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ is A.P and a+ b + c ≠ 0 prove that the sequence

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ is also A.P.

24. In a A.P, if a is the first term and *l* is the last term, prove that the sum of r^{th} term at the beginning and r^{th} term at the end is equal to the sum of the 1^{st} and last term.

25. If r is the sum of first p numbered terms, s is the sum of q numbered terms and d is the common difference, prove that $\frac{r}{q} - \frac{s}{q} = (p-q)\frac{d}{2}$.

26. The sum of the first p, q, r terms of an A.P. are a, b, c respectively. Show that $\frac{a}{q}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$.

27. In a A.P, tp = q, tq = p, prove that tm = p + q – m.
(Note : solve a+(p–1)d = q and a+(q–1)d = p, find a and d, find t_{pq})

28. If $S_m = n$, $S_n = m$ is in A.P, prove $S_{m+n} = -(m+n)$.

3.3 DIFFERENCE FORMULA

Earlier we have discussed about the reverse addition method to get the sum of terms of A.P. , now we will discuss another method i.e. Difference Formula to get the sum of terms of A.P.

Difference Formula : $\frac{1}{n(n+1)} = (\because \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)})$

This is known as difference formula as one term is shown as two term difference. We get the following from this formula – $\frac{1}{1x2} = \frac{1}{1} - \frac{1}{2}$ and $\frac{1}{2x3} = \frac{1}{2} - \frac{1}{3}$

Example – 10 :

Find the sum of the : $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + ... + \frac{1}{n(n+1)}$

Solution : Using Difference Formula

$$\frac{1}{1x2} = \frac{1}{1} - \frac{1}{2}$$
$$\frac{1}{2x3} = \frac{1}{2} - \frac{1}{3}$$
$$\frac{1}{3x4} = \frac{1}{3} - \frac{1}{4}$$
$$\dots\dots\dots$$
$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

On Addition : $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + ... + \frac{1}{n(n+1)} = 1 - \frac{1}{n(n+1)}$

$\therefore S_n = \frac{n+1-1}{n+1} = \frac{n}{n+1}$ (Ans)

(i) SUM OF N TERMS OF NATURAL NUMBERS

Suppose $S_n = 1 + 2 + 3 + ... n$

Here 1^{st} term = 1, common difference = 1, terms = n

$S_n = \frac{n}{2}\{2x1+(n-1)1\} = \frac{n}{2}(2+n-1) = \frac{n(n+1)}{2}$ ………………(1)

Formula : $1 + 2 + 3 + ... + n^{th}$ Term = $\frac{n(n+1)}{2}$

(ii) SUM OF N TERMS OF ODD NATURAL NUMBERS

Suppose $S_n = 1 + 3 + 5 + ... n$

Here 1^{st} term = 1, common difference = 2, terms = n

$S_n = \frac{n}{2}\{2x1+(n-1)2\} = \frac{n}{2}(2+n-2) = \frac{n}{2}.2n = n^2$ ……………(2)

Formula : $1 + 3 + 5 + ... + n^{th}$ Term = n^2

(iii) SUM OF N TERMS OF EVEN NATURAL NUMBERS

Suppose $S_n = 2 + 4 + 6 + ... n$

$= 2(1 + 2 + 3 + …………n)$

$= 2.\frac{n(n+1)}{2} = n(n+1)$ ………………………..(3)

Formula : $2 + 4 + 6 + ... + n^{th}$ Term = n (n+1)

Now we will discuss about the sum of squares and cubes of n natural numbers. For this let us recall the common difference which is discussed before in this chapter.

(A) SUM OF SQUARES OF n NATURAL NUMBERS

$S_n = 1^2 + 2^2 + 3^2 + ... + n^2$

As we know $n^3 - (n-1)^3 = n^3 - (n^3 - 3n^2+3n-1) = 3n^2 - 3n + 1$

If we replace n with 1, 2, 3, ……………. etc, the result will

$1^3 - 0^3 = 3.1^2 - 3.1 + 1$

$2^3 - 1^3 = 3.2^2 - 3.2 + 1$

$3^3 - 2^3 = 3.3^2 - 3.3 + 1$

....

....

$(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1$

$n^3 - (n-1)^3 = 3.n^2 - 3.n + 1$

$n^3 = 3(1^2 + 2^2 + 3^2 + ... + n^2) - 3(1 + 2 + 3 + ... + n) + n$

On solving RHS and LHS

⇨ $n^3 = 3S_n - 3.\frac{1}{2}n(n+1) + n$ (as per 1^{st} formula)

⇨ $-3S_n = -n^3 + n - \frac{3n}{2}n(n+1) \Rightarrow 3S_n = n^3 - n + \frac{3n}{2}n(n+1)$

$= n(n^2-1) + \frac{3n}{2}n(n+1)$

$= n(n+1)\{(n-1)+\frac{3}{2}\} = n(n+1)\left(\frac{2n-2+3}{2}\right) = \frac{n(n+1)(2n+1)}{2}$

$S_n = \frac{n(n+1)(2n+1)}{6}$ ……………….. (4)

Formula : $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

(B) SUM OF CUBES OF n NATURAL NUMBERS

$S_n = 1^3 + 2^3 + 3^3 + ... + n^3$

As we know $(r+1)^2 - (r-1)^2 = 4r$

$r^2(r+1)^2 - (r-1)^2 r^2 = 4r^3$ (On multiplying both sides with r^2)

If we replace r with 1, 2, 3, ……………. etc, the result will

$1^2.2^2 - 0^2.1^2 = 4.1^3$

$2^2.3^2 - 1^2.2^2 = 4.2^3$

$3^2.4^2 - 2^2.3^2 = 4.3^3$

....

....

$(n-1)^2 .n^2 - (n-2)^2 . (n-1)^2 = 4(n-1)^3$

$n^2 (n+1)^2 - (n-1)^2 . n^2 = 4n^3$

$n^2 (n+1)^2 = 4 (1^3 + 2^3 + 3^3 + ... + n^3)$ (on addition)

$\therefore 4S_n = n^2 (n+1)^2$

$\therefore S_n = \frac{n2\ (n+1)2}{4} = \left[\frac{n(n+1)}{2}\right]^2$ …………………. (5)

$$\boxed{\text{Formula : } 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$$\boxed{\text{Note : } 1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + …..n)^2}$$

Hence the sum of cubes of natural numbers is equal to square of sum of natural numbers.

N.B – We can obtain $S_n = n^4 - (n-1)^4 = 4n^3 - 6n^3 + 4n - 1$using the methods given above

∑ Sigma notation :

The Greek word sigma (∑) is used to represent the sum of terms of Arithmetic Progression.

$1+2+3 + ... + n = \sum = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 + ... + n^2 = \sum = \frac{n(n+1)(2n+1)}{6}$

$1^3 + 2^3 + 3^3 + ... + n^3 = \sum = \frac{n(n+1)^2}{2}$ etc.

To solve remaining problems, we use formulae sited above from 1 – 5

Note : $\sum n(n+1) = \sum(n^2 + n) = \sum n^2 + \sum n$,

$\sum(n+1)(n+2) = \sum(n^2 + 3n + 2) = \sum n^2 + 3\sum n + \sum 2 = \sum n^2 + 3\sum n + 2n$

Example 11 : Find the sum of 1 . 2 + 2 . 3 + 3. 4 + ... + n(n+1)

Solution : Here tn = n(n+1), where the sum of n terms = S_n

$\therefore S_n = \sum t_n = \sum n(n+1) = \sum (n^2 + n) = \sum n^2 + \sum n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$

$= \frac{n(n+1)}{2} . \frac{n(n+2)}{3} = \frac{1}{3}(n+1)(n+2)$

$\therefore S_n = \frac{n(n+1)(n+2)}{3}$ …………………………… (Ans)

Note : Formula $\sum n^2$ and $\sum n$ is used.

Example 12 : Find the sum of 1. 2. 3 + 2 . 3 . 4 + 3 . 4 . 5 + ... n

Solution : Here tn = n (n + 1) (n + 2) = $n(n^2+3n+2) = n^3+3n^2+2n$

$\therefore S_n = \sum t_n = \sum (n^3+3n^2+2n) = \sum n^3 + 3\sum n^2 + 3\sum n$

$= \left\{\frac{n(n+1)}{2}\right\}^2 + 3.\frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$

Formula $\Sigma n^3, \Sigma n^2, \Sigma n$ is used

$= \frac{\{n(n+1)\}^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1) = \frac{n(n+1)}{4}\{n(n+1)+2(2n+1)+4\}$

$= \frac{n(n+1)}{4}(n^2+n+4n+2+4) = \frac{n(n+1)(n^2+5n+6)}{4}$

$= \frac{n(n+1)(n^2+2n+3n+6)}{4}$

$= \frac{n(n+1)\{n(n+2)+3(n+2)\}}{4} = \frac{n(n+1)(n+2)(n+3)}{4}$

$\therefore S_n = \frac{n(n+1)(n+2)(n+3)}{4}$ (Ans)

NB – Had it been asked to get the sum of 10 terms, then the equation would had been

N=10 and

$S_n = \frac{10 \times 11 \times 12 \times 13}{2} = 8580$

Example 13 : Find the sum of 1+ (1+2) + (1+2+3) + (1 + 2 + 3 + 4) + ... n

Solution : n^{th} term = $t_n = (1 + 2 + ... + n) = \frac{n(n+1)}{2} = \frac{1}{2} n^2 + \frac{n}{2}$

$\therefore S_n = \sum t_n = \frac{1}{2} \sum n^2 + \frac{1}{2} \sum n$

$= \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2} = \frac{1}{4} n(n+1) \left(\frac{2n+1}{3} + 1\right)$

$= \frac{1}{4} \frac{n(n+1)(2n+4)}{3} = \frac{1}{6} n(n+1)(n+2)$

$\therefore S_n = \frac{n(n+1)(n+2)}{6}$ …………………(Ans)

Example 14 : Find the sum of $1^2 + 3^2 + 5^2 + 7^2 + ...$ n

Solution : If n^{th} term of AP is t_n

$t_n = \{1 + (n-1) 2\}^2 = (2n - 1)^2 = 4n^2 - 4n + 1$

$\therefore S_n = \sum t_n = 4 \sum n2 - 4 \sum n + \sum 1$

$= 4 \frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n = 2n(n+1) \left(\frac{2n+1}{3} - 1\right) + n$

$= \frac{2n(n+1)2(n-1)}{3} + n = \left\{\frac{4n(n^2-1)}{3} + n\right\} = n \left(\frac{4n^2-4)}{3} + 1\right) = \frac{n}{3}(4n^2 - 1)$

$S_n = \frac{n}{3}(4n^2 - 1)$ …………… (Ans)

Example : Find the sum of 1 + 3 + 6 + 10 + 15 + ...n

Solution : In the above case the sequence is not A.P, but the terms of series are chronologically differ, hence in A.P. (i.e. 2,3,4,5,.... etc).

$S_n = 1 + 3 + 6 + ... + t_n - 1 + t_n$

$S_n = 1 + 3 + ... + t_n - 2 + t_n - 1 + t_n$

On subtraction $0 = 1 + (3 - 1) + (6 - 3) + (10 - 6) + ... + (t_n - t_{n-1}) - t_n$

$\therefore t_n = 1 + 2 + 3 + ... + n$

$\Rightarrow t_n = \frac{1}{2} n(n + 1) = \frac{1}{2} n^2 + \frac{1}{2} n$

$S_n = \sum t_n = \frac{1}{2} \sum n^2 + \frac{1}{2} \sum n = \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2}$

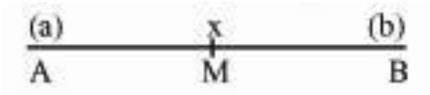
$= \frac{1}{4} n(n+1) \left\{\frac{2n+1}{3} + 1\right\} = \frac{1}{4} \frac{n(n+1)(2n+4)}{3} = \frac{1}{6} n(n+1)(n+2)$

$\therefore S_n = \frac{1}{6} n(n+1)(n+2)$ …………..(Ans)

3.4 ARITHMETIC MEAN

If two digits like a and b is given, then the arithmetic mean of it is $\frac{a+b}{2}$

If we study it geometrically – a and b are two points of $\overline{AB}$ (b>a)



The position of middle point M of $\overline{AB}$ is $x=\frac{a+b}{2}$

Here a, $\frac{a+b}{2}$, *b form AP as* $\frac{a+b}{2} - a = b - \frac{a+b}{2} = \frac{b-a}{2}$ = d (common difference)

If a, $\frac{a+b}{2}$, b form AP, then $\frac{a+b}{2}$ forms Arithmetic mean for a and b

Hence

$$\boxed{A.M. = \frac{a+b}{2} \text{ (where } a \quad \frac{a+b}{2} \quad b \text{ are three terms of A.P.}}$$

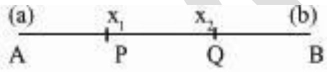
For example the AM of 7 and 15 is $\frac{7+15}{2}$ = 11, similarly AM of -1 and 10 is $\frac{-1+10}{2}$ = 4.5 etc.

3.4.1 Arithmetic Mean position of n number between the two points a and b

(i) Suppose a and b are two given variables. First, fix two mean positions like x_1 and x_2. Draw a straight line $\overline{AB}$ and divide it equally in to three parts like $\frac{b-a}{3}$ which gives a, x_1 , x_2 , b and their distances will be equal to common diffence d of A.P. Hence d= $\frac{b-a}{3}$ ($\because$ length of $\overline{AB}$ = b – a)

Hence $x_1 = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$ and

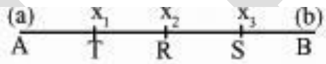
$$x_2 = a + 2d = a + 2\left(\frac{b-a}{3}\right) = \frac{a+2b}{3}$$



A.M between the a and b is $x_1=\frac{2a+b}{3}$ and $x_2=\frac{a+2b}{3}$ ………………………….(iii)

(ii) Now to find three mean position between a and b.

Suppose x_1 , x_2, x_3 are the three mean positions between a and b. There will be 5 terms of A.P like a, x_1 , x_2, x_3, b. In order to know mean position of x_1 , x_2, x_3 between points a and b we have to divide the straight line in to 4 equal parts using d = $\frac{b-a}{4}$. ($\because$ length of $\overline{AB}$ = b – a)



$$x_1 = a + d = a + \frac{b-a}{4} = \frac{3a+b}{4}, \quad x_2 = a + 2d = a + 2 \times \frac{b-a}{4} = \frac{a+b}{2}$$

and $x_3 = a + 3d = a + 3 \times \frac{b-a}{4} = \frac{a+3b}{4}$

$\therefore$ The three A.M position between a and b $\frac{3a+b}{4}$, $\frac{a+b}{2}$ and $\frac{a+3b}{4}$ ...... (iv)

(iv) Similarly, to find n number of A.M. positions between a and b, we have to divide straight line $\overline{AB}$ in to (n+1) equal parts where the length of each part is $\frac{b-a}{n+1}$. If the mean positions are x1, x2, x3, ...x_n, then

$$x_1 = a + \frac{b-a}{n+1}, \ x_2 = a + \frac{2(b-a)}{n+1}, \ x_3 = a + \frac{3(b-a)}{n+1}, \ldots\ldots\ldots, \ x_n = a + \frac{n(b-a)}{n+1}$$

Here the sequence a, x1, x2, x3 .... xn, b is in AP and their common difference d= $\frac{b-a}{n+1}$

Example 16 – Find the A.M positions (i) one (ii) two (iii) three (iv) four between 2 and 62.

Here a = 2 and b=62. $\therefore$ b – a = 62 – 2 = 60

(i) The A.M position = $x_1 = a + \frac{b-a}{2} = 2 + \frac{60}{2} = 2 + 30 = 32$

$\therefore$ 32, is the A.M position between 2 and 62

(ii) Let two A.M x_1 x_2 and $2, x_1, x_2, 62$ are A.M. positions

comm.diffence $d = \frac{b-a}{3} = \frac{60}{3} = 20$

$\therefore x_1 = a + d = 2 + 20 = 22$ $\quad x_2 = a + 2d = 2 + 2 \times 20 = 42$

$\therefore$ 22 and 42 are two mean positions between 2 and 62

(iii) Three mean position are x_1, x_2 x_3

$2, x_1, x_2, x_3, 62$ are in A.P and common difference $d = \frac{b-a}{4} = \frac{60}{4} = 15$ Hence

$x_1 = a + d = 2 + 15 = 17$, $x_2 = a + 2d = 2 + 2 \times 15 = 32$ $\quad x_3 = a + 3d = 2 + 3 \times 15 = 47$

$\therefore$ 17, 32 and 47 are the A.M. positions between 2 and 62

(iv) Let the four A.M positions be x_1, x_2 , x_3, x_4

The series 2, x_1, x_2 , x_3, x_4 , 62 is A.P and the common diffence $d = \frac{b-a}{5} = \frac{62-2}{5} = \frac{60}{5} = 12$

$x_1 = a + d = 2 + 12 = 14$, $x_2 = a + 2d = 2 + 2 \times 12 = 26$, $x_3 = a + 3d = 2 + 3 \times 12 = 38$, and $x_4 = a + 4d = 2 + 4 \times 12 = 50$

$\therefore$ 14, 24, 38 and 50 are the mean positions between 2 and 62.

Exercise 3 (b)

1. Fill in the blanks

(a) $\frac{1}{15 \times 16} = \ldots.. - \frac{1}{16}$ (b) $\frac{1}{12 \times 11} = \frac{1}{11} - \ldots\ldots$

(c) $\frac{1}{n(n+1)} = \ldots\ldots\ldots - \frac{1}{n+1}$ (d) $\frac{1}{(n+1)n} = \frac{1}{n} - \ldots\ldots$

(e) Arithmetic Mean between 5 and 9 ……………

(f) If 5 is the Arithmetic mean between x and 7, the value of x = ………..

(g) Arithmetic Mean between (a+b) and (a–b)

(h) The AM of two numbers is 11, if one number is 7, find the other number ………

2. Find the sum of the following sequence

(a) $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4}$ ……………… 20^{th} *term*

(b) $\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8}$ ……………… 16^{th} *term*

3. (a) Find the t_n for the following sequence 7 x 15 + 8 x 20 + 9 x 25 +...

(b) Simplify $6\sum n^2 + 4\sum n^3$

(c) Find S_n and S_{20} for the sequence 1 x 2 + 2 x 3 + 3 x 4 ... + n (n + 1)

(d) Find S_n and S_{10} for the sequence 1 x 3 + 2 x 4 + 3 x 5 ... t_n .

4. Find the sum of n^{th} terms for following sequence

(a) 1. 1. + 2. 3. + 3. 5 + 4. 7 + ... (b) 1 . 3 + 3 . 5 + 5 . 7 + 7 . 9 + ...

(c) 3 . 8 + 6 . 11 + 9 . 14 + ... (d) 1 + (1 + 3) + (1 +3 + 5) + ...

(e) $1^2 + 4^2 + 7^2 + 10^2 + \ldots\ldots$ (f) $2^2 + 4^2 + 6^2 + 8^2 + \ldots\ldots\ldots$

(g) 1 + 5 + 12 + 22 + 35 + ...

(h) $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + (1^2 + 2^2 + 3^2 + 4^2) + \ldots\ldots\ldots$

5. Find Arithmetic mean positions (i) one (ii) two between 15 and 27.

6. Find Arithmetic mean positions (i) two (ii) three between 12 and 36.

7. Find Arithmetic mean positions (i) two (ii) four between 6 and 46.

8. Find Arithmetic mean positions (i) three (ii) five between 5 and 65.

9. Find 5 Arithmetic mean positions between 11 and 71.

10. The Arithmetic mean position between 20 and 80 is n. If the ration of first mean position and last mean position is 1:3, find the value of n.

11. Find the four terms of AP whose sum is 2 and the product of first term and the last term is 10 times the product of middle terms.

CHAPTER 4 : PROBABILITY

4.1 What we have learnt in the chapter on probability in class IX was experimental or impirical approach to probability. In this approach, as we have seen, the probabilities were based on actual experiments and adequate recording of the happening of events. In this chapter and higher classes, we will study about theoretical approach to probability. The basic difference between these two approaches to probability is that in the experimental approach to probability, the probability of an event is based on what has been actually happened while in theoretical approach to probability, we try to predict what will happen without actually performing the experiment.

It has been observed that the experimental probability of an even approaches to its theoretical probability if the number of trials of an experiment is very large.

4.2 Experimental and Theoretical Probability

In the theory of probability we deal with events which are outcomes of an experiment and its observation. That is probability of events is decided basing on outcomes of experiment. This kind of probability is known as **Empirical** Probability. In class IX, Experiment -1, we have studied about tossing of coin. With the increase in number of tosses, note down the number of Heads P(H) and Tails P(T). It has been observed that the difference between the number we get and 0.5 i.e. $\frac{1}{2}$ is reduced. Similarly in Experiment – 2, a cubical die marked with 1, 2, 3, 4, 5 and 6 is thrown, one of the six faces come upward, hence the probability result is 0.166 or $\frac{1}{6}$.

The result of two experiments is $\frac{1}{2}$ and $\frac{1}{6}$ respectively. It is result of experimental probability.

$$\therefore \text{Empirical probability of an event} = \frac{favrourable\ number\ of\ events}{total\ number\ of\ events}$$

Few examples of Empirical Probability is given below :

Example – 1: Find P(T) and P(H), if a coin is tossed 20 times and we get T seven times.

Solution : Total numbers of events = 20 and number of favourable events of T = 7

$$\therefore P(T) = \frac{number\ of\ event\ of\ T}{Total\ events} = \frac{7}{20} \text{ and } P(H) = \frac{number\ of\ event\ of\ H}{Total\ events} = \frac{13}{20}$$

Example – 2 : If a die is thrown 30 times where the outcome of 1 and 2 is 4 times each and outcome of 3,4 and 5 is 5 times each, find the P(6).

Solution : Given – outcome of 1 = 4 outcome of 2 = 4 outcome of 3 = 5
outcome of 4 = 5 outcome of 5 = 5

$$\therefore \text{outcome of } 6 = 30 - (4 + 4 + 5 + 5 + 5) = 7$$

$$P(6) = \frac{number\ of\ event\ of\ 6}{Total\ events} = \frac{7}{30}$$

Example – 3 : 15 goals are scored in a football match. If 5 goals are scored by one team, find the probability of scoring goals by other team.

Solution : Suppose event of goals scored by other team = E

$$\therefore \text{outcome of E} = 15 - 5 = 10$$

$$P(E) = \frac{number\ of\ event\ of\ E}{Total\ events} = \frac{10}{15} = \frac{2}{3}$$

Example – 4 : The probability of crossing check-gate by various vehicles is given below :

P(car) = $\frac{1}{4}$ P(truck) = $\frac{1}{8}$ P(two wheelers) = $\frac{1}{2}$ P(tractor) = $\frac{1}{8}$

If total number of vehicles is nearly 4000 vehicles cross the gate, find the number of each type of vehicles.

Solution : Suppose x, y, z and w are names given to the vehicles car, truck, two wheelers and tractors respectively then n = x + y + z + w = 4000

As per the given question $\frac{x}{n} = \frac{1}{4}$ $\frac{y}{n} = \frac{1}{8}$ $\frac{z}{n} = \frac{1}{2}$ $\frac{w}{n} = \frac{1}{8}$

$\frac{x}{4000} = \frac{1}{4}$ $\frac{y}{4000} = \frac{1}{8}$ $\frac{z}{4000} = \frac{1}{2}$ $\frac{w}{4000} = \frac{1}{8}$

x=$\frac{4000}{4}$ = 1000 x=$\frac{4000}{8}$ = 500 x=$\frac{4000}{2}$ = 2000 x=$\frac{4000}{8}$ = 500

∴ Hence, 1000 cars, 500 trucks, 2000 two wheelers and 500 tractors cross the gate every day.

REMARKS

1. The eighteenth century French naturalist Comte de Buffon tossed a coin 4040 times and got 2048 heads. The experimental probability of getting a head, in this case was $\frac{2048}{4040}$ i.e. 0.507.
2. J.E. Kerrich, from Britain, recorded 5067 heads in 10000 tosses of a coin. The experimental probability of getting a head, in this case, was $\frac{5067}{10000}$ 0.5067.
3. Statistician Karl Pearson spent some more time, making 24000 tosses of a coin. He got 12012 beads and thus, the experimental probability of a head obtained by him was $\frac{12012}{24000} = 0.5005$.

Hence, from above experiment, it is concluded that the probability of getting Heads is 0.5 or $\frac{1}{2}$ similarly in a die, it is $\frac{1}{6}$. It is known as theoretical probability. **Theoretical probability is also known as Classical Probability.**

Example – 5 :If a die is thrown in an event, what is the probability of getting number less than 4.

Solution : The event getting less than 4 will occur if we get one of the numbers 1,2 and 3 as an outcome.

$\therefore$ favourable number of outcomes = 3

Total number of outcomes of a die once it is thrown = 6

$\therefore$ Probability of an event $= \frac{favrourable\ number\ of\ events}{total\ number\ of\ events} = \frac{3}{6} = \frac{1}{2}$

The above type of probability is known as Theoretical probability or Classical Probability.

Example – 6 : A bag contains red, blue and yellow stones of one each and they are all of same size and shape. Anindita takes out a stone from the bag without looking into it. What is the probability that she takes out the red stone, blue stone and yellow stone.

Solution : Let Y be the event 'the ball taken out is yellow', B be the event 'the ball taken out is blue' and R be the event 'the ball taken out is red.

Here, the number of possible outcomes = 3

(i) The number of outcomes favourable to be the event Y = 1

So $P(Y) = \frac{1}{3}$

Similarly,

(ii) $P(R) = \frac{1}{3}$ and (iii) $P(B) = \frac{1}{3}$

REMARKS : (i) $P(Y) + P(B) + P(R) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

An event having only one outcome of the experiment is called an elementary event. All the three events, Y,B and R are elementary events and the sum of the probability of all the elementary events of this experiment is 1.

Rememeber – sum of all the elementary events in an event is 1.

Example – 7 : Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Solution : Here, let E be the event 'getting a number greater than 4'. The number of possible outcomes is six : 1,2,3,4,5 and 6, and the outcomes favourable to E are 5 and 6. Therefore, the number of outcomes favourable to E is 2. So,

$$P(E) = P\text{ (number greater than 4)} = \frac{2}{6} = \frac{1}{3}$$

Let F be the event 'getting a number less than or equal to 4'.

Number of possible outcomes = 6

Outcomes favourable to the event F are 1,2,3,4.

So, the number of outcomes favourable to F is 4.

Therefore, $P(F) = \frac{4}{6} = \frac{2}{3}$

Hence $\frac{1}{3} + \frac{2}{3} = 1$ ……………………(i)

Remarks

1. Study the events 'E' and 'F'.
   Where E is event 'getting a number > 4' and F is the event 'getting a number ≤ 4'. Note that getting a number not greater than 4 is same as getting a number less than or equal to 4 and vice versa.
   ∴ P(E) + P($\bar{E}$) = 1 [(from (i)]

⇨ P($\bar{E}$) = 1 – P(E)

Remember : for any event E, P($\bar{E}$) = 1 – P(E)

The event **$\bar{E}$,** representing 'not E', is called the Complement of the event E. We also say that E and **$\bar{E}$** are complementary events.

Example – 8 : Two unbiased coins are tossed simultaneously. Find the probability of getting at least one head.

Solution : If two unbiased coins are tossed simultaneously we obtain any one of the following as an outcome - HH , HT, TH, TT. Hence total number of events = 4

If E be the event 'getting a number at least one head', we obtain HH, HT, TH and the number of events is 3.

$\therefore$ P(E) $=\frac{3}{4}$

$\therefore$ Probability of getting at least one head= P(E) $=\frac{3}{4}$.

Alternative solution = $\mathrm{P(E)} = 1 - \frac{1}{4} = \frac{3}{4}$ $\quad$ $(\because \mathrm{P(E)} = 1 - \mathrm{P}(\bar{E}))$

Where P($\bar{E}$) is the event where there is no probability of getting at least one head $=\frac{1}{4}$

Observe that in tossing two unbiased coins, probability of getting at no head is equal to probability of getting at least one head are complementary events.

Exercise 4 (a)

1. (i) A die is thrown once. What is the probability of getting 8?
   (ii) A die is thrown once. What is the probability of getting less than 7?
   (iii) A die is thrown once. What is the probability of getting ≤ 3?
   (iv) Mili and Lima were playing tennis. If the probability of winning Mili is 0.62, find the probability Lima loosing the game.
   (v) Two coins are tossed together. What is the probability of getting at most one T.
   (vi)Find the sum of the pro0babilities of all the elementary events of an experiment?
   (vii)If P(E)=0.05, find P($\bar{E}$).

2. A coin is tossed 30 times. The probability of getting H is 16. Find P(H) and P(T).

3. A coin is tossed 30 times. The probability of getting T is twice that of H. Find P(H) and P(T).

4. A die is tossed 30 times. The probability of getting 1 is 4, 2 is 5, 3 is 6, 4 is 7 and 5 is 8; then find the probability of getting 6.

5. 20 saplings were sown, out of which 8 saplings remain alive rest died. Find the probability of each died sapling.

6. There are total 100 students studying in a school appeared matriculation examination. Out of which 10 students passed in first division, 15 students in second division, 50 students in third division. Rest of the students failed. Find the probability of getting various divisions and probability of getting fail. Find the sum of the probabilities.

7. Pumpkin seeds of 40 numbers are sown. 15 seeds germinated and 10 seeds germinated but died and rest seeds did not germinate at all. Find the probability of number of seeds germinated and did not germinate at all.

8. A box contains 3 blue, 2 white, and 4 red marbles. A ball is drawn at randomly. What is the probability that the ball drawn is :
   (i) white marble drawn (ii) blue marble drawn (iii) red marble drawn

9. A bag contains 5 white, 4 red, and 3 black balls of same shape. What is the probability that the ball drawn is :
(i) white ball (ii) not red (iii) not white

10. A bag contains 60 electric bulbs. 12 bulbs are defective and rest are good. One bulb is drawn randomly from the bag. What is the probability of the bulb drawn :
(i) good bulb (ii) defective bulb

4.3 SOME STATEMENTS BASED ON SET THEORY

To solve the probability using set theory, we should know some basic concepts of Set Theory. First take an example tossing a coin. We get H or T when we toss a coin. As per set theory concept, we can represent it

S = {H, T} .... (i)

The above example is known as **Sample Space**. Similarly if we throw a die, it will give either of these numbers i.e. 1, 2, 3, 4, 5, and 6

Sample space of die = S = {1, 2, 3, 4, 5, 6} ....(ii)

If a coin is tossed twice or a pair of coins tossed simultaneously, we obtain

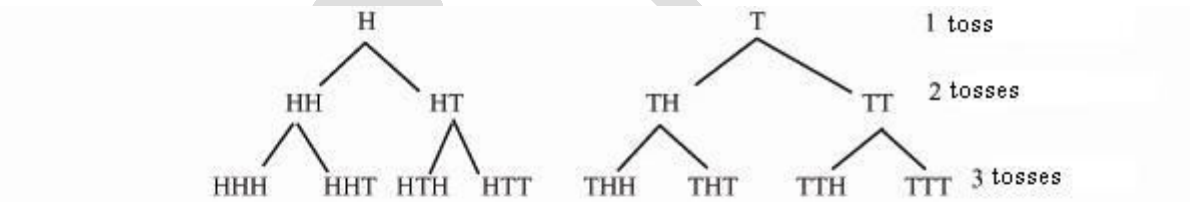
S = {HH, HT, TH, TT} ....(iii)

(In HT, H is for first toss or coin and T is for second toss or coin)

Similarly, if you throw a die twice or two dice are thrown together, we get the following result

S = {11, 12, 13, 14, 15, 16,
21, 22, 23, 24, 25, 26,
31, 32, 33, 34, 35, 36,
41, 42, 43, 44, 45, 46,
51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66} .... (iv)

From (i) and (ii), we come to know that if the coin is tossed n times then probability number = 2^n and from (iii) and (iv), the probability of die is = 6^n.



If we divide the result of each event into two and rename it as H and T, we get 8 terms

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} and it is known as 3 term space sample.

4.3.1 EVENT

If Space sample S, then subsets E will be each event of S. For an example if a coin is tossed twice

S = {HH, HT, TH, TT}

Suppose it is asked to get 'at least one T' then event E= {HT, TH, TT}

Example - 9 : If a die is thrown twice. Find the events given below.
(i) E1 : sum $\leq$ 3 (ii) E2 : sum = 9 (iii) E3 : sum = 13

Solution : if a die is thrown twice, we get 36 terms

(i) Event 1 E1 : sum $\leq$ 3, the favourable outcomes are 12, 21 and 11
$\therefore$ E1 = {12, 21, 11}

(ii) Event 2 E2 : sum = 9, the favourable outcomes are 63, 36, 45 and 54
$\therefore$ E2 = {63, 36, 45, 54}

(iii) Event 3 E3 : sum = 13, it is not a favourable outcome. $\therefore$ E3 = $\emptyset$
(Zero set is known as subset of a set and taken as an event)

Now we have come to know about the set and subsets. If S is the sample space and E is the event then E $\subset$ S.

(i) **Simple or Elementary Event -** An outcome of a random experiment is called an elementary event. Consider the random experiment of tossing of a coin. The possible outcomes of this experiment are {H} and {T}. If we toss the coin twice {HH}, {HT}, {TH} and {TT} are the simple or elementary events.

(ii) **Compound Event** – An event associated to a random experiment is a compound event if it is obtained by combining two or more elementary events associated to the random experiment. {TH, HT}{HH,TT} are events occur if two coins are tossed where the each event is compound event. If the coin is tossed twice S = {TH, TT, HH, HT}

(iii) **Mutually Exclusive Event -** E1 and E2 (E1 , E2 $\subset$ S) are two events which are mutually exclusive i.e. E1 $\cap$ E2 = $\emptyset$. If coin is tossed once {T} {H} are two events and if it is tossed twice the events {HH,TH} and {TT} are mutually exclusive events.

(iv) **Complementary Events -** If E1 and E2 are complementary to each other they are mutually exclusive and they are $E_1 \cup E_2$ then the outcome is Sample space. For Example E1 = {H}and E2 = {T} events are complement when a coin is tossed and E1 = {HH}, E2 = {HT, TH, TT} are complementary when a coin is tossed twice.

4.3.2 **Probability of an Event**

If E be the event, S is the Sample Space, then probability of event E

$$P(E) = \frac{\text{the number of outcomes of E}}{\text{the number of outcomes of S}} = \frac{|E|}{|S|}$$

For an Example if a coin is tossed once then sample space S = {H, T}i.e. |S|=2, S has got two events.

We can represent E_1, E_2, E_3, and E_4 in the following manner.

E1 = H={H}, E2 = T = {T}

E3 =H, T = {H,T} E4 = H andT both not present = f

$\therefore$ | E1 | = 1 , | E2 | = 1, | E3 | = 2 | | E4 | = 0

$P(E_1) = \frac{|E_1|}{|S|} = \frac{1}{2}$, $P(E_2) = \frac{|E_2|}{|S|} = \frac{1}{2}$, $P(E_3) = \frac{|E_3|}{|S|} = 1$ and $P(E_4) = \frac{|E_4|}{|S|} = \frac{0}{2}$

4.3.3 **Some rule related to Probability**

(i) If an event is E $\subset$ S, then P(f) = 0, P(S)=1 and 0 $\leq$ P(E) $\leq$ 1, where $\emptyset$ is known as Impossible Event and S is a Sure Event.

(ii) If an event (E) has its complementary events $\overline{E}$, and $E^{\backslash}$ are subsets of S, $P(E)+P(E^{\backslash})= 1$.

(iii) If a pair of events E_1 and E_2 i.e. $E_1 \subset S$ and $E_2 \subset S$ then $E_1 \cup E_2$ is also an event. Because both E_1 and E_2 are subsets of S. As we know

$|E_1 \cup E_2| = |E_1| + |E_2| - |E_1 \cap E_2|$ (When E_1 and E_2 subsets intersect each other)

$$\therefore P(E_1 \cup E_2) = \frac{|E_1 \cup E_2|}{|S|} = \frac{\left| |E_1| + |E_2| - |E_1 \cap E_2| \right|}{|S|} = \frac{|E_1|}{|S|} + \frac{|E_2|}{|S|} - \frac{|E_1 \cap E_2|}{|S|}$$

$$= P(E1) + P(E2) - P(E1 \cap E2)$$

Note : (i) Simple outcomes or favourable outcomes present in the events E_1 and E_2.

(ii) E_1 and E_2 are not mutually exclusive events.

(iii)If E_1 and E_2 are mutually exclusive events i.e. E1 $\cap$ E2 = $\emptyset$, then P(E1 $\cap$ E2) = 0 and here

$$\boxed{P(E_1 \cup E_2) = P(E1) + P(E2)}$$

Remember : for the events E_1 and E_2 : $P(E_1 \cup E_2) = P(E1) + P(E2) - (E1 \cap E2)$ and if events E_1 and E_2 are mutually exclusive : $P(E_1 \cup E_2) = P(E1) + P(E2)$

Example 10 – Two coins are tossed simultaneously. If the favourable event E is "H and T", find the probability of E.

Solution : the outcome of events for tossing two coins simultaneously or a coin is tossed twice is same and it is known Sample Space. S = {HH, HT, TH, TT} $\therefore$ | S | = 4

And out of 4 outcomes of toss, 2 events are favourable i.e. contains H and T

$\therefore$ E = {TH, HT} and | E | = 2

$\therefore P(E) = \frac{|E|}{|S|} = \frac{2}{4} = \frac{1}{2}$ ………………….. (Ans)

Example 11 - Two coins are tossed simultaneously. If the favourable event E is "at least one H", find the probability of E.

Solution : the outcome of events for tossing two coins simultaneously or a coin is tossed twice is same and it is known Sample Space. S = {HH, HT, TH, TT} $\therefore$ I S I = 4

And out of 4 outcomes of toss, 3 events are favourable HH, HT, TH ; $\therefore$ I S I = 4

$\therefore$ E = {HH, TH, HT} and I E I = 3

$\therefore P(E) = \frac{|E|}{|S|} = \frac{3}{4}$ ………………….. (Ans)

Example 12 – A pair of ludo dice are thrown simultaneously. Find the probability of getting a sum of the digits ≥ 11.

Solution : the outcome of events of two dice thrown simultaneously $|S|=6^2=36$ (as shown in (iv).
The favourable outcomes out of 36 events E

E = {56, 65, 66} and I E I = 3

$\therefore P(E) = \frac{|E|}{|S|} = \frac{3}{36} = \frac{1}{12}$ ………………….. (Ans)

Example 13 - A ludo die is thrown. Find the probability of getting "an even number and an odd number".

Solution : the outcome of events if a dice thrown i.e

Sample space S = {1, 2, 3, 4, 5, 6}

The favourable outcomes out of E_1 for Even numbers and E_2 for odd numbers, and E_1, E_2 are subsets of S then E1 = {2,4,6} and E2 = {1,3,5}

$\therefore$ I S I = 6, I E1 I = 3, I E2 I = 3, (they are mutually exclusive events)

$\therefore$ The probability of getting an "even number and an odd number" is

$= P(E_1 \cup E_2) = P(E1) + P(E2)$

$= \frac{|E_1|}{|S|} + \frac{|E_2|}{|S|} = \frac{3}{6} + \frac{3}{6} = \frac{1}{2} + \frac{1}{2} = 1$

Example 14 - A die is thrown. Find the probability of getting "an even number or number ≥ 4".

Solution : Here Sample space S = {1,2,3,4,5,6}

Let E_1 be the event of getting an even number E1 = {2, 4,6} and for number ≥ 4 E_2={4, 5,6}

$\therefore |E_1| = 3, |E_2| = 3$

Both E_1 and E_2 are not mutually exclusive as they don't have common outcomes.

$E1 \cap E2 = \{4, 6\} \Rightarrow |E_1 \cap E_2| = 2$

The probability of getting "an even number or number ≥ 4"

$= P(E_1 \cup E_2) = P(E1) + P(E2) - (E1 \cap E2) = \frac{|E_1|}{|S|} + \frac{|E_2|}{|S|} - \frac{|E_1 \cap E_2|}{|S|} = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$ …..(i)

Note : $E1 \cup E2 = \{2,4,6\} \cup \{4,5,6\} = \{2,4,5,6\} \Rightarrow$ I $E1 \cup E2$ I = 4

L.H.S. $= P(E_1 \cup E_2) = \frac{|E_1 \cup E_2|}{|S|} = \frac{4}{6} = \frac{2}{3}$

R.H.S $= P(E1) + P(E2) - P(E1 \cap E2) = \frac{2}{3}$ (as per (i))

$\therefore P(E_1 \cup E_2) = P(E1) + P(E2) - (E1 \cap E2)$……………………………(proved)

Exercise 4 (b)

1. Which of the following statements are correct, explain.

(i) If an event is $\emptyset$, the probability is zero.

(ii) An event E = S, where S is the sample space, then P(E) < 1.

(iii) If a coin is tossed once, the outcome of the sample space is 4.

(iv) The probability of "i" in the word "probability" is $\frac{2}{11}$.

(v) The sum of probability of two mutually exclusive events E1 and E2 ($E1, E2 \subset S$) is 1.

(vi) If a die is thrown twice, then the outcome of Sample Space is 36.

(vii) If a coin is tossed thrice, then the outcome of Sample space is $3^2 = 9$.

(viii) To get a letter randomly from the word "mathematic", the Sample space = {m,a,t, h, e, i, c, s}

(ix) If E_1 and E_2 are exclusive events, $P(E_1 \cup E_2) = P(E1) + P(E2)$

(x) If toss a toss a coin once, E2 = {H, T} is the complementary event of E2 = {H}

2. E1 , E2 , E3 and E4 are the four exclusive events of an experiment. The occurrence of $E1 \cup E2 \cup E3 \cup E4$ is definite. If the events have equal probability, find the probability of each event.

3. If a die is thrown once. What is probability of getting
(i) outcome ≤ 3 (ii) outcome < 3 (iii) outcome ≤ 4 (iv) outcome < 6 (v) outcome ≤ 6
(vi) outcome > 6

4. What is the probability of letter S if picked randomly from the world 'School'.

5. A jar contains 5 red, 4 green, and 3 black marbles. What is the probability that the green ball drawn randomly.

6. If a die is thrown once. If E is an "outcome of an Even number", find the probability of the event E.

7. If a die is thrown once. Find the probability of the getting an event "Odd number".

8. If a die is thrown once. If E is an "outcome of an Even$\leq$5", find the probability of the event E.

9. If a coin is tossed twice. Find the probability of getting
   (i) at least one H
   (ii) only T
   (iii) one H
   (iv) no H

10. If coin is tossed thrice. Write Sample Space for the following statements and find the probability of getting
    (i) Only T
    (ii) At least two H
    (iii) Maximum two T
    (iv) Only H or only T
    (v) No T at all

11. If a die is thrown twice. What is the probability of
    (i) obtaining total of two numbers=6
    (ii) obtaining total of two numbers=4
    (iii) each number of the two numbers present in an event is a square
    (iv) obtaining total of two numbers ≥ 10
    (v) obtaining total of two numbers < 6
    (vi) getting first number odd and second number 6

12. The mutually exclusive events E_1 and E_2 are of an experiment where P(E1) = 2P(E2) and P(E1)+ P(E2) = 0.9. Find the probability of obtaining $E1 \cup E2$ and the event E_1.

13. If E_1 and E_2 are events where $P(E1) = \frac{5}{8}$ and $P(E2) = \frac{2}{8}$ and $E1 \cap E2 = \frac{1}{8}$, find the following
    (i) (i) $P(E1 \cup E2)$ (ii) $P(E1')$ (iii) $P(E2')$ (iv) $P(E1' \cup E2')$

14. Find the probability of getting the letter 'A or T' which is picked up randomly from the word 'MATHEMATICS'.

15. A lude die is thrown once. Find the probability of obtaining "5 or an odd number".

16. A lude die is thrown once. Find the probability of getting "an odd number or outcome $\geq$ 3".

CHAPTER 5 ; STATISTICS

5.1 INTRODUCTION

In class IX, we have learnt about definition of statistics and representation of data like Numeric Data, Primary Data, Secondary Data etc. The data collected was represented by Frequency Distribution Table. You have also learnt to represent the data pictorially in the form of various graphs such as bar graphs, histograms, pie-chart, pictograph and frequency polygon. Now we will discuss about the numerical expression which represent the characteristics of a group i.e. a large collection of numerical data are call Central Tendency.

5.2 Central Tendency

We know about various subjects and resources from different print media. It is very much essential to represent the collected data in single digit. Note the data given in table where numbers secured in five subjects by two students is mentioned below.

	MIL	ENGLISH	SCIENCE	MATHS	SOCIAL
LIZA	70	60	78	90	87
PUJA	78	68	75	87	86

From the above table, it is clear that Liza has done better than Puja in 3 subjects whereas Puja did well 2 subjects as compare to Liza. The marks of both the students are more or less equal in one subject. Hence it is very difficult to derive a formula about the comparison of marks obtained by both students. Hence to compare the marks we need to get the **Mean or Average** marks of each student.

Hence the average marks of Lisa = $\frac{total\ number\ of\ marks\ obtained}{total\ number\ of\ subjects} = \frac{385}{5} = 77.0$

the average marks of Puja = $\frac{total\ number\ of\ marks\ obtained}{total\ number\ of\ subjects} = \frac{386}{5} = 77.2$

The numerical expressions which represent the characteristics of a group (a large collection of numerical data) are called Measures of Central Tendency (or, Averages). **Mean, Mode and Median** is the three types of measures to represent central tendency.

5.2.1 Arithmetic Mean – (or, simply, mean)

(a)Mean of individual Series : of a set of numbers is obtained by dividing the sum of numbers of the set by the number of numbers.

For example :

The mean of n numbers $x_1, x_2, x_3, \ldots\ldots\ldots ; x_n$ is

$$M= \frac{x1+ x2+ x3+\cdots\ldots\ldots+xn}{n} = \frac{1}{n}\sum_{k=1}^{k=n} x_k$$

Here M=mean, $\sum$=represents sum of the numbers,

$\sum_{k=1}^{k=n} x_k$: sum of the numbers $x_1 \ldots\ldots x_n$ where n=number of numbers

$$M= \frac{sum\ of\ the\ total\ numbers}{total\ number\ of\ numbers}$$

$$\text{i.e. } M=\frac{\sum x}{n}$$

Example : Marks obtained by a students in six subjects are 65, 67, 85, 78, 69, 78. Find the arithmetic mean of the numbers

$M=\frac{\sum x}{n}$ (where $\sum x$ is the sum of numbers and n = total numbers of numbers)

$M=\frac{65+67+ 85+ 78+ 69+ 78}{6} = \frac{442}{6} = 73.66 \ldots\ldots = 73.67$

(b) **Mean of a frequency distribution :**

Example : 2 – Find the mean of the following frequency distribution table where the heights of the children are given.

Table A

Heights (cm) x:	69	70	71	72	73
Frequency f:	4	2	3	2	1

Solution

Table A_1

Height (cm)(x)	Frequency (f)	Freq X Height (fx)
69	4	276
70	2	240
71	3	213
72	2	144
73	1	73
	$\sum f = 12$	$\sum fx=846$

$$M=\frac{\sum fx}{\sum f} = \frac{846}{12} = 70.5 \text{ cm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(Ans)}$$

Short-cut Method or Deviation Method :
In the above table we either have multiply or add long digits which is time consuming and lengthy. In order to minimize the time consumption and tedious lengthy calculation, deviation method or short-cut method is used.

93, 98, 112, 103, 97, 109 = $\frac{1}{6}$ (93 + 98 + 112 + 103 + 97 + 109)

$= \frac{1}{6}\{(100 - 7) + (100 - 2) + (100 + 12) + (100 + 3) + (100 - 3) + (100 + 9)\}$

$= \frac{1}{6}$ [6 x 100 + {(-7) +(-2) + 12 + 3 + (-3) + 9}]

$= \frac{1}{6}$ x 6 x 100 + $\frac{1}{6}$ x 12 = 100 + $\frac{12}{6}$

Calculate the given data either by subtracting it from 100 or adding it to 100. The resultant is known as deviation whereas 100 is known as working zero. Hence the deviated (x) numbers are -7, -2, 12, 3, -3, 9.
The sum of the deviated numbers = (-7) +(-2) + 12 + 3 + (-3) + 9 = 12

$\therefore$ M = 100 + $\frac{12}{6}$

M=assumed mean + $\frac{sum\ of\ the\ deviations}{total\ number\ of\ numbers}$

Note – the deviation number will not affect if we take any number in place of 100. The AM we obtain from the assumed mean and deviation number is called short-cut method.

Example 3 : Using Short-cut Method, calculate Table A.

Height (cm)(x)	Frequency (f)	Deviation (y) Assumed mean:70	Freq X Height (fy)
69	4	-1	-4
70	2	0	0
71	3	1	3
72	2	2	4
73	1	3	3
	$\sum f = 12$		$\sum fy=6$

$$M=\text{Assumed mean} + \frac{\sum fy}{\sum f} = 70 + \frac{6}{12} = 70 + 0.5 = 70.5 \ \ldots\ldots\ldots\ldots\text{ (Ans)}$$

(C) **Mean of a Grouped frequency distribution :**
In this method, we have to find out the mean (y) of each class interval and it is multiplied by the frequency (f) i.e. (fy). Later find the sum (∑fy) of (fy) i.e. and sum of the number of frequencies (∑f).

Example 4 : 100 days wages of a labour is displayed in the following group frequency distribution table. Find the mean wages of the labour.

Table B

(x):	0 - 10	10 – 20	20 – 30	30 - 40	40 - 50	50 - 60	60 - 70
(f):	1	7	24	36	25	6	1

Note : the mid value of each class interval is obtained by dividing the sum of its lower and upper limits by 2 i.e. $\frac{l1+l2}{2}$ where l_1 is lower limit and l_2 is upper.

Table B$_1$

Class interval	Frequency (f)	Mid-value (y= $\frac{l1+l2}{2}$)	Frequency X mid value (fy)
0 - 10	1	5	5
10-20	7	15	105
20-30	24	25	600
30-40	36	35	1260
40-50	25	45	1125
50-60	6	55	330
60-70	1	65	65
	∑f = 100		∑fy=3490

$$M = \frac{\sum fy}{\sum f} = \frac{3490}{100} = 34.9 \qquad \text{…………….. (Ans)}$$

Example 5: we also solve the above given frequency distribution table using assumed value method or short cut method.

Table B$_2$

Class interval	Frequency (f)	Mid-value (x)	Deviation (y)	Frequency X mid value (fy)
0 - 10	1	5	-30	-30
10-20	7	15	-20	-140
20-30	24	25	-10	-240
30-40	36	35	0	0
40-50	25	45	10	250
50-60	6	55	20	120
60-70	1	65	30	30
	∑f = 100			∑fy= -10

$$M = A + \frac{\sum fy}{\sum f} = 35 + \frac{-10}{100} = 34.9 \qquad \text{…………….. (Ans)}$$

Step - deviation method

This method is also a very easy method for calculating the average mean value. Like previous methods, assumed mean value (A), value of deviation is required.

$$M = A + \frac{\sum fy\prime}{\sum f} \text{ x c}$$

Where A=Assumed mean

$\sum fy'$; f=frequency, y' = $\frac{deviation\ (y)}{common\ multiple\ (c)}$

$\sum f$ = sum of the frequency

Example 6 : Find the mean of the following distribution table using step-deviation method.

Table C

x:	5	10	15	20	25
Frequency f:	3	4	5	2	1

\

Table C_1

x	f	x-A=y (A=15)	c=5; $y' = \frac{y}{5}$	fy'
5	3	-10	-2	-6
10	4	-5	-1	-4
15	5	0	0	0
20	2	5	1	2
25	1	10	2	2
	∑f =15			∑fy'=-6

$$M = A + \frac{\Sigma fy'}{\Sigma f} \text{ x } c = 15 + \frac{-6}{15} \text{ x } 5 = 15 + (-2) = 13\text{.......... (Ans)}$$

Observe that the common deviation in the above table is 5. It is further simplified by dividing the Deviation by 5.

Example 7 – Find the mean for the distribution table B using step-deviation method.

Table B_2

Class interval	Frequency (f)	Mid-value (x)	Deviation y= x – A (A=35)	$\frac{deviation\ (y')}{class\ interval}$	(fy')
0 - 10	1	5	-30	-3	-3
10-20	7	15	-20	-2	-14
20-30	24	25	-10	-1	-24
30-40	36	35	0	0	0
40-50	25	45	10	1	25
50-60	6	55	20	2	12
60-70	1	65	30	3	3
	∑f = 100				∑fy = -1

$$\therefore \quad M = A + \frac{\Sigma fy'}{\Sigma f} \text{ x } i = 35 + \frac{-1}{100} = 35 - 0.1 = 34.9 \quad \text{................ (Ans)}$$

Note : If M be the mean of the numbers $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, $\sum_{i=1}^{n}(xi - M) = 0$

Some Useful Results on Mean

If Mean of the numbers $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, is M then,

(i) Mean of $x_1 + a$, $x_2 + a$, $x_3 + a$, ... x_n is M + a.

(ii) Mean of $x_1 - a$, $x_2 - a$, $x_3 - a$ ....x_n- a is M – a.

(iii) Mean of ax, ax_2, ax_3 .... ax_n is aM, when a≠0.

(iv) Mean of $\frac{x1}{a}, \frac{x2}{a}, \frac{x3}{a}, \ldots\ldots\ldots\ldots\ldots \frac{x2}{a}$ is $\frac{M}{a}$ when a≠0.

Example 8 : If M be the mean of the numbers $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, show that $\sum_{i=1}^{n}(xi - M) = 0$.

$$M = \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots\ldots + x_n}{n} \Rightarrow x_1 + x_2 + x_3 \ldots\ldots + x_n = n \,.\, M$$

$$\sum_{i=1}^{n}(x_i - M) = (x_1 - M) + (x_2 - M) + (x_3 - M) \ldots\ldots + (x_n - M)$$

$$= (x_1 + x_2 + x_3 \ldots\ldots + x_n) - (M + M + M \ldots\ldots n \quad)$$

$$= (x_1 + x_2 + x_3 \ldots\ldots + x_n) - n.M$$

$$= n \,.\, M - n \,.\, M = 0 \qquad \text{Proved}$$

Example 9 : Prove M be the mean of numbers x_1, x_2, x_3,…………. x_n.

If $\sum_{i=1}^{n}(x_i - 12) = -10$ and $\sum_{i=1}^{n}(x_i - 3) = 62$, find the value of .n and M

$: \sum_{i=1}^{n}(x_i - 12) = -10 \Rightarrow (x_1 - 12) + (x_2 - 12) + \ldots\ldots + (x_n - 12) = -10$

$\Rightarrow (x_1 + x_2 + x_3 \ldots\ldots + x_n) - 12n = -10$

$\Rightarrow nM - 12n = -10$ ....(i) $\qquad [\because \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots\ldots + x_n}{n} = M]$

Similarly $\sum_{i=1}^{n}(x_i - 3) = 62 \Rightarrow nM - 3n = 62$ ....(ii)

On subtracting (ii) from (i) $\quad -9n = -72 \Rightarrow n = 8$

On using value of n in (i) = $\quad 8M - 12 \times 8 = -10$

$\Rightarrow 8M = 12 \times 8 - 10 = 86 \Rightarrow M = \frac{86}{8} = 10.75$ ………… Proved.

Example 10 : Prove M be the mean of numbers x_1, x_2, x_3,…………. x_n.

If $\sum_{i=1}^{n}(x_i - 2) = 110$ ଏବଂ $\sum_{i=1}^{n}(x_i - 5) = 80$ find n and m

Solution $\quad : \sum_{i=1}^{n}(x_i - 2) = 110$

$\Rightarrow \sum_{i=1}^{n}(x_i - 2) = (x_1 - 2) + (x_2 - 2) \ldots\ldots\ldots + (x_n - 2) = 110$

$\Rightarrow (x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_n) - 2n = 110$

$\Rightarrow nM - 2n = 110$ ....(i) $\qquad [\because \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots\ldots + x_n}{n} = M]$

$\sum_{i=1}^{n}(x_i - 5) = 80 \Rightarrow nM - 5n = 80$ ....(ii)

On subtracting (ii) from (i) $\quad 3n = 30 \Rightarrow n = \frac{30}{3} = 10$

On putting the value of n in eqn (i) $\quad 10M - 2 \times 10 = 110$

$\Rightarrow 10M = 110 + 20 = 130 \Rightarrow M = \frac{130}{10} = 13$

$\therefore n = 10$ ଓ $M = 13$ ....(Ans)

EXERCISE 5(a)

Part A

1. Write T for True and F for false

(i) The mean of two consecutive odd numbers is equal to the mid of their even numbers.
(ii) The mean of three consecutive numbers of an Arithmetic progression is equal to mid- term of the numbers.
(iii) The average of the group data is equal to their mean.
(iv) We get varied answers if we take different Assumed mean values of given data.
(v) If the Assumed Mean Value is 20 and its class term is 15, then deviation will be 5.
(vi) The mean of first *n* natural numbers is $\frac{n+1}{2}$.
(vii) The mean of first *n* even natural numbers is 2n+1 .
(viii) The mean of first 10 natural odd numbers is 10.
(ix) The mean of 15 numbers is 17. If each number is multiplied by 2, their mean will be 8.5.
(x) The mean of first 20 even natural numbers is equal to the mean of first 20 natural numbers.

2. Choose the correct answer from the choices given below for each question.

(i) The assumed mean value for the data 61, 62, 68, 56, 64, 72, 69, 51, 71, 67, 70, 55, 63 is
(A) 55 (B) 60 (C) 70 (D) 72
(ii) The mean of the first 20 natural numbers
(A) 10 (B) 10½ (C) $\frac{21}{20}$ (D) 210
(iii) The mean of *n* numbers of whole numbers
(A) $\frac{n-1}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) n
(iv) The mean of the first *n* even natural numbers
(A) (n - 1) (B) n (C) n + 1 (D) n + 2
(v) The mean of the first *n* even natural numbers
(A) (n - 11) (B) n (C) n + 1 (D) n + 2
(vi) If the mean of 10 observations (terms) is M, find the mean of same data if each term is increased by 2
(A) m (B) 2m (C) m^2 (D) m + 2
(vii)If the mean of *n* observations (terms) is M, find the mean of same data if each term is multiplied by 4
(A) $\frac{M}{4}$ (B) M (C) 4M (D) $\frac{4}{M}$
(viii)If mean of *n* terms of data is M, find the mean if each term is subtracted by *x*.
(A) M (B) (M + x) (C) Mx (D) (M - x)
(ix) If M is the mean of *n* terms, find the mean if each term is divided by 5.
(A) M (B) $\frac{M}{5}$ (C) 5M (D) M – 5
(x) If mean age of the '*a*' number boys is 12 and '*b*' numbered girls is 10, find the mean age of both boys and girls.
(A) $\frac{10a+12b}{a+b}$ (B) $\frac{12a+10b}{a+b}$ (C) $\frac{10a+12b}{10+12}$ (D) $\frac{12a+10b}{10+12}$
(xi) Find the mean of 998.9, 999.1, 1000.3, 1000.6, 1000.1
(A) 998 (B) 999 (C) 1000 (D) 1001
(xii) Find the value of *x*, if the mean of 6, 8, 5, 7, x and 4 is 7
(A) 10 (B) 11 (C) 12 (D) 13
(xiii) If mean of $x_1, x_2, x_3, x_4, x_5, x_6$ is M, then find $\sum_{i=1}^{6}(xi - 13)$.
(A) 0 (B) 6 (C) 36 (D) -6
(xiv) Find the mean of x, x + 2, x + 4, x + 6, x + 8
(A) x + 2 (B) x + 4 (C) x + 6 (D) x
(xv) Find the mean of all multiples of 18 :
(A) 5 (B) 6 (C) 6.5 (D) 7

Part B

3. A player scores 47, 41, 50, 39, 45, 48, 42, 32, 60 and 20 runs each time after playing ten matches. Find the mean score using short-cut method.

4. The weights of 30 students are 21, 30, 40, 25, 26, 22, 26, 31, 22, 36, 30, 25, 25, 33, 30, 25, 27, 27, 25, 31, 33, 22, 21, 36, 40, 31, 33, 30, 37, 36 respectively. Prepare the frequency distribution table for above data and find the mean weight.

5. The weight of a chemical substance is taken for 30 times and displace in the following frequency distribution table. Find the mean weight.

Weight (gms)	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6
Frequency	1	1	6	6	7	5	2	1	1

6. The average age of 30 students is 12 years. Find the age of the class teacher if the average age of students and class teacher is 13 years.

7. The mean of x_1, x_2, x_3 .....x_n is m. If (a+b) is added to each term of the data prove that their mean will be (m-a+b).

Part B

8. The heights of the plants of a garden is given below. Find the mean weight of the plants in cm.

heights (cms)	70-65	65-60	60-55	55-50	50-45	45-40	40-35	35-30	30-25
Frequency	4	7	8	10	7	5	2	1	1

9. Find the mean of the following frequency distribution table using Short-cut method.

Class interval	84-90	95-96	96-102	102-108	108-114	114-120
Frequency	8	10	16	23	12	11

10. Find the mean of the following frequency distribution table using Step-Deviation method.

Class interval	0-4	4-8	8-12	12-16	16-20	20-24
Frequency	5	7	10	15	9	4

11. Find the mean of the following frequency distribution table using Short-cut and Step-Deviation method.

Class interval	0-50	50-100	100-150	150-200	200-250	250-300
Frequency	4	10	12	10	8	8

12. Find the mean of the following frequency distribution table using Step-Deviation method.

Class interval	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

13. (i) Find the value of f if mean of the given frequency distribution is 7.5

Class	5	6	7	8	9	10	11	12
Frequency	20	17	f	10	8	4	7	6

(ii) Find the value of P if mean of the given frequency distribution table is 7.5

Class	3	6	7	4	P+3	8
Frequency	5	2	3	2	4	6

14. Find the value of f_1 and f_2 if the mean of the given frequency distribution table is 50 and the sum of the frequencies is 120.

Class interval	0-20	20-40	40-60	60-80	80-100
Frequency	17	f_1	32	f_2	190

15. Find the mean of the following frequency distribution using step-deviation method.

Class interval	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	5	65	222	112	53	40	3

MEDIAN (M_d):

The median is the middle value of a distribution i.e. median of a distribution is the value of the variable which divides it into two equal parts. It is the value of the variable such that the number of observations above it is equal to the number of observations below it.

Arrange the observations x_1, x_2, x_3,………… x_n in ascending or descending order of magnitude. Determine the total number of observation say, *n*.

1. **If n is odd, then median is the value of $(\frac{n+1}{2})^{th}$ observation.**
2. **If *n* is even, then median is the AM of the values of $(\frac{n}{2})^{th}$ and $(\frac{n}{2}+1)^{th}$ observations.**

(a) Determination of Median of Numeric Table :

Example 11: (i) Let the weight of 7 children be 40, 42, 44, 45, 46, 48, 49 Kgs. respectively.

(here the total number of numbers is odd and are arranged in ascending order.

Median (M_d) = $\frac{7+1}{2}$ = 45 i.e. 4^{th} term of the numbers.

(ii) Let the marks obtained in Mathematics by 6 students is 87, 95, 63, 53, 69, and 72 respectively.

(here the total number of numbers is **even** and are not arranged in ascending or descending order hence arrange them in ascending order i.e. 53, 63, 69, 72, 87, 95)

M_d= Mean of $(\frac{n}{2})^{th}$ **and** $(\frac{n}{2}+1)^{th}$ i.e 3^{rd} term and 4^{th} term 69 and 72 respectively

Median (M_d) = $\frac{69+72}{2} = \frac{141}{2}$ = 70.5

(b) Determination of Median of Discrete Frequency Distribution Table :

Example 12 : **Table D**

Weight (Kg)	46	48	50	52	53	54	55
Frequency	7	5	8	12	10	2	1

Arrange in frequency distribution table

Table D_1

Weight (x) in kg	Frequency	Cumulative frequency (cf)	Place of the each term
46	7	7	1^{st} to 8^{th}
48	5	12	9^{th} to 12^{th}
50	8	20	13^{th} to 20^{th}
52	12	32	21^{st} to 32^{nd}
53	10	42	33^{rd} to 42^{nd}
54	2	44	43^{rd} to 44^{th}
55	1	45	45^{th} term
	$\sum f$=45		

As number of observations is odd number i.e. 45 therefore M_d=$\frac{n+1}{2} = \frac{45+1}{2} = \frac{46}{2}$ **= 23.**

$\therefore$ M_d= 23 which is between 21^{st} to 32^{nd} term, hence M_d = 52Kg.

Example 13 : Find Median value of weights of 60 people is given in the following frequency distribution table.

Table E

Weight (Kg)	37	38	39	40	41
frequency	10	14	18	12	6

Here n = 60 which is an even number.

$\therefore$ M_d= Mean of $(\frac{n}{2})^{th}$ **and** $(\frac{n}{2}+1)^{th}$ i.e. $\frac{60}{2}$ **and** $\frac{60}{2}+1$ i.e. 30^{th} and 31^{st} terms.

$\therefore M_d = \frac{30+31}{2} = \frac{61}{2} = 30.5$ which is between

Table E_1

Weight (Kg)	Frequency (f)	Cumulative frequency
37	10	10
38	14	24
39	18	42
40	12	54
41	6	60
	$\sum f = 60$	

$$\text{Median (m)} = \frac{n+1}{2} = \frac{60+1}{2} = 30.5$$

We find the cumulative frequency just greater than 30.5 is 42 and the value of m corresponding to 42 is 39 kgs.
∴ Median weight is 39 kgs............................(Ans)

(c) Determination of Median of a Continuous Frequency Distribution Table :
In the above section, we have obtained median of the discrete grouped data by determining the cumulative frequency which is greater than $\frac{n}{2}$.
In this section, first we obtain the frequency distribution. Prepare the cumulative frequency column and obtain $\sum f$. Find $\frac{n}{2}$. See the cumulative frequency just greater than $\frac{n}{2}$ and determine the corresponding class. This class is known as median class. Use the following formula :

$$\text{Median } (M_d) = l + \frac{m-c}{f} \times i$$

Where m=median, l=lower limit of median class, f=frequency of the Median class, i=size f the median class, c=cumulative frequency of the class just preceding the Median class.
Example 14 : The marks obtained by students of a class in Physical science is shown in the table given below. Find the Median.

Table F

Marks (x)	0 - 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	7	10	8	5

Table F_1

Number (x)	Frequency (f)	Cumulative Frequency (cf)
0 – 10	5	5
10 – 20	7	12
20 – 30	10	22
30 – 40	8	30
40 – 50	5	35

n = 35

$m = \frac{n}{2} = \frac{35}{2} = 17.5$
the cumulative frequency just greater than 17.5 is 22
∴ Class interval is (20 – 30)
Hence l=20, f=10, c=12, i=10
$M_d = l + \frac{m-c}{f} \times i = 20 + \frac{17.5-12}{10} \times 10 = 20 + 5.5 = 25.5$…………(Ans)

Example 15 : Find the median of the following distribution table.

Table G

Class interval	4 - 7	8 – 11	12 – 15	16 – 19	20 – 23	24-27	28-31	32-35
Frequency	4	11	25	47	56	29	20	8

Here the frequency table is given in inclusive form. So, we first transform it into exclusive form by subtracting and adding $\frac{h}{2}$ to the lower and upper limits respectively of each class, where h denotes the difference of lower limit of a class and the upper limit of the previous class. Here the difference (h) is 1.
$\therefore \frac{1}{2} = 0.5$ is subtracted from the lower class and adding to upper class.
∴ table can also be represented as follows –

Table G_1

Class interval	Frequency (f)	Cumulative frequency(cf)
3.5 - 7.5	4	4
7.5 - 11.5	11	15
11.5 - 15.5	25	40
15.5 - 19.5	47	57
19.5 - 23.5	56	143
23.5 - 27.5	29	172
27.5 - 31.5	20	192
31.5 - 35.5	08	200

n= 200

Median (m) = $\frac{n}{2} = \frac{200}{2} = 100$

The cumulative frequency just greater than 100 is 143

∴ Class interval is 19.5 - 23.5

Hence l=19.5, f=56, c=87, i=4

$M_d = l+\frac{m-c}{f} \text{ x } i = 19.5+\frac{100-87}{56} \text{ x } 4 =$

$= 19.5 +\frac{13}{14} = 19.5 + 0.93 = 20.43$………..(Ans)

(d) Determination of Median through Ogive or frequency distribution curve

Example 18 : Find the median of given Table H through Ogive.

Table H

Class	5	6	7	8	9	10	11	12	13	14
Frequency	6	8	8	11	22	36	59	28	21	3

Solution : Note – if we plot the points taking the upper limits of the class intervals as x-co-ordinates and their corresponding cumulative frequencies as y-co-ordinates and then join these points by a free hand curve, the curve so obtained is called cumulative frequency curve.

1. Construct a cumulative frequency table.
2. Mark the actual class limits along x-axis.
3. Mark the cumulative frequencies of respective classes along y-axis.
4. Find the points corresponding the cumulative frequency at each upper limit point.
5. Join the points plotted by a free hand curve.

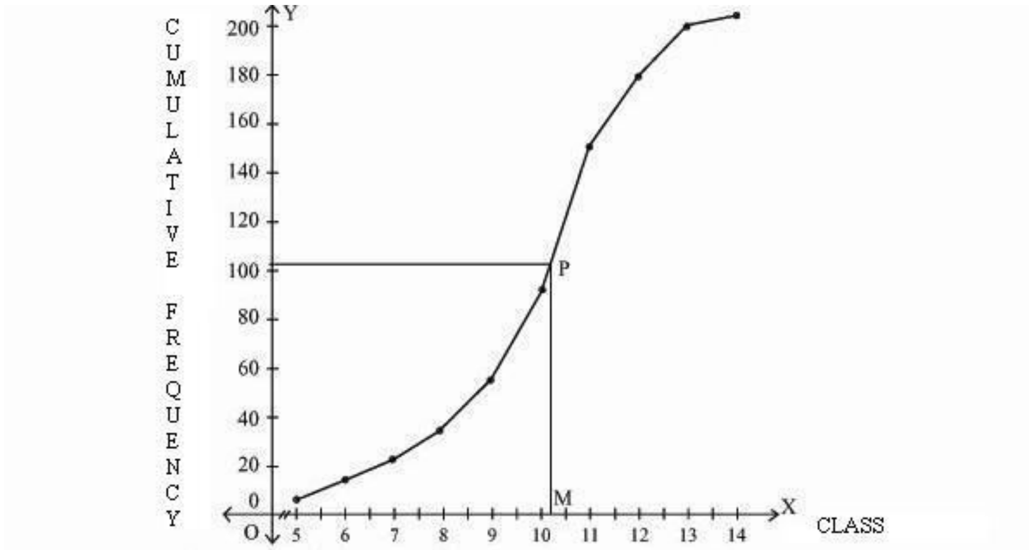
Table H_1

Class	5	6	7	8	9	10	11	12	13	14
Frequency	6	8	8	11	22	36	59	28	21	3
Cumulative frequency(cf)	6	14	22	33	55	91	150	179	200	203

Method to determine Median

Median m = $\frac{n+1}{2} = \frac{203+1}{2} = 102$

The cumulative frequency curve or ogive (P) whose cumulative frequency = 102



Through mark 102 on y axis, draw a horizontal line which meets the curve at P and a vertical line drawn on x axis which meets at 10.2.

Example – 17 : The marks obtained by 120 students in an examination are given below in table I. Draw an Ogive for the given distribution and find

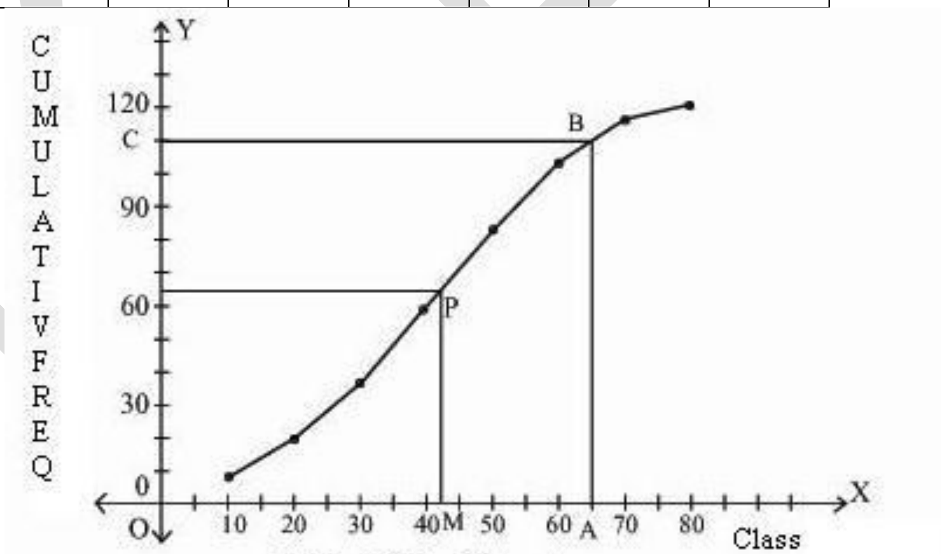
(i) The median
(ii) The number of students who obtained more than 65% marks

Table I

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	12	18	22	24	20	13	4

Table I$_1$

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	12	18	22	24	20	13	4
Cumulative Frequency	7	19	37	59	83	103	116	120



Median $= \frac{1}{2}\left[\frac{120}{2}+\left(\frac{120}{2}+1\right)\right] = \frac{1}{2}\ (60+61) = 60.5$

The cumulative frequency curve or ogive (P) whose cumulative frequency = 60.5

Through median 60.5 on y axis, draw a horizontal line which meets the curve at P and a vertical line drawn on x axis which meets at M.

(i) From point M we get 40.2
(ii) 65% marks means 65 of 100 = 65

Through Marks 65 on x axis, draw a vertical line which meets the curve at point B and a horizontal line drawn on y axis which meets at point C and the cumulative frequency is 110.

∴ Number of students who score marks more than 65% = 120-110 =10……..(Ans)

Exercise 5 (b)
Section (A)

1.**(a) Write T fr True and F for False**

(i) The Median of any frequency distribution table is equal to the Mean of it.
(ii)In a frequency distribution table having 13 terms is arranged in an ascending order, the Median of it is equal to its 7^{th} term.
(iii) The median of any frequency distribution table is equal to one of its terms.
(iv)The median of a frequency distribution table having 30 terms is equal to 15.
(v) The median of a frequency distribution table of 5, 8, 3, 7, 11, 27, 16, is 8.

(b) Answer the following questions

(a) Find the median of first 9 natural numbers.
(b) Find the median of first 10 prime numbers.
(c) Find the median of x when $1 \leq x \leq 7$
(d) If x is the median of 7, 3, 10, 5, x, then find the value of x where ($x \in N$).
(e) How the median of first 6 natural numbers is less than median of first 7 natural numbers.

Section (B)

2. **Find the median of the following**

(i) 7, 8, 4, 3, 10
(ii) 11, 27, 36, 58, 65, 72, 80, 95
(iii) 7, 12, 15, 6, 20, 8, 4, 10
(iv) 18, 32, 37, 25, 31, 19, 25, 29, 31

3. **Find the median of the following**

(i)

Class (x)	11	12	13	14	15	16
Frequency(f)	2	4	6	10	8	7

(ii)

Class (x)	1	2	3	4	5	6	7	8
Frequency(f)	5	8	15	24	14	9	5	4

(iii) The marks obtained in Mathematics by students is given below in table. Calculate median.

Marks (x)	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
Frequency(f)	3	12	27	57	75	80

4. Find the Class Interval for the following frequency distribution table.

Mid value	55	65	75	85	95	105	115	125	135
Frequency	4	21	35	42	70	28	10	25	15

5. Find the Class Interval for the following frequency distribution table.

Height (cm)	more than 0	more than 10	more than 20	more than 30	more than 40
Frequency(f)	55	50	40	20	5

Section (C)

6. Find the median of the following frequency distribution table.

Interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	9	15	14	8

7. Calculate the median of the following frequency distribution table using any two methods and find the difference between two.

Class (x)	4	5	6	7	8	9	10
Frequency(f)	8	12	21	31	18	13	5

8. Calculate the median of the following table

Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	12	22	18	10	6

9. Using given frequency distribution table, draw ogive or frequency distribution curve. Find the (i) the median (ii) find the number of students secured more than 65%.

Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	10	20	25	15	12	9	8

10. Draw Frequency distribution curve (Ogive) using the data given below. Find Median.

Interval	0-8	8-16	16-24	24-32	32-40	40-48	48-56
Frequency	4	8	14	23	15	11	5

11. If the median of the following distribution table is 36 and sum of the frequencies is 74, find the missing frequencies.

Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	8	?	20	12	?	4	3

12. The marks obtained in Mathematics by 200 students is given in following frequency distribution table.

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Frequency	2	8	?	20	12	?	4	3

(i) Find the median by drawing Frequency Distribution curve (Ogive)
(ii) Find the number of students secured 45% marks in mathematics

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5.2.3 Mode

(i) 4, 2, 6, 4, 4, 0; are the runs scored by Sachin Tendulkar after facing 6 balls where the occurrence of number of 4s is maximum i.e. 3 times. Hence the Mode (Mo) = 4.

(ii) Observe the frequency distribution table mentioned below

Class (x)	2	3	4	6
Frequency (f)	25	15	12	10

In the above table number of occurrences of 2 is 25 times, hence Mode (Mo) = 25.

(iii) If a die is thrown ten times we get 3, 6, 3, 2, 5, 5, 1, 3, 2, 2 terms. The occurrence of face 2 and 3 is three times, therefore Mode (Mo) = 2 and 3.

The Mode or Modal Value of a distribution is that value of the variable for which the frequency is maximum. Hence Mode is the value which occurs most frequency in a set of observations. It is the point of maximum frequency.

Remarks – If the number of occurrences of an observation in a series of data is same, then it is not a Modal value. There is no modal value for the data 3, 5, 7, 3, 8, 5, 8, 7.

Example 18 : Find the Modal Value for the following distribution table.

Class	8	9	10	11	12	13	14	15	16
Freq.	3	8	12	15	14	17	12	8	6

Solution : Occurrence of class 13 is maximum i.e. 17 times.

$\therefore$ Mode Mo = 13

Example 19 : The height of 10 saplings planted in a garden (cm) is 22, 24, 19, 21, 33, 21, 24, 22, 20, 22. Find the mode.

Solution : first arrange the above series of data in ascending order

19, 20, 21, 21, 22, 22, 22, 23, 24, 24.

Here Mode (Mo)=22 ($\because$ occurrence of 22 is more)

Example 20 : Find the mode for the following frequency distribution table.

Class	5	6	7	8	9	10	11	12
Freq	7	18	25	24	20	25	19	13

Solution : The frequency of occurrence of 7 and 10 is maximum.

Hence the mode is 7 and 10.

Remarks – Relationship among Mean, Median and Mode

We have learnt about three measure of central value, namely, arithmetic mean, median and mode. These three measures are closely connected by the following relations.

Mode (Mo) = 3 Median (M_d) – 2 Mean

Exercise 5 (c)

1. Write T for true and F for false

(i) If the frequency of occurrence of each term of a frequency distribution table is equal then it is not a mode data.

(ii) The highest frequency of a frequency distribution table is the Modal value of the data.

(iii) In a frequency distribution table, there will be only one modal value.

2. Find the Mode of the following data:

(i) 5, 6, 7, 7, 8, 9, 9, 9, 10, 10, 11, 12, 12

(ii) 12, 8, 15, 9, 11, 8, 10, 11, 13, 9, 12, 10, 14, 11, 13, 10

3. Find the Mode of the following distribution.

Height (in cm)	120	121	122	123	124
Frequency	5	8	18	10	9

4. If a die is thrown 15 times we get 7, 8, 10, 10, 11, 7, 12, 9, 7, 9, 8, 12, 11, 10, 7 terms. Find the Mode of the series.

5. The sale of shoes of various sizes of a Shoe shop is given below

Size of shoe	5	6	7	8	9	10
Sale	20	33	40	85	15	8

(i) Find the size of the shoe to be kept in stock for sale depending upon its sale.

(ii) Find which method of central tendency can be used for the above data.

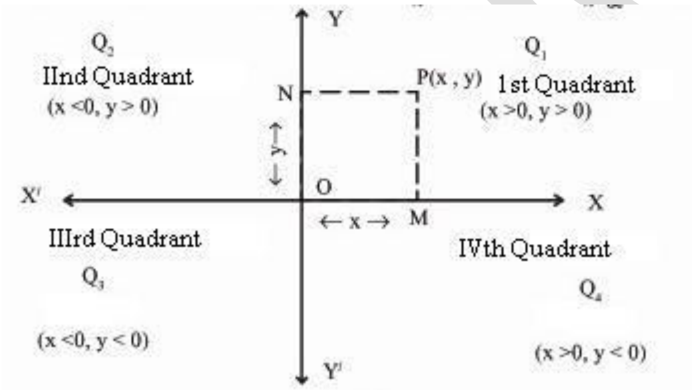
CO-ORDINATE GEOMETRY

6.1 Introduction

In Class IX, we have seen that to locate the position of a point on a plane, we require a pair of mutually perpendicular lines which are known as the coordinate axes. The horizontal line is known as the x-axis and the vertical line is known as the y-axis. The intersection point of the coordinate axes is known as the origin. The distance of a point from the y-axis is called x-coordinate, or abcsissa and the distance from x-axis is called its y-coordinate., or ordinate. We have seen that the coordinates of a point on the x-axis are of the form (x,0) and that of a point on y-axis are of the form (0,y).

6. 2 Cartesian plane and Cartesian co-ordinates :

Let X'OX and Y'OY be the coordinate axes, and let P be any point in the plane. Draw perpendicular PM and PN from P on x and y-axis respectively. The length of the directed line segment OM in the units of scale chosen is called the x-coordinate or abscissa of point P. similarly, the length of the directed line segment ON on the same scale is called the y-coordinate or ordinate of point P. Let OM = x and ON = y. Then the position of the point P in the plane with respect to the coordinate axes is represented by the ordered (x,y). The ordered pair (x,y) is called the coordinates of point P. This system of coordinating an ordered pair (x,y) with every point in a plane is called Rectangular Cartesian Coordinate system.



It follows from the above discussion that corresponding to every point P in the Euclidean plane there is a unique ordered pair (x,y) of real numbers called its **Cartesian coordinates**. Controversely, when we are given an ordered pair (x,y) and a Cartesian co-ordinate system, we can determine a point in the Eucldean or **Cartesian plane** having its coordinates (x,y). for this we mark-off a directed line segment OM=x on the x-axis and another directed line segment ON=y on y-axis. Now, draw perpendicular at M and N to X and Y axes respectively. The point of intersection of these two perpendiculars determines point P in the Euclidean space having coordinates (x,y).

Thus, there is one-to-one correspondence between the set of all ordered pairs (x,y) of real numbers and the points in the Euclidean plane. The set of all ordered pairs (x,y) of real numbers is called the Cartesian plane and is denoted by R^2. Let X'OX and Y'OY be the coordinate axes. We observe that the two axes divide the Euclidean plane into four regions called the quadrants. The regions XOY, X'OY, X'OY' and Y'OX are known as the first Q_1, second Q_2, third Q_3 and fourth Q_4 quadrants respectively.
In view of the above sign convention the four quadrants are characterized by the following signs of abscissa and ordinates –

I quadrant : *x>0, y>0*
II quadrant : *x<0, y>0*
III quadrant : *x<0, y<0*
IV quadrant : *x>0, y>0*

The coordinates of the origin are taken as (0,0). The coordinates of any point on x-axis are of the form (x,0) and the coordinates of any point on y-axis are of the form (0,y). Thus, if the abscissa of a point is zero, it would lie somewhere on the y-axis and it its ordinate is zero it would lie on x-axis.

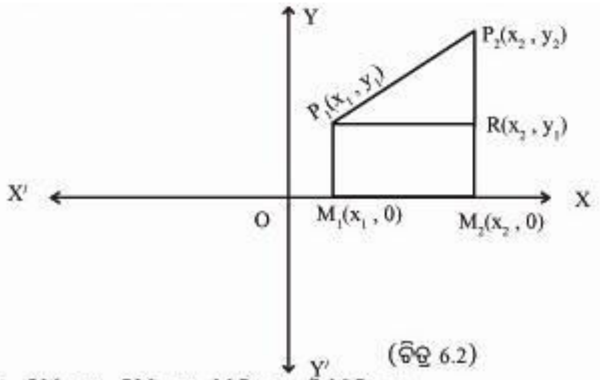
6.3 Distance between two given points

The distance between any two points in the plane is the length of the line segment joining them.

Theorem 1 : The distance between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

$$P_1P_2 = \sqrt{(x2-x1)^2+(y2-y1)^2}$$

Draw : P_1 and P_2 are two points on a plane. (x_1,y_1) and (x_2,y_2) are the coordinates. Join P_1 and P_2. Draw perpendiculars $\overline{P1M1}$ and $\overline{P2M2}$ from P_1 and P_2 on x-axis. From P_1 draw P_1R perpendicular to P_2M_2.



(ଚିତ୍ର 6.2)

Proof : $OM_1 = x_1$, $OM_2 = x_2$, $M_1P_1 = y_1$ ଓ $M_2P_2 = y_2$

Hence $P_1R = M_1M_2 = OM_2 - OM_1 = x_2 - x_1$

and $RP_2 = M_2P_2 - M_2R = M_2P_2 - M_1P_1 = y_2 - y_1$

Since P_1RP_2 Δ ରେ $m\angle P_1RP_2 = 90°$, Hence as per Pythogoras theoram

$(P_1P_2)^2 = (P_1R)^2 + (RP_2)^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

Since distance is a positive number

Hence $P_1P_2 = +\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ or $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Hence distance between any two points is given by

$$\sqrt{(diff.of\ abscissae)^2+(diff.of\ ordinates)^2}$$

Note 1 : If O (0,0) is the origin and P(x,y) is any point, then from the above formula, we have OP=$\sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2}$

Note 2 : If P_1P_2 are the points on x-axis, then $P_1P_2 = |x2-x1|$ and if it is on y-axis then $P_1P_2 = |y2-y1|$.

Example 1 : Find the distance between the points P(0, –5) and Q(4, –6).

Solution : Here $x_1 = 0$, $y_1 = -5$, $x_2 = 4$, $y_2 = -6$

$$PQ = \sqrt{(x1-x2)^2+(y1-y2)^2}$$
$$= \sqrt{(0-4)^2+(-5-(-6)^2} = \sqrt{-4^2+(-5+6)^2} = \sqrt{17} \text{ .....(Ans)}$$

Example 2 : Show that the points A(0,6), B(2,3) and C(4,0) are collinear.

Solution : $AB = \sqrt{(x1-x2)^2+(y1-y2)^2}$

$AB = \sqrt{(0-2)^2+(6-3)^2} = \sqrt{4\ +9} = \sqrt{13}$

$BC = \sqrt{(2-4)^2+(3-0)^2} = \sqrt{4+9} = \sqrt{13}$

$AC = \sqrt{(0-4)^2+(6-0)^2} = \sqrt{16+36} = 2\sqrt{13}$

$AC = AB + BC = \sqrt{13}+\sqrt{13} = 2\sqrt{13}$ Hence A, B, C are collinear..proved

Example 3 : Show that the points A(–2,3), B (5, –2), C(3,–4) are the vertices of an isosceles Δ.

Solution $AB = \sqrt{(-2-5)^2 + (3-(-2)^2} = \sqrt{49 + 25} = \sqrt{74}$

$BC = \sqrt{(3-5)^2 + ((-2)-(-4))^2} = \sqrt{4+4} = \sqrt{8}$

$AC = \sqrt{(-2-3)^2 + (3-(-4))^2} = \sqrt{25+49} = \sqrt{74}$

$\because AB = AC = \sqrt{74}$; $\therefore$ Δ is isosceles……………………….Proved

Exampale 4 : Find a point on the y-axis which is equidistant from the point A(6,5) and B(-4,3).

Solution : We know that a point on y-axis is of the form (0,y). So, let the required point be P(0,y). Then, AP = BP

$AP = \sqrt{(0-6)^2 + (y-5)^2} = BP = \sqrt{(-4-0)^2 + (3-y)^2}$

$= \sqrt{36 + (y-5)^2} = \sqrt{16 + (y-3)^2} = 36 + y^2 + 25 - 10y = 16 + y^2 + 9\ 6y$

$= 10y - 6y = 36 + 25 - 16 - 9 \Rightarrow 4y = 36 \Rightarrow y = 9$

So, the required point is (0, 9)………………………… Ans.

Example 5 : Prove that A(1,0), B(5,3) and C(4, –4) are the vertices of a right angles triangle isosceles triangle.

Solution : The given three points are A(1,0), B(5,3) are C(4, –4)

$AB = \sqrt{(1-5)^2 + (0-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(5-4)^2 + (3-(-4))^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$CA = \sqrt{(4-1)^2 + (-4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\because AB = CA = 5$; $\therefore$ right angles Δ is isosceles……………………….Proved

$\because BC^2 = AB^2 + AC^2 \Rightarrow 5^2 + 5^2 = 50$

Or $BC = \sqrt{50} = 5\sqrt{2}$ $\therefore$ right angles Δ is isosceles……………………….Proved

Example 6 : A(3,5) and B(–2,4) are two points. A bisector intersects the line $\overline{AB}$ at a point C on Y-axis. Find the co-ordinates of C.

Solution : Since the point C is lying on Y-axis, its co-ordinates are (0,y). As C is the point of bisector which intersects line $\overline{AB}$, it is equidistant from point A and B. That means AC=BC.

$AC = \sqrt{(3-0)^2 + (5-y)^2}$ and $BC = \sqrt{(-2-0)^2 + (4-y)^2}$

$\therefore AC = BC \Rightarrow \sqrt{(3-0)^2 + (5-y)^2} = \sqrt{(-2-0)^2 + (4-y)^2} \Rightarrow 3^2 + (5-y)^2 = (-2)^2 + (4-y)^2$

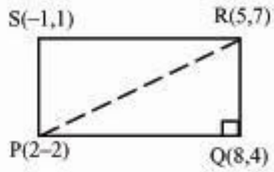
$\Rightarrow 9 + 25 - 10y + y^2 = 4 + 16 - 8y + y^2 \Rightarrow 2y = 14 \Rightarrow y = 7$

$\therefore$ Coordinates of C are (0,7) ……………………………………(Ans)

Example 7 : Prove that four point P(2–2), Q(8,4), R(5,7) and S(–1,1) are the vertices of a rectangle.

Solution : $PQ = \sqrt{(8-2)^2 + (4-(-2))^2} = \sqrt{6^2 + 6^2} = 6\sqrt{2}$;

$QR = \sqrt{(5-8)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$;



$RS = \sqrt{(-1-5)^2 + (1-7)^2} = \sqrt{(-6)^2 + (-6)^2} = 6\sqrt{2}$ and

. $SP = \sqrt{(2-(-1))^2 + (-2-1)^2} = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$

i.e PQ = RS & QR = SP

again $PR^2 = (5-2)^2 + \{7-(-2)\}^2 = 3^2 + 9^2 = 90$

and $PQ^2 + QR^2 = (6\sqrt{2})^2 + (3\sqrt{2})^2 = 90 = PR^2 \Rightarrow m\angle PQR = 90^0$

$\therefore$ PQRS is a rectangle (Proved)

Prove of PQ=RS is enough for proving PQRS is a rectangle.

EXERCISE 6 (a)

1. Find the distance between the following two points.

(i) (0, 0) and (4, 3)
(ii) (0, 2) and (-6, 2)
(iii) (-3, 0) and (5, 6)
(iv) (2, 4) and (1, 3)
(v) (-2, -2) and (-3, -5)
(vi) (a, -b) and (-a, b)

2. Find a point which is equidistant from the point

(i) (0, 1) and (-1, 0)
(ii) (2, 3) and (4, $\frac{3}{2}$)
(iii) ($\sqrt{17}, \sqrt{19}$) and ($-\sqrt{17}$, - $\sqrt{19}$)
(iv) (4, –2) and (2, 4)
(v) (0, 4) and (2, 2)

3. Show the points given in below are vertices of a right angled triangle. Find that which point forms the right angle.

(i) A (3, 3), B(9, 0) and C(12, 21)
(ii) A(1, 1) B(3, 4) and C(0, 6)
(iii) A(–1, –2), B(5, –2) and C(5, 6)
(iv) A(12, 8), B(–2, 6) and C(6, 0)
(v) A(1, 6), B(5, –1) and C(7, 2)

4. Show the points given in below are vertices of an isosceles triangle.

(i) A (8, 2), B(5, –3) and C(0, 0)
(ii) A(0, 6) B(–5, 3) and C(3, 1)
(iii) A (8, 9), B(–6, 1) and C(0, –5)
(iv) A(7, 1) B(11, 4) and C(4, –3)
(v) A (0, 0), B(4, 0) and C(0, – 4)
(vi) A(2, 2) B(–2, 4) and C(2, 6)

5. Do the points given below form a triangle mentioned in brackets.

(i) (1, 1), (–1, –1), (- $\sqrt{3}$, $\sqrt{3}$) (Equilateral triangle)
(ii) (3, –3), (–3, 3), (3$\sqrt{3}$, 3$\sqrt{3}$) (Equilateral triangle)
(iii) (1, 2), (3, 4) and (5, 8) (Scalene triangle)
(iv) (1, 2), (2, 4) and (3, 5) (Scalene triangle)
(v) (–2, 3), (8, 3) and (6,7) (Right angle triangle)
(vi) (–6, –8), (–16, 12) and (–26, –18) (Right angle isosceles triangle)

6. Do the points given below form the figures given in brackets.

(i) (–8, 3), (–2, –1), (6, –2) and (0, 2) (Parallelogram)
(ii) (–2, –1), (1, 0), (4, 3) and (1, 2) (Parallelogram)
(iii) (0, –1), (2, 1), (0, 3) and (–2, 1) (square)
(iv) (0, 5), (–1, 2), (–4, 3) and (–3, 6) (square)
(v) (–2, 3), (–4, –1), (–6, 0) and (–4, 4) (rectangle)

7. Show that point P(1,1) is equidistant from points A (0,2), B (2, 0) and C (0, 0).
8. If the point C (x, 3) is equidistant from points A (2, 4) and B (3, 5), find the value of x.
9. Find the value of y if P(2,y) is 5 units away from Q(-1,2).
10. Show that the points A (1, 1), B (2, 2) and (C (3, 3) are collinear.
11. Show that the points A (1, 4), B (–1, 6), C (2, 3) are collinear.
12. Prove that the points (1, 0), (2, –3) (–1, 6) are collinear, and the point (1, 0) is the mid points of other two points.
13. Find a point on x axis which is equidistant from the points (5, 4) and (–2, 3).
14. If O (0, 0), A (1, 2), B (3, 8) and C (3, –1), show that AB = 2CO.
15. The two vertices of an equilateral triangle is (0, 3), (4, 3). Find the third vertex.

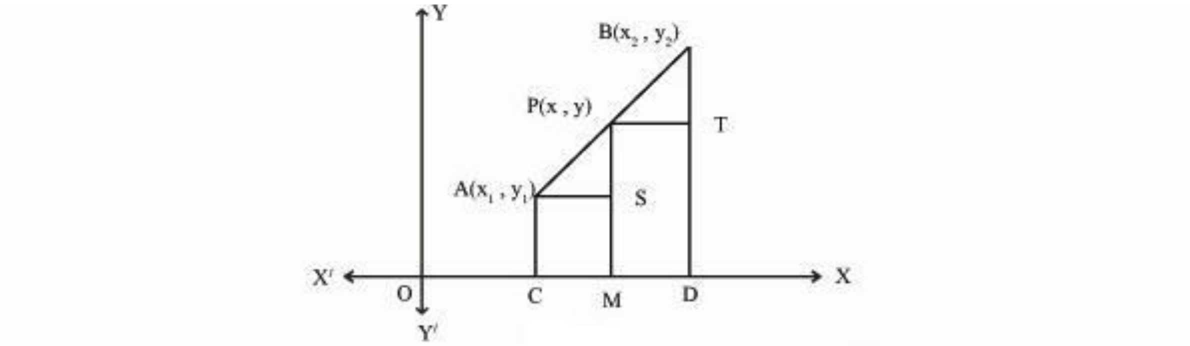
6.4 Division Fermulae

Let A and B be two points in the plane of the paper as shown below and P be a point on the segment joining A and B such that where AP + PB = AB and AP:BP = m:n. Then, we say that the point P divides segment AB internally in the ration of m:n, hence $\frac{PA}{PB} = \frac{m}{n}$.

But point P divides the line segment BA internally in the ratio of r:s then $\frac{PB}{PA} = \frac{r}{s}$

Theoram 2 :
Prove that the coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m:n are given by

$$(x= \frac{mx2+nx1}{m+n}, \ y = \frac{my2+ny1}{m+n})$$



Given : P is the point divides the line segment AB such that $\frac{PA}{PB} = \frac{m}{n}$. The coordinates of A B and P are (x_1, y_1), (x_2, y_2) and (x,y) respectively.
Proof : Coordinates of P(x,y) = $(\frac{mx2+nx1}{m+n}, \frac{my2+ny1}{m+n})$
Draw $\overline{AC}, \overline{PM}, \overline{BD}$ from the point A, P and B on x- axis such that $\overline{AS} \perp \overline{PM}$, $\overline{PT} \perp \overline{BD}$.
In a Δ ASP and Δ PTB

$m\angle PSA = m\angle BTP = 90^0$

$m\angle PAS = m\angle BPT$ (Corresponding angles)

∴Δ ASP and Δ PTB are similar hence Δ ASP ~ Δ PTB
Hence $\frac{AS}{PT} = \frac{PS}{BT} = \frac{PA}{PB} = \frac{m}{n}$ i.e. $\frac{AS}{PT} = \frac{m}{n}$ and $\frac{PS}{BT} = \frac{m}{n}$
AS = CM = $x - x_1$, PT = MD = $x_2 - x$ and PS = PM – SM = PM – AC = $y - y_1$
BT = BD – TD = TD – PM = $y_2 - y$
$\frac{AS}{PT} = \frac{x-x1}{x2-x1} = \frac{m}{n}$ => $mx_2 - mx = nx - nx_1$ => $mx_2 + nx_1 = mx + nx$
=> x(m + n) = mx2 + nx1 => $x = \frac{mx2+nx1}{m+n}$

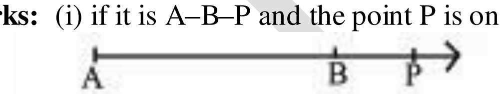
$\frac{PS}{BT} = \frac{y-y1}{y2-y1} = \frac{m}{n}$ =>my2 – my = ny – ny1=> my2 + ny1 = my + ny
=> y (m + n) = my2 + ny1=> $y = \frac{my2+ny1}{m+n}$

Hence the coordinates of the point P (x,y) which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio m:n are given by

$$P(x,y) = (\frac{mx2+nx1}{m+n}, \frac{my2+ny1}{m+n})$$

Note : Point A, B and P may present in any of the quadrant, the coordinates of P(x,y) is above formulae only.

Remarks: (i) if it is A–B–P and the point P is on $\overrightarrow{AB}$, the $\overline{AB}$ is divided by point P into $\overline{AP}$ and $\overline{BP}$.



(ii) Here the ratio of external segment AP : BP and AP–PB = AB
(iii) If $\frac{AP}{BP} < 1$, then P –A–B and $\frac{AP}{BP} > 1$, then A–B–P
(iv) If the point A(x1, y1) and B(x2, y2) join together and form $\overline{AB}$ and P(x,y) divides it into m:n externally then P(x,y) = $(\frac{mx2+nx1}{m-n}, \frac{my2+ny1}{m-n})$
Note : If P is the mid point of line segment $\overline{AB}$, them m=n and the coordinates of P is

$$P(x,y) = (\frac{x1+x2}{2}, \frac{y1+y2}{2})$$

Example 8 : Find the coordinates of the point which divides the line segment joining (1,–2) and (–3,–4).
Solution : The coordinates of the given points is A(1, –2) and B(–3, –4) and let the mid-point which divides the line segment be P(x,y).

Here $x_1 = 1$, $y_1 = -2$, $x_2 = -3$, $y_2 = -4$

The "x" coordinates of mid-point = $\frac{x1+x2}{2} = \frac{1-2}{2} = -1$

"y" coordinates of mid-point = $\frac{y1+y2}{2} = \frac{-2-4}{2} = -3$

∴ The coordinates of mid-point = -1 and -3

Example 9 : Find the coordinates of the end point of the line segment, if the coordinates of the point at the beginning and mid-point is (3, 5) and (2, 1).

Solution : Let $P(x_2, y_2)$ be the end point of the line segment.

Coordinates of point at the beginning is (x1, y1) = (3, 5) and the mid-point is (x, y) = (2, 1)

$x=\frac{x1+x2}{2}$ or $x_2 = 2x - x_1 = 2\,x\,2 - 3 = 1$

$y=\frac{y1+y2}{2}$ or $y2 = 2y - y1 = 2 \times 1 - 5 = -3$

∴ coordinates of end point of line segment is (1, -3)

Example 10 : Find the coordinates of the point which divides the line segment joining the points A(2, 3) and B(5, -3) in the ration1:2 internally.

Here $x_1 = 2$, $y_1 = 3$; $x_2 = 5$, $y_2 = -3$; $m = 1$, $n = 2$

Let P(x,y) be the required point, then,

$$x=\frac{mx2+mx1}{m+n} = \frac{1x5+2x2}{1+2} = 3$$

$$y=\frac{my2+my1}{m+n} = \frac{1x(-3)+2x3}{1+2} = 1$$

Hence the coordinates of P are (3, 1) which divides the line segment $\overline{AB}$

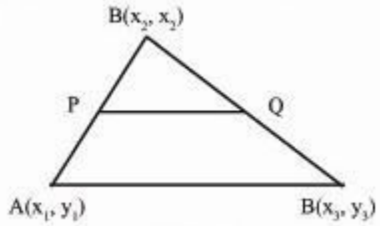
Example 11 : Using co-ordinate geometry, prove that the length of the line joining the mid points of the two sides of a triangle is half of its third side.

Let ABC be the triangle and the co-ordinates for points A, B, C are (x1, y1), (x2, y2) and (x3, y3)

P and Q are the mid points on line $\overline{AB}$ and $\overline{BC}$

coordinates of P $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and

coordinates of Q $= \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$



$$AC = \sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$$

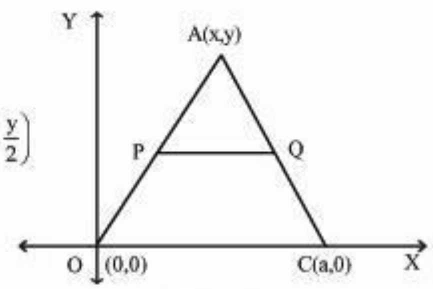
and $PQ = \sqrt{\left\{\frac{(x_2+x_3)}{2}-\frac{(x_1+x_2)}{2}\right\}^2+\left\{\frac{(y_2+y_3)}{2}-\frac{(y_1+y_2)}{2}\right\}^2}$

$= \sqrt{\frac{1}{4}(x_3-x_1)^2+\frac{1}{4}(y_3-y_1)^2} = \frac{1}{2}\sqrt{(x_3-x_1)^2+(y_3-y_1)^2} = \frac{1}{2}AC$ (Proved)

Alternate Method

In ΔOAC O(0,0), C(a,0) and A(x,y)

Coordinate of P and Q $P\left(\frac{x}{2},\frac{y}{2}\right)$ $Q\left(\frac{x+a}{2},\frac{y}{2}\right)$



$$PQ = \sqrt{\left(\frac{x}{2}-\frac{x+a}{2}\right)^2+\left(\frac{y}{2}-\frac{y}{2}\right)^2}$$

$= \sqrt{\left(-\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{4}} = \frac{1}{2}a = \frac{1}{2}OC$ $\therefore PQ = \frac{1}{2}OC$ (Proved)

Exercise 6(b)

1. Choose the correct answer

i. If $(1, -2)$ is the coordinates of a point which divides the line joining the coordinates of $(4, 2)$ and $(K, -6)$, find the value of k. [-2, 2, -4, 4]

ii. _______________ is the coordinates of mid-point of the line joining the coordinates $(-2, 3)$ and $(3, -2)$. [$(1,1)$, $(\frac{1}{2}, \frac{1}{2})$, $(\frac{5}{2}, \frac{5}{2})$, $(-\frac{1}{2}, -\frac{1}{2})$]

iii. The mid-point of a line segment is its starting point. If the coordinates of one end of the line segment is $(2, 3)$, find the coordinates of other end of the line segment. [$(-2, 3)$, $(2, -3)$, $(-2, -3)$, $(\frac{1}{2}, \frac{3}{2})$]

iv. Find the coordinates of the point which divides the line segment joining $(0, 2)$ and $(2, 0)$ internally in the ratio 3:2. [$(\frac{4}{3}, \frac{2}{3})$, $(\frac{2}{3}, \frac{4}{3})$, $(-2,4)$, $(4,-2)$]

2. Find the coordinates of the mid-point of in each of the following given points which divides the line segment joining :

(i) $(3, 4)$, $(1, -2)$ (ii) $(-1, 3)$, $(4, 0)$, (iii) $(\frac{1}{2}, \frac{1}{3})$, $(\frac{1}{3}, \frac{1}{2})$
(iv) $(0, -3)$, $(-4, 0)$ (v) $(-1, -2)$, $(3, -1)$ (vi) (a, b), (c, d)
(vii) $(-2, 1)$, $(-3, -4)$ (viii) $(at_1^2, 2at)$, $(at_2^2, 2at_2)$

3. The coordinates of the mid-point $(-1, 2)$ which joins the two points of a line segments given below. Find the value of h and k in each line point.

(i) $(h, -1)$, $(2, k)$ (ii) $(5, 3)$, (h, k)
(iii) $(1 + h, k)$, $(k, -h - 1)$ (iv) $(h - k, k - h)$, $(2h, 2k)$

4. $(0, 0)$ is the coordinates of a mid-point of a given line segment. If the coordinates of one end of the line segment is $(2,3)$, find the coordinates of other end.

5. If the one end point and mid-point of a line segment is $(-2, 4)$ and $(1, 2)$ respectively, find the coordinates of other end point of line segment.

6. If the one end point and mid-point of a line segment is $(3, 5)$ and $(2, 1)$ respectively, find the coordinates of other end point of line segment.

7. For what value of X and Y, the line joining the coordinates $(6, -2)$ and $(2, -4)$ and the line joining the coordinates $(x, 1)$ and $(-2, y)$ bisect each other.

8. Find the coordinates of the point which divides the line segment joining the points $(2, 3)$ and $(1, 4)$ in the ratio 3:2 internally.

9. Find the coordinates of the point which divides the line segment joining the points $(-2, 3)$ and $(5, -7)$ in the ratio 3:4 internally.

10. If the coordinates of the point $(5, 9)$ which divides the line segment joining the points $(7, -3)$ and $(4, k)$ in the ratio 1:2 internally. Find the value of k.

11. Using co-ordinate geometry, prove that the medians of any triangle are concurrent.
NB : The point of intersection of the medians of a triangle is called centroid.
The centroid divides the medians of a triangle in 2:1 ratio internally.

12. A triangle is made of points $(h,5)$, $(-4,k)$ and $(8,9)$ respectively and centroid is $(-2, 6)$, find the value of h and k.

13. The centroid of a ΔABC is $(1,1)$. If the coordinates of $A(3,-4)$ and $B(-4, 7)$, find the coordinates of C.

14. If the vertices of a ΔABC are (-4, 1), (3, -4) and (1, 3), show that the centroid of triangle is a starting point.

15. The coordinates of A and B are (1, 2) and (5, -4). Put a point on the line segment $\overline{AB}$ such that the distance between the point from A is three times of point B.

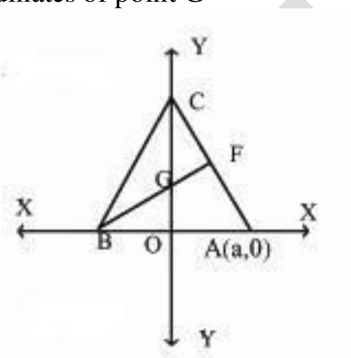
16. Find the coordinates of the point which divides the line segment joining the coordinates (1,5) and (7,2).

17. Show that O(0,0), A(2a, 0) and B(0, 2b) form a right angle triangle and the median of the hypotenuse is equidistant from its vertices.

18. Using coordinate geometry, prove that the diagonals of a parallelogram bisect each other.

19. Using coordinate geometry, show that the diagonals of a rectangle are equal and bisect each other.
    Note : In a quadrilateral ABCD, take (0,0), (a,0), (a,b) and (0,b) as the coordinates of point A, B, C, and D respectively.

20. In given picture ΔABC is an equilateral triangle. If (a,0) is the coordinate of point A,
    i. Find the coordinates of other two vertices.
    ii. Find the lengths of the sides.
    iii. Find the lengths of the median $\overline{BE}$.
    iv. Find the coordinates of point G



6.5 Area of a Triangle :

As we know that the area of a Trapezium = $\frac{1}{2}$ * altitude * (sum of the lengths of parallel sides)

We can obtain the area of a triangle using the above formula, if the vertices of a triangle are given.

Theorem – 3

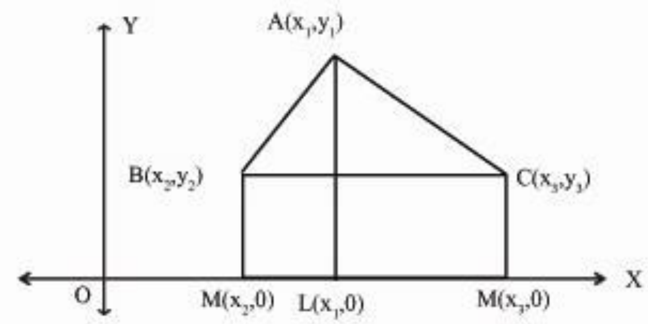
The area of a triangle, the coordinates of whose vertices are (x1, y1) (x2, y2) and (x3, y3) is

$$\frac{1}{2}\left|\{x1\,(y2 - y3\,) + x2\,(y3 - y1\,) + x3\,(y1 - y2\,)\}\right|$$

($\because$ Area of the triangle is positive modulus | | is used.)

Take a triangle ABC on a plane surface, the coordinates of vertices ABC are (x1, y1) (x2, y2) and (x3, y3)

Proof : the area of the ΔABC = $\frac{1}{2}\left|\{x1\,(y2 - y3\,) + x2\,(y3 - y1\,) + x3\,(y1 - y2\,)\}\right|$



Draw $\overline{AL}$, $\overline{BM}$ and $\overline{CN}$ perpendicular from A,B,C on the x-axis, clearly ABML, ALNC and BMNC are all trapeziums.

As per coordinate geometry, $OL = x_1$, $OM = x_2$, $ON = x_3$ and $AL = y_1$, $BM = y_2$, $CN = y_3$,

$ML = OL - OM = x_1 - x_2$ and $MN = ON - OM = x_3 - x_2$.

From the above figure, it is clear that the

Area of ΔABC= Area of trapezium ALMB + area of trapezium ALNC – Area of trapezium BMNC

$= \frac{1}{2} ML(LA + MB) + \frac{1}{2} LN(LA + NC) - \frac{1}{2} MN(MB+NC)$

($\because$ area of a Trapezium = $\frac{1}{2}$ * altitude * (sum of the lengths of parallel sides))

$= \frac{1}{2} [(x_1 - x_2)(y_1 + y_2) + (x_3 - x_1)(y_1 + y_3) - (x_3 - x_2)(y_2 + y_3)]$

$= \frac{1}{2} [x_1(y_1 + y_2 - y_1 + y_3) - x_2(y_1 + y_2 - y_2 - y_3) + x_3(y1 + y_3 - y_2 - y_3)]$

$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2} |\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$ ………………………..(Proved)

Remark 1: Three points A(x1, y1), B(x2, y2) and C(x3, y3) are collinear if

Area of Δ ABC = 0 i.e. $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

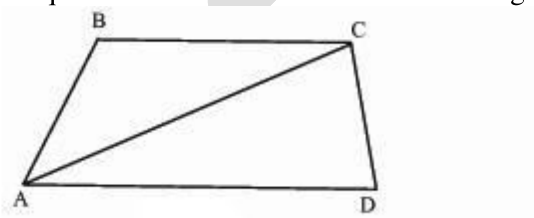
Remark 2: In a triangle, if the coordinates of a vertex is ______________,

Area of the triangle = $\frac{1}{2} [x_1y_2 - x_2y_1]$

The coordinates of vertices ABC will be (x_1, y_1), (x_2, y_2) and (0, 0).

Remark 3 : Let ABCD be a quadrilateral and $\overline{AC}$ be the diagonal. We get two triangles i.e. ΔABC and ΔACD respectively. Hence the

Area of the quadrilateral = Sum of the areas of triangles.



Note : As we have already discussed about 2 x 2 Matrix in our first lesson, where

Area of the triangle = $\frac{1}{2} |\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$

If 3 x 3 determinant of matrix,

Area of the triangle = $\frac{1}{2} |\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$

$$\text{Area of a Triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left\{x_1\begin{vmatrix} y_2 & 1 \\ y_3 & 1 \end{vmatrix} - x_2\begin{vmatrix} y_1 & 1 \\ y_3 & 1 \end{vmatrix} + x_3\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix}\right\},$$

Example 12 : Find the area of a triangle, coordinates of whose vertices are (1, 3),(–7, 6) and (5, –1).

Solution : Here $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (-7, 6)$, $(x_3, y_3) = (5, -1)$

Area of the ΔABC = $\frac{1}{2}\left|\{\mathbf{x1}\,(\mathbf{y2} - \mathbf{y3}) + \mathbf{x2}\,(\mathbf{y3} - \mathbf{y1}) + \mathbf{x3}\,(\mathbf{y1} - \mathbf{y2})\}\right|$

Area of the ΔABC = $\frac{1}{2}\left|1\{6 - (-1)\} + (-7)(-1-3) + 5(3-6)\right|$

$= \frac{1}{2}\left|(7 + 28 - 15)\right| = 10$ ………………………………….. (Ans)

Example 13 : Prove that the points A(1, 2), B(0, 5), and C(2, –1) are collinear.

Solution : Here (x1, y1) = (1, 2), (x2, y2) = (0, 5) , (x3, y3) = (2, –1)

Area of the ΔABC = $\frac{1}{2}\left|\{\mathbf{x1}\,(\mathbf{y2} - \mathbf{y3}) + \mathbf{x2}\,(\mathbf{y3} - \mathbf{y1}) + \mathbf{x3}\,(\mathbf{y1} - \mathbf{y2})\}\right|$

Area of the ΔABC = $\frac{1}{2}\left|1\{5 - (-1)\} + 0(-1, -2) + 2(2-5)\right|$

$= \frac{1}{2}\left|(6 + 0 - 6)\right| = 0$ ………………………………….. (Proved)

Hence, the points A(1, 2), B(0, 5), and C(2, –1) are collinear.

Example 14 : Find the area of the quadrilateral whose vertices are respectively A (–2, 1), B (1, 0), C (2, 3) and D (0, 4).

Solution : Let ABCD be the quadrilateral and diagonal $\overline{AC}$ is drawn, we get triangles ΔABC and + ΔACD.

∴ Area of the quadrilateral = Area of ΔABC + Area of ΔACD

In the area of ΔABC = (x1, y1) = (–2, 1),

(x2, y2) = (1, 0), (x3, y3) = (2, 3)

In the area of ΔACD = (x1, y1) = (–2, 1),

(x2, y2) = (2, 3), (x3, y3) = (0, 4)

Hence the area of quadrilateral ABCD

$$= \frac{1}{2} \mid (-2)(0-3) + 1(3-1) + 2(1-0) \mid + \frac{1}{2} \mid (-2)(3-4) + 2(4-1) + 0(1-3) \mid$$

$$= \frac{1}{2} \mid 6 + 2 + 2 \mid + \frac{1}{2} \mid 2 + 6 + 0 \mid$$

$$= \frac{1}{2} * 10 + \frac{1}{2} * 8 = 5 + 4 = 9 \text{ square units}$$

$$\text{Area of a Triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left\{ x_1\begin{vmatrix} y_2 & 1 \\ y_3 & 1 \end{vmatrix} - x_2\begin{vmatrix} y_1 & 1 \\ y_3 & 1 \end{vmatrix} + x_3\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \right\},$$

Exercise C

1. Choose the correct answer from the brackets given below.

(i) The area of the triangle is ____________, if coordinates of the vertices of triangle are (2,5), (–3,5) and (0,5). [–5, 3, 0, 10]

(ii) The vertices are (a, –2), (2, 5) and (2, 10) are collinear, if a=________. [0, 3, 2, –2]

(iii) If (–2, –2), (0, y) are (3, 3) are three vertices which are collinear, find y=________ [0, 2, 2, 3]

(iv) If vertices (k, –2), (1, 4) and (–2, 7) are collinear, find k = _________. [3, –3, 2, –2]

(v) The coordinates of a triangle (4, –5), (1, a) and (–2, 7) are not the vertices, if value a=______. [1, 2, 3, 4]

2. Find the areas for the following given vertices of the triangles.

(i) (3, 0), (4, 5) and (2, 0) (ii) (0,0), (1, 0) and (1, 1) (iii) (–2, 1), (2, –3) and (4, –4) (iv) (5, 7), (6, 4) and (2, –5) (v) (5, 2), (–1, 3) and (1, –2)

3. Prove that the given three vertices of each are collinear.

(i) (1, 1), (4, 3) and (–2, –1) (ii) (–1, –5), (0, –3) and (4, 5)

(iii) (1, 4), (3, -2) and (-3, 16) (iv) (–4a, –6a), (–a, –2a) and (5a, 6a)

(v) (–a, 2b), (0, b) and $\left[\frac{a}{2}, \frac{b}{2}\right]$

4. Find the value of x, if vertices of a triangle are (1, -3), (2, –5) and (x, 1) and area of triangle is 4 square units.

5. For what value of k, the area of the triangle is $\frac{19}{5}$ square units whose vertices are (3, -5), (k, 0) and (-4, 7).

6. Find the value of y, if the points (2, 3), (0, 5) and (1, y) lie on a line.

7. For what value of k, the points (2, 3), (3, k), and (x, y) lie on a line.

8. Find that on which formula, the three points (1, 1), (3, 5) and (y, y) lie on a line.

9. Find the area of the quadrilateral whose vertices are (1, 0), (2, 4), (0, 5) and (-2, 1).

10. Find the area of the quadrilateral whose vertices are (-2, 3), (3, 2), (7, 4) and (1, 5).

11. In a ΔABC, the coordinates of A is (1, 1) and if D(-1, -2) and E(3, 2) are the mid-points of $\overline{AB}$ and $\overline{AC}$, find the area of the ΔABC.

12. Find the value of P, if the points (3, 0), (5, -1) and (p, p) are collinear.

13. Find the value of P, if the points (p, 2p), (3p, 3p) and (3, 1) are collinear.

14. Find the value of x, if the points (x, -1), (2, -1) and (2, 1) are collinear.

15. If (x, y) is any point on the line joining the points (a, 0) and (0, b), then show that $\frac{x}{a} + \frac{y}{b} = 1$.

16. Prove that three points (a, b), (a, b) and (a-a, b-b) are not collinear.

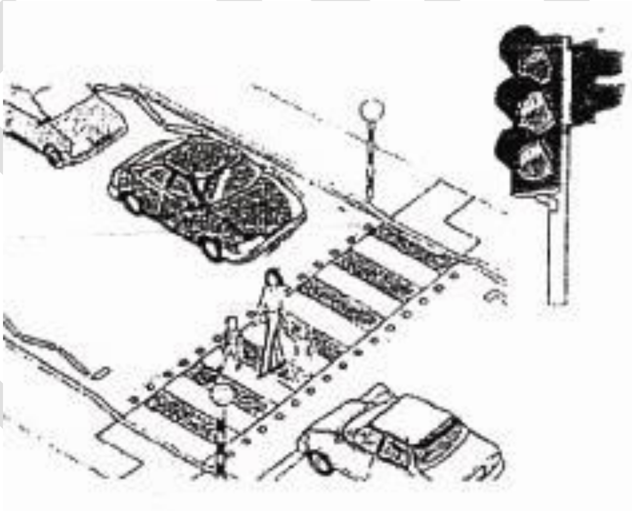
17. Find the value of p, if the points A(p+1, 1), B(2p+1, 2) and C(2p+2, 2p) are collinear.

18.If three points (x, y), (3, 4) and (-5, -6) are collinear, prove that 5x – 4y + 1 = 0.

Chapter 7
ROAD SAFETY EDUCATION

7.1 **Purpose** - All traffic signs have to be crossed while travelling on the road. The purpose of this textbook is to provide a mathematical overview of the process by creating an Arithmatic Sequence based on the distance between the traffic signals and the time it takes to cross them. As the topic progresses in parallel, we discuss the sequence of numbers and their parallel classes, as well as time and distance. For example, when a car or light vehicle or goods van crosses a distance from one place to another on the road and we can make a arithmetic progression based on the time it takes to cross that distance.

Example 1 - The distance between two points A and B is 150 km. There are 10 traffic signs between A and B. A car is travelling at a speed of 60 km per hour. Car starts from point A at high speed, cross all traffic signals and reach B in 2 hours and 30 minutes. But on other days, due to the overcrowding, the car has to be parked near the various traffic lights below. First Traffic Sign: 1 Minute, Second Traffic Sign: 2 Minutes and 10 Minutes Up to the 10th Traffic Sign it is 10 minutes - Picture [2]



If the speed of the car is 60 km / h. If the car is in compliance with all traffic laws, determine the time taken by the car.

Answer: The traffic stop time of the car in 1 to 10 signals are in arithmetic progression like 1, 2, 3, ..., 10

A.P. a = 1, d = 1 and n = 10
.: Total 'Stop Time = $\frac{10\ (10+1)}{2}$ = 55 minutes
The car takes 2 hours 30 m to cover the distance between A to B at speed of 60 km / h. without stopping at any Traffic signal.
= 2 hrs 30 m + 55 m = 3 hours 25 minutes

Example 2 - Ashok was driving on a road when the first, second and third traffic Cross the lights in 5, 12 and 19 seconds respectively. Which number of traffic lights will be crossed by the car in 75 seconds if you continue to cross the traffic light in a sequence?

Answer: Given timing is in Arithmetic progression i.e. 5, 12, 19 ....
Hence a= 5 and d = 12 - 5 = 7

Suppose Ashok passes the n traffic light in 75 seconds.
t = a + (n -1) d
=> 75 = 5 + (n -1) 7
=> 7n -2 - 75 = 0
=> 7n = 77 = >n = 11
.: Ashok will cross 11 traffic lights in 75 seconds.

Example 3: The first, second and third traffic signals on any straight road are situated at 3 km, 4 km and 7 km respectively. How far will the 10th traffic signal be in this order ?

Answer: The distance of a given traffic signal is Arithmetic Progression. E.g., 3, 5, 7 ....
Here a = 3, d = 5 - 3 = 2

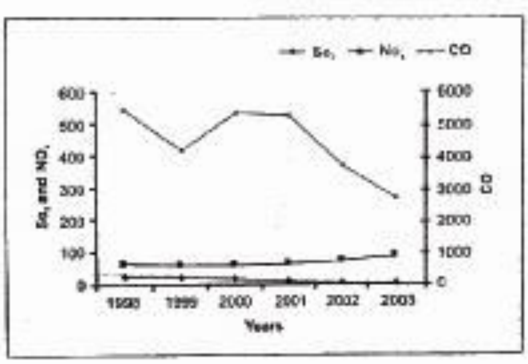
The distance to the 10th traffic signal = t_{10} = a + (n -1) d
= 3 + (10-2) 2 = 21
.: The distance to the 10th traffic signal is 21 km.

Example - 4: The distance of traffic lights installed on any road is in Arithmetic Progression. If the distance of the third light is 1500 meters and the distance of the eighth light is 3000 meters, and determine the distance of the 15th light.

Answer: The n^{th} position of the Arithmetic Progress t_n = a + (n -1) d
As per question, t_3 = 1500 and t_8 = 3000
a + (3–1) d = 1500
=> a + 2d = 1500................(i) and
a + (8 -1) d = 3000
=>a + 7d = 3000 ....(ii)
Subtract (i) from (ii),
5d = 1500 = d = 300
Substitute the value of 'd' in (i),
a + 2 x 300 = 1500 => a = 900
. : Distance of 15th light: t_{15} = a + (15 -1) d
= 900 + 14 x 300 = 5100 meters.

7.2 Application of Statistics: - Purpose: Pollution caused by vehicles reaches different levels at different times. It is imperative to reduce the level of air pollution in terms of environmental protection. Similarly, road accidents are on the rise for various reasons. It is also important to address this. Various Pollution and Road Accidents

The purpose of the textbook is to collect information about the subject and create a statistical document based on it to create public awareness through it.



Issue: The Pollution Control Certificate (PUC) is required for all types of diesel and petrol-powered vehicles under the 1989 Motor Vehicles Act of the Government of India. It was not so strict before. In major cities like Delhi, Bombay, Madras, Hyderabad and Bangalore, the government's concerns have escalated as pollution levels reach critical levels. The government has recently decided to strictly enforce the law to control the situation. Previously, low-pollution paper-based control letters were issued (Figure 4). If it sometimes lost or destroyed, online pollution control certificates have been issued since October 1, 2019. In which there is no fear of being lost and destroyed. The pollution certificate for new vehicles is valid for one year and for old vehicles for 6 months. Driving without a pollution control certificate carries a fine of Rs 2,000 or 3 months in jail (for the first time), a fine of Rs 4,000 for a second violation or imprisonment for up to 4 months. The main reasons for the increasing number of road accidents are: (i) reckless driving without complying with traffic rules (ii) driving under the influence of alcohol. (iii) speeding (iv) driving without a helmet, etc.

The government has taken steps to reduce the number of accidents. Traffic fines have been increased more than ever before. The following is a list of new traffic fines.

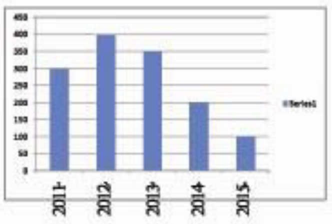
(i) Rs.1000 for driving without a helmet (ii) Rs.1000 for driving without seat belts (iii) Rs.5000 for non-compliance with the signal (iv) Rs.5000 for three people in the bay (v) Rs.5000 for driving without a license (vi) Rs 5,000 for under age (vii) Rs 5,000 reckless driving (viii) Rs 10,000 for not giving way to ambulance.

(ix) Driving under the influence of alcohol Rs 10,000 (x) Rs 10,000 talking on mobile phone while driving.

Example - 5: The diagram given shows the amount of pollutants in the atmosphere. In which year did the level of major pollutants reach its lowest level? Who will be credited with bringing pollution to a lower level?

Answer: In 2003, CO was the lowest pollution. The government's program is to (i) strictly enforce pollution control certificates. (ii) The government should strictly enforce CNG on vehicles. (iii) to raise awareness by advertising large-scale documentaries on pollution levels in crowded places.

Example - 6: The figure depicts the death toll from a road accident in a city in recent years. (a) How much increase or decrease of number of people died in accidents between 2011 and 2013? (b) Determine the percentage reduction or increase in the number of fatalities in accidents between 2012 and 2014.



Answer: (a) Death rate in 2011 = 300

Number of deaths in 2013 = 350

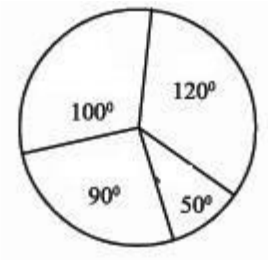
increase = 350 - 300 = 50%

increased rate = $\frac{50}{300}$ x 100 = $16\frac{2}{3}$ %

(b) Number of deaths in 2012 = 400

Number of deaths in 2014 = 200
Reduced = 400 - 200 = 200
Decrease rate= $\frac{200}{400}$ x 100 = 50%

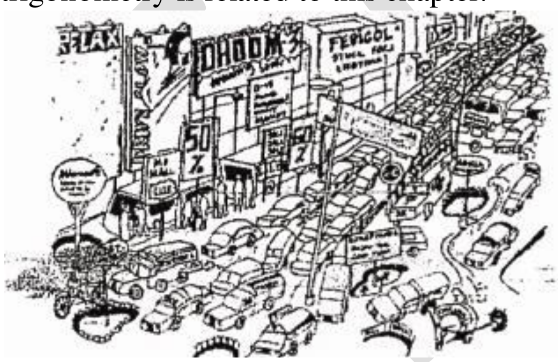
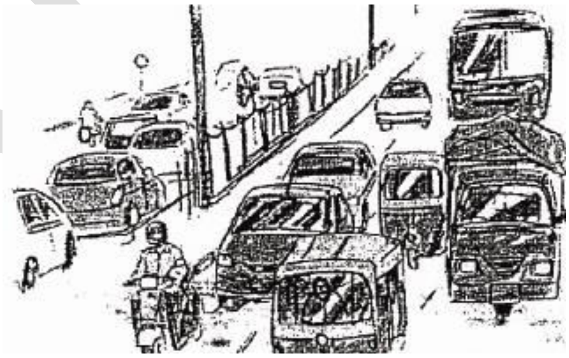
Example - 7: In the given circle, the number of road accidents in any city in 2018 is different. The death toll has risen sharply. If 10800 people died in road accidents that year, answer the following questions.



(a) What is the death toll from alcohol drinking?
(b) What is the death toll from speed driving?

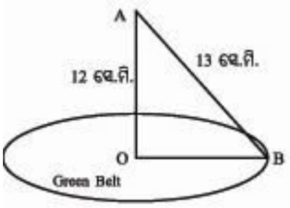
Answer: Total number of fatalities in road accidents = 10800
Degree measure = 360°
(a) Degree of persons involved in an accident while driving under the influence of alcohol = 120 °
Hence, the total number = $\frac{10800}{360}$ x 120 = 3600
(b) Degree of persons involved in an accident while speed driving = 90°
Accidents due to speeding vehicles = $\frac{10800}{360}$ x 90 = 2700 (b) Speed Degree of death by driving = 20 = 10800 360 X 90 = 2700

7.3 Application of Trigonometry:
Objectives: To avoid day by day increase in road accidents, lights are installed on the roads for lighing. CCTV cameras are also installed in various parts of the road to detect traffic violators. The purpose of this article is to describe how triangles can be applied in all these systems. **Topics:** Large buildings on the main street or on the road side, lights and CCTV are usually installed. Heights and distance of trigonometry is related to this chapter.

Example - 8: A CCTV camera was mounted on a high pillar of 12 cm high so that the 13 cm long line of sight could see all the traffic/vehicles moving.

In this case
(i) determine the distance from the foot of the column to the point where the traffic is visible.
(ii) How much spaces around the pillar can be covered by grass? (green belt)
(iii) Do you think CCTV cameras are really helpful in controlling traffic? If so, why?

Answer: (i) 

AB = 13 cm. (LOS)

OA = column height = 12 cm.

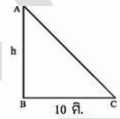
The distance from the foot of the column to the point where the traffic is visible = OB = $\sqrt{(AB^2 - OA^2)}$= 5 cm

(ii) Traffic will not be visible in the circular area covering around the pillar with a radius OB where the green belt can be made.

∴ Area of green belt = $\pi\ OB^2$= 25 π sq.m. .

(iii) CCTV cameras play an important role in traffic control. Traffic violators were caught by CCTV cameras.

Example 9: A four-way /square crossing where a CCTV camera is installed in the country. The angle of depression from the place of the pillar to foot of the car is 45°. What is the height of the pillar if the car is standing 10 meters away from the foot of the pillar?



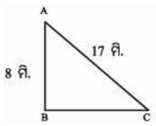
Answer: According to the picture, $\frac{h}{10}$ = tan 45°

=> $\frac{h}{10}$ = 1.

=> h = 10 m.

∴ The height of the pole is 10 meters.

Example: - 10: A 8-meter-high pole has a camera for traffic control. The camera can see traffic up to 17 meters from the top of the pillar. How much area can the camera control the movement of people around the pole?



Answer: According to the picture, the height of the pole = AB 8 meters

view point = AC = 17 meters

Right angle triangle AB

$AC^2 = BC^{\ 2} + AB^2$

=> 17^2 m = $BC^2 + 8^2$

=> $BC^2 = 17^2 - 8^2 = 15^2$

=> BC = 15

Area of viewable area around the pillar

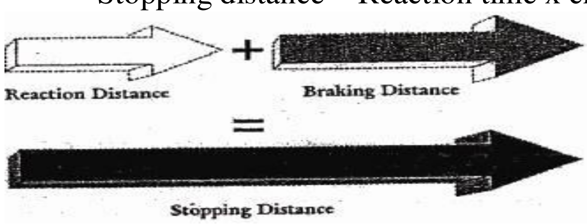
= $\pi(BC)^2$ = 3.14 x 225 = 706.5 square meters

7.4 Two-wheeled problems:

The purpose of this text is to solve the following equations with the help of road-based problems. –

Equation: Reaction distance + Breaking distance = stopping distance

Stopping distance = Reaction time x chasing distance



Chasing distance: The chasing distance of the vehicle in front is usually determined in seconds. This is calculated by the reaction distance and the stability distance. The easy way to count seconds was to count 19, 20, 21, etc. in one go. Each sequence is taken in a second. In the given figure, how much is the chasing distance in seconds between you and the vehicle you are chasing?

Example 11:

Speed km. / Hour (km / h)	Stopping distance (meters)	Reaction Distance (meters)	Chasing Distance (Seconds)
(i)	(ii)	(iii)	(iv)
30 60 90	18 54 108	9 18 -	2 - 4

Answer:

Total stability distance = reaction distance x Chase distance

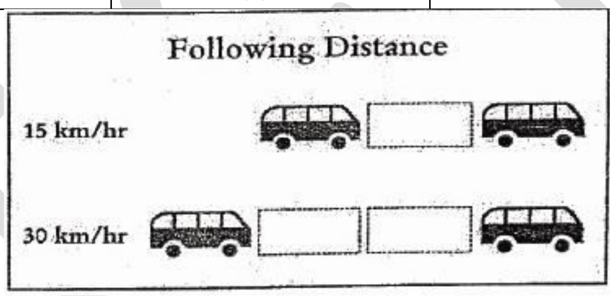
(i) 9 x 2 = 18

(ii) 54 = 18 x X => $\frac{54}{18}$ = 3 (The first blank will be filled)

(iii) 108 = X x 4 => = $\frac{108}{4}$ = 27 (The second blank will be filled)

After filling the blanks the table will look like

Speed km. / Hour (km / h)	Stopping distance (meters)	Reaction Distance (meters)	Chasing Distance (Seconds)
(i)	(ii)	(iii)	(iv)
30 60 90	18 54 108	9 18 27	2 3 4



Some important equations related to velocity :

(i) v = u + at

(ii) $v^2 = u^2 + 2as$

Example 12: A car has a speed of 50 km / h. If the stopping distance is 40 meters and the slow speed is 4.4 meter/s. Find the time taken by car to stop.

Answer: The starting speed of the car is u = 50 km / h.

= 50 x $\frac{5}{18} = \frac{125}{9}$ = meter/s.

The Stopping time S = 40 m

Slow velocity = 4.4 m./ Second

=> a = - 4.4 meter/s.

As per first equation of velocity : v = u + at

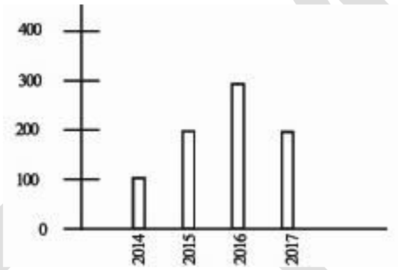
Where v, final velocity , stop time = 0

=>0=$\frac{125}{9}$ - 4.4t

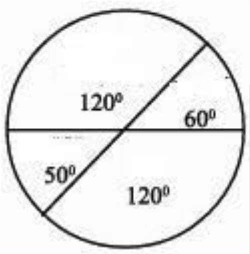
=> t = $\frac{125}{44x9}$ => t = 3.16 sec. Hence, it will take 3.16 seconds for the car to stop.

Exercise 7

1. Write down how Arithmetic progression can be applied in road safety education.
2. There are all traffic signs on one road at a distance of one kilometer. A car crossed 15 traffic signals in 15 minutes. Determine the speed of the car per hour.
3. A truck crossed the traffic light on the road in 10 minutes, 20 minutes, 30 minutes. How long did it take the truck to cross 15 lights?
4. The distance from the starting point of the LED light posts on a road in an Arithmetic Progression. The distance of the 5th light post is 45 km. and the distance of the 8th light poster is at 75 km. If a bus takes 2 hours to cross 10 light posts, determine the speed of the bus per hour.
5. The column details shows the number of people killed in road accidents in any city in recent years.
   (a) What is the growth rate of people who die in 2014-15?
   (b) What is the rate of growth or decline of households in 2016-2017?



6. The Helmet Rate Circle states that the number of other alcoholics who died in road accidents in 120 ° 60 ° of a city in 2018 due to various reasons is in degrees. 120 ° fast speed



   If a total of 720,000 people die in the city that year,
   (a) What is the death toll from alcoholism?
   (b) What is the death toll for other reasons?
   (c) What is the death toll due to no helmets?
7. A CCTV camera is mounted on a pole at a bus stand. The angle elevation of the CCTV camera from the point place on a platform situated 30 m distance from the foot of the pillar is 60°. Find the height of the pillar.
8. At the top of a 24-meter-high pole, a CCTV camera is installed so that 25 meters from the top of the pole can be seen moving traffic.
   (a) How far is the distance from the foot of the pole from where the traffic is visible?
   (b) What is the area of the green belt zone (invisible area) around the pole?
9. A CCTV camera is mounted on a pole 10 meters high on a square. A car was approaching the pillar. If the angle of depression of the car from the camera is changing from 45° to 60°, find the distance covered by the car.
10. A scooter was seen coming towards the pole of height 8 meter through CCTV camera. If the scooter takes 1 minute for crossing the angle of elevation of CCTV camera from 30° to 45°, determine the speed per hour of the scooter.
11. The speed of the car 60 km per hour. If the car stops after applying breaks at a distance of 50 meters and the de-accelerated speed of the car becomes is 5 meters per second, determine the arrival time of the car.
12. A CCTV camera is mounted on a pillar. The camera captures the traffic up to 25 meters from the foot of the pillar. The distance between the foot of the pillar and traffic is 24 meters. Find

the height of the pillar and the area around the pillar where the CCTV camera is able be capture traffic.

13. Fill in the blanks in the table below.

Speed km. / Hour (km / h)	Stopping distance (meters)	Reaction Distance (meters)	Chasing Distance (Seconds)
(i)	(ii)	(iii)	(iv)
30	20	5	-
45	64	16	4
60	98	-	7

14. Explain how you can use statistics for road safety.
15. Give ideas about government programs to prevent air pollution.
16. Write how trigonometry can be applied to road safety.
17. Answer the following questions.
    (a) What is the line of sight?
    (b) What is the green belt zone around the CCTV camera?
    (c) What is the reaction distance?
    (d) What is the breaking distance?
    (e) What is the stopping distance?
    (f) What is the chasing distance?

18. Fill in the blanks.
    (a) A fine of _____________ amount is levied for driving without a helmet.
    (b) _______________ amount fined for not wearing the seatbelt.
    (c) A fine of ___________amount is charged for not obeying the signal rules.
    (d) Fines of _____________ will be charged for driving without a license.
    (e) Driving under the influence of drugs, _______________ amount is charged.
    (f) Fines of _____ amount will be charged for driving while talking on a mobile phone.
    (g) ___________ amount is fined for not giving way to the ambulance.

ANSWERS

1(a)

1.(i) (–4, 4) (ii) (4,2), (iii) (0,–2), (iv) 4y–1, (v) 2x + 2, (vi)$\frac{1}{2}$(x+3); 2. Unique : (i), (v); infinite : (ii), (iv); impossible : (iii), (vi); 4. (i) 2:1, (ii) 1, (iii) 3, (iv) 12, (v) $\pm\sqrt{6}$; 5.(i) x = 2, y = 2, (ii) x = 1, y = 1, (iii) x = –1, y = 1, (iv) x = 1, y = 1, (v) x=–1, y= –1, (vii) x = 1, y = 4, (viii x = 7, y = –6, (ix) x = 3, y = 2, (x) impossible , (xi) x=1, y = –4, (xii) x = 1, y =1; 6. (iv) $-1, \frac{1}{2}$; 7. (i) $k \neq -6$, (ii) $k \neq -3$, (iii) $k \neq \frac{36}{5}$ (iv) $k \neq 6$, (v) $k \neq \frac{-2}{3}$, (vi) $k \neq 6$; 8.(i) k = 15, (ii) k = 16, (iii) k = $\frac{8}{3}$ (iv) k = 9, (v) k = $\frac{3}{2}$, (vi) k = $\frac{-8}{3}$; 9. (i) k = 16, (ii) k = –15, (iii) k = 2, (iv) k = 3, (v) k = 16, (vi) k = $\frac{-9}{4}$

1(b)

1.(i) (5, 3) (ii) (3,–2), (iii) (1,2), (iv) (2, –3), (v) (1, –1), (vi) (–b, a+b); 2. (i) (4, 1) (ii) (2,1), (iii) ($\frac{-1}{3}$, –1), (iv) (5, –3), (v) (0,0), (vi)$\left[\frac{bc}{b-a}, \frac{ac}{b-a}\right]$, 3.(i)(3, –2), (ii) (3, –1), (iii) (–5,$\frac{2}{3}$), (iv) (a^2, b^2), (v)(2, –3), (vi) (9,4) ; 4. (i) $\left(\frac{1}{4}, \frac{1}{3}\right)$, (ii) $\left(\frac{41}{25}, \frac{68}{41}\right)$, (iii) (3, –1), (iv) (3,4), (v)$\left(a+b, \frac{-2ab}{a+b}\right)$, (vi) (a,b), (vii) (3,2), (viii) (2,3), (ix) (3,2), (x) (2,6), (xi) (18,6), (xii) (a,b); 5.(i) –30, (ii) 7, (iii) –20, (iv)$\frac{-13}{20}$; 6. (i) (1,1), (ii) (1,2), (iii) (1,1), (iv) (2,1)

1(c)

1.90, 47; 2. 4.5cm; 3. 88 m^2; 4. 24, 5. 63 or 36; 6. 5 or 3; 7. 37; 8. 12, 17; 9. $\frac{7}{9}$; 10.$\frac{12}{25}$; 11.Rs $\frac{3}{2}$, $\frac{1}{2}$;
12. 36years, 12 yeas; 13. length 17cm and width 9 cm; 14. 20 days or 30 days 15. 12 days or 24 days ;
16. Rs. 6000 or 5250 ; 17. 40 years or 10 years ; 18. 253m^2 ; 19. 20, 30; 20.$\frac{2}{7}$

2(a)

1 (i) roots are real and equal
(ii) discriminant is 1
(iii) sum roots is $-\frac{b}{a}$
(iv) product of roots $\frac{c}{a}$
(v) 1 and -1 are the roots of quadratic eq^2 x^2-1=0
(vi) x^2=0 roots are equal
(vii) sum of roots $\frac{2}{3}$
(viii) product of roots $-\frac{1}{3}$

2.(i) $x^2 + 2x - 15 = 0$, (ii) m = –1 (iii) p = 3, (iv) c = $\frac{1}{4}$ (v) k = – 16, (vi) 5, (vii) $2\sqrt{6}$

3. (i) a (ii) a (iii) b (iv) d (v) c (vi) b (vii) b

4. (i) –3 ও 2 (ii) 4 ও $\frac{1}{2}$ (iii) $\frac{3}{7}$ ও $-\frac{1}{2}$ (iv) $\frac{1}{3}(16+\sqrt{220})$ ও $\frac{1}{3}(16-\sqrt{220})$
(v) –2p ও 3q (vi) $\frac{-4\sqrt{3}}{3}$ ও $-2\sqrt{3}$ (vii) $\frac{-3+\sqrt{2}}{5}$ ও $\frac{-3-\sqrt{2}}{5}$ (viii) $\frac{-2b}{a}$ ও $\frac{-2b}{3a}$
(ix) $\frac{-a \pm \sqrt{a^2-4b}}{2}$ (x) –a, (a – b)

5. (i) 2, $\frac{3}{4}$ (ii) $\frac{1}{2}$, 2 (iii) $\sqrt{2}$, 1 (iv) a, $\frac{1}{a}$ (v) $-\frac{1}{3}, -\frac{-3}{2}$ (vi) –23, $\frac{5}{2}$
(vii) $\frac{2}{3}, \frac{-3}{4}$ (viii) 2, $\frac{-5}{6}$ (ix) $-\frac{4}{3}, \frac{7}{5}$ (x) 8, –8

6. k=3, 7. P= 4, 8.$\frac{15}{4}$, 9.$\frac{11}{2}$, 10. p=2, 12.$\frac{229}{36}$; 13. 2p, 14. m=$\frac{1}{2}$, 2; 16. $x^2-3x-10=0$

2(b)

1. (i) $x^2 - 2x + 1 = 0$ (ii) $y^2 + y - 20 = 0$ (iii) $x^2 - 18x + 72 = 0$ (iv) 0, 1 (v) $n^2+n-240=0$ (vi) $x^2-13x+28=0$, (vii) $x^2 - 7x = 0$ or if $t = x + 9$ then $t2 - t - 12 = 0$ (viii) $x2-12x+32 = 0$

2. (i) 16 (ii) $\frac{5}{4}$ or $\frac{4}{5}$ (iii) 5, 6 (iv) 11, 12 (v) 9, 42

3. 24 4. 12 5. 48 or 16 6. 5 cm, 7. 15 cm, 8cm 8. 12 9. 18m, 12m. 10. 3 Km per Hr 11. 5 Km per Hr 12. 100 13. 56 m 14. 25 Km per Hr 15. 2.5 m 16. 24

17. (i) –6, 1 (ii) 27, $\frac{25}{147}$, (iii) $\frac{1}{4}, \frac{5}{12}$, (iv) $\frac{-3}{4}, \frac{-3}{2}$, (v) $\pm 2, \pm 3$, (vi) –1, 1 (vii) –1, 3, $1 \pm \sqrt{2}$, (viii) $\pm 1, \pm\frac{1}{2}$, (ix) 2, $\frac{1}{2}$, (x) $\frac{1}{8}$, (xi) $\frac{3}{2}, -\frac{5}{2}$ (xii) $3, -\frac{3}{2}$ (xiii) –4, 9, (xiv) 1, –1, $\frac{-2 \pm \sqrt{13}}{3}$ (xv) 0, 2 , (xvi) 8, (xvii) 6

3(a)

1.(i) 8, (ii) 14, (iii) 13, (iv) 3, (v) 2, (vi) 11, (vii) 0.4, (viii) 6, (ix) 0.5,, (x) –5; 2. (ii) (vi) ଏବଂ (viii); 3. (ii) 7, (iii) d ଏବଂ (viii) 3; 4. (i) 10, 15, 20, 25, (ii) 9, 13, 17, 21, (iii) 7, 9, 11, 13, (iv) 3, 1, –1, –3, (v) 2, –1, –4, –7; 5. (i) 3, 4.5, 5.5 (ii) 0, 6, 10, (iii) 55, 85, 105, (iv) 14, 26, 34; 6. (i) 7, 10, 13, (ii) –10, –12, –14, (iii) 3, –1, –5, (iv) 15, 20, 25, (v) 2, $\frac{7}{2}$, 5, (vi) $-\frac{1}{2}, \frac{-3}{2}, \frac{-5}{2}$; 7. T: (a), (d), (e), (f), (i); 8.(a) 465, (b) 100, (c) 240, (d) –15, (e) 21, (f) 89, (g) 312, (h) –777, (i) –270, (j) –2800, (k) $\frac{n}{2}(n+1)$, (l) $-26\frac{2}{3}$; 9. (a) 210, (b) –493, (c) 1,3,5,7,9 (d) 5,8,11,5795, (e) 3575, (f) $\frac{n}{2}(1-3n)$, (g) 29, (h) 21, (i) 5, (j) 102; 10. (i)(a) 5565, (b) 4071,

(c) 18648; (ii) (a) 210, (b) 1275, (iii) 3159, (iv) 2450, (v) 5625; 11. 6 ବା 12; 12.(i) 4.6,8 ବା 8,6,4, (ii) 3,5,7,9,11, 13 ବା 13, 11, 9, 7, 5, 3; 13. 5,7,9 ବା 9,7, 5; 15. 950; 16. 13267; 17. 6,5,4; 18. 3, 5, 7 ବା 7,5,3; 19. 1,3,5,7 ବା 7,5,3,1;

3(b)

1.(a) $\frac{1}{15}$, (b) $\frac{1}{12}$, (c) $\frac{1}{n}$, (d) $\frac{1}{n+1}$, (e) 7, (f) 3 (g) a (h) 15; 2. (a) $\frac{20}{11}$, (b) $\frac{16}{105}$;

3.(a) $5n^2+40n+60$, (b) $n(n+1)(n^2+3n+1)$, (c) $\frac{2n^3+9n^2+4n}{6}$, 3080, (d) n^2+2n, $\frac{2n^3+9n^2+7n}{6}$, 495; 4.(a) $\frac{1}{6}n(n+1)(4n-1)$, (b) $\frac{1}{3}(4n^2+6n-1)$ (c) $3n(n+1)(n+3)$ (d) $\frac{n(n+1)(2n+1)}{6}$ (e) $\frac{1}{2}n(6n^2-3n-1)$ (f) $\frac{2}{3}n(n+1)(2n+1)$ (g) $\frac{1}{2}n^2(n+1)$ (h) $\frac{1}{12}n(n+1)^2(n+2)$; 5.(i) 21 (ii) 19 ଓ 23; 6.(i) 20 ଓ 28, (ii) 18, 24 ଓ 30; 7.(i) $\frac{58}{3}$ ଓ $\frac{98}{3}$, (ii) 14, 22, 30 ଓ 38; 8.(i) 20, 35 ଓ 50, (ii) 15, 25, 35,45 ଓ 55; 9. 11; 10. –4, –1, 2, 5 କିମ୍ବା 5, 2, –1, –4

4(a)

1.(i) 0, (ii) 1, (iii) $\frac{1}{2}$, (iv) 0.38, (v) $\frac{3}{4}$, (vi) 1, (vii) 0.95; 2. $\frac{8}{15}, \frac{7}{15}$; 3. $\frac{1}{3}, \frac{2}{3}$; 4. 0; 5. $\frac{3}{5}$; 6. $\frac{3}{4}, \frac{1}{4}$, ସମଷ୍ଟି 1; 7. $\frac{5}{8}, \frac{3}{8}$; 8. (i) $\frac{2}{9}$ (ii) $\frac{1}{3}$ (iii) $\frac{4}{9}$; 9.(i) $\frac{1}{4}$ (ii) $\frac{2}{3}$ (iii) $\frac{7}{12}$; 10. (i) $\frac{4}{5}$ (ii) $\frac{1}{5}$

4(b)

1. ଠିକ୍ ଉକ୍ତି; (i) (vi)(viii)(ix); 2. $\frac{1}{4}$; 3. (i) $\frac{1}{2}$, (ii) $\frac{1}{3}$, (iii) $\frac{2}{3}$, (iv) $\frac{5}{6}$, (v) 1, (vi) 0; 4. $\frac{1}{5}$; 5. $\frac{2}{5}$; 6. $\frac{1}{2}$; 7. $\frac{1}{2}$; 8. $\frac{5}{6}$; 9.(i) $\frac{3}{4}$ (ii) $\frac{1}{4}$ (iii) $\frac{3}{4}$, (iv) $\frac{1}{4}$; 10. (i) $\frac{1}{8}$ (ii) $\frac{1}{2}$, (iii) $\frac{7}{8}$, (iv) $\frac{1}{8}$,(iv) $\frac{1}{8}$; 11. (i) $\frac{5}{36}$ (ii) $\frac{1}{12}$, (iii) $\frac{1}{18}$ (iv) $\frac{1}{6}$ (v) $\frac{11}{36}$ (vi) $\frac{1}{12}$; 12. 0.9, 0.6; 13. (i) $\frac{3}{4}$ (ii) $\frac{3}{8}$ (iii) $\frac{3}{4}$ (iv) $\frac{7}{8}$; 14. $\frac{1}{2}$; 15. $\frac{1}{2}$; 16. $\frac{5}{6}$

5(a)

1. T - (i) (ii) (iii) (vi)(viii); 2. (i) (B) 60, (ii) (B) $10\frac{1}{2}$ (iii) (A) $\frac{n-1}{2}$ (iv) (c) n +1) (v) (B) n, (vi) (D) m + 2 (vii) (C) 4m (viii) (D) (M –x) (ix) (B) $\frac{M}{5}$ (x) (B) $\frac{12a+10b}{a+b}$ (xi) (C) 1000 (xii) (C) 12 (xiii) (A) 0 (xiv) (B) x + 4 (xv) (C) 6.5

3. 42.4, 4. 29.2, 5. 4.17 gm , 6. 30, 8. 49.6; 9. 103.5, 10. 12.24, 11. 151, 10. 43, 12. 49.6, 13(i). 16, 14. $f_1 = 28$, $f_2 = 24$, 15. 40, 16. n = 12, m = 10

5(b)

1. T - (ii) (v); 2. (i) 7 (ii) 61.5 (iii) 9, (iv) 29; 3.(i) 14, (ii) 4, (iii)34.3; 4. 93.3; 5. 26.25; 6. 28; 7. 7; 8. 25; 9. 36, 8; 10. 30.0 ପ୍ରାୟ, 11. 15, 10, 12. (i) 52.2 (ii) 140

5(c)

1.T : (i); 2. (i) 9, (ii) 10,11, 3. 122, 4. 7, 5. (i) 8, (ii) ଗରିଷ୍ଠକ 6. 5;

6(a)

1. (i) 5, (ii) 6, (iii) 10, (iv) $\sqrt{2}$ (v) $\sqrt{10}$, (vi) $2\sqrt{a^2+b^2}$; 2. (i) (iii) ଏବଂ (iv) ମୂଳ ବିନ୍ଦୁଠାରୁ ସମଦୂରବର୍ତ୍ତୀ । 8. 4; 9.6 କିମ୍ବା –2; 13(2,0); 15. $(2,3+2\sqrt{3})$

6(b)

1. (i) –2, (ii) $\left(\frac{1}{2},\frac{1}{2}\right)$, (iii) (–2, –3), (iv) $\left(\frac{2}{3},\frac{4}{3}\right)$; 2.(i) (2, 1), (ii) $\left(\frac{3}{2},\frac{3}{2}\right)$, (iii) $\left(\frac{5}{12},\frac{5}{12}\right)$, (iv) $\left(-2,\frac{-3}{2}\right)$, (v) $\left(1,\frac{-3}{2}\right)$, (vi) $\left(\frac{a+c}{2},\frac{b+d}{2}\right)$, (vii) $\left(-\frac{5}{2},\frac{-3}{2}\right)$, (viii) $\frac{a(t_1+t_2)}{2}$, $a(t_1+t_2)$; 3. (i) h = –4, k = 5, (ii) h = –7, k = 1, (iiii) h = –4, k = 1, (iv) h = $-\frac{1}{4}$, k = $\frac{5}{4}$; 4. (–2, –3); 5. (–4,0); 6. (1, –3); 7. x = 10, y = –7; 8. $\left(\frac{7}{5},\frac{18}{5}\right)$; 9. $\left(1,-\frac{9}{7}\right)$; 10. k = 15; 12. h = –10, k = 4; 13. c(4,0); 15$(4,-\frac{5}{2})$ 16. (3, 4), (5, 3); 20. (i) B(–a, 0), c$(0,\sqrt{3}a)$ (ii) 2a, (iii) $\sqrt{3}a$ (iv) $\left(0,\frac{\sqrt{3}a}{3}\right)$

6(c)

1. (i) 0, (ii) 2, (iii) 0, (iv) 3 (v)1; 2. (i) $\frac{5}{2}$ (ii) $\frac{1}{2}$ (iii) 18, (iv) 10.5, (v)14; 4.3; 5.8; 6.4; 7. 5; 8. 2x –y = 1; 9. 11.5 ବର୍ଗ ଏକକ; 10. 32.5 ବର୍ଗ ଏକକ; 11. B(–3, –5), C(5,3), 8 ବର୍ଗ ଏକକ; 12. 0.6; 13. 0 କିମ୍ବା 1; 14. 2; 17. –7 କିମ୍ବା 1 ।