LESSON 1
(SIMILARITY IN GEOMETRY)
EXERCISE 1 (a)

1. Fill in the blanks

(a) In fig 1.21, $L_1 \parallel L_2 \parallel L_3$ and T_1 and T_2 are transversal lines find

(i) If AB = 2 cm, BC = 3 cm, DE = 3 cm, find EF = __________

(ii) If DE = 6 cm, EF = 8 cm, BC = 6 cm, find AC = __________

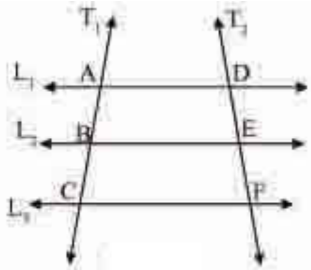


Fig 1.21

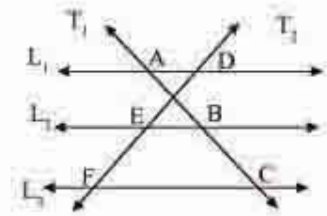


Fig 1.22

(b) In fig 1.22, L1 II L2 II L3 and T1 and T2 are transversal lines find

(i) If AB = 1.5 x BC, find $\frac{EF}{FD}$ = ...

(ii) If B is the midpoint of $\overline{AC}$, EF is ________ times of FD.

2. In fig 1.23, $L_1 \parallel L_2 \parallel L_3$ and T_1 and T_2 two transversal lines. G and H are two points present on L_2 and L_3 respectively such that BG = AD and CH = BE;

Prove that (i) DG : EH = DE : EF (ii) (DG + EH) : EH = DF : EF

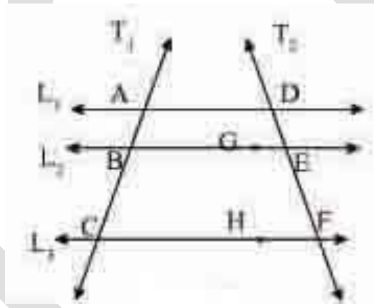


Fig 1.23

3. In fig 1.24, $L_1 \parallel L_2 \parallel L_3$ and T_1 and T_2 are transversal, find

If AB = BC, Prove that 2BE = AD + CF

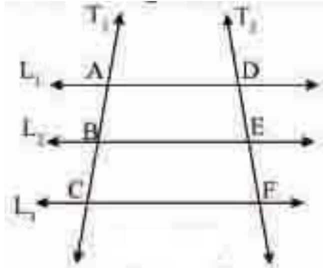


Fig 1.24

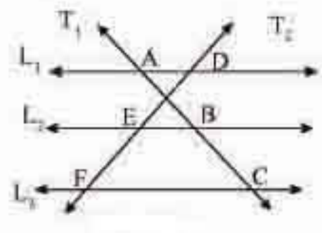
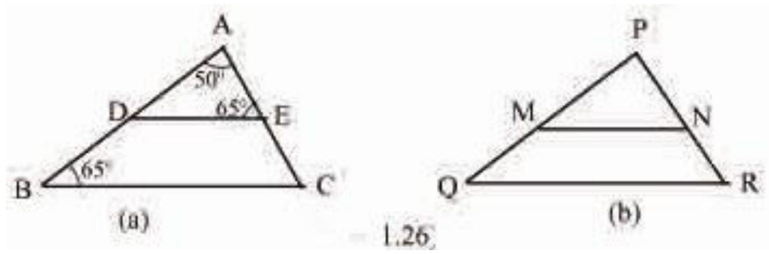


Fig 1.25

4. In fig 1.25, $L_1 \parallel L_2 \parallel L_3$ and T_1 and T_2 are transversal lines. T_1 intercepts L_1, L_2, L_3 at point A, B and C respectively whereas T_2 intercepts L_1, L_2 and L3 at points D, E and F. If DE= EF prove that CF – AD = 2 EB.

5.



(i) In fig 1.26(a), A-D-B and A-E-C $m\angle DAE = 50^{o}$, $m\angle AED = m\angle ABC = 65^{o}$. If AD = 3 cm. and AE:EC = 2:1, find the length of $\overline{DB}$ and $\overline{AB}$.

(ii) In fig 1.26(b), If $\overline{MN} \parallel \overline{QR}$, NR $= \frac{2}{5}$ PR and PQ = 10 cm., find PM and QM.

(iii) In fig 1.26(b) if PM $= \frac{2}{3}$ PQ, NR = 1.2 cm and $\overline{MN} \parallel \overline{QR}$, determine PR.

6.

(i) In ΔABC, if X and Y are the midpoints on $\overline{AB}$ and $\overline{AC}$ show that $XY \parallel BC$.

(ii) Prove that, a parallel line drawn from the midpoint of one side of the triangle to other side bisects the third side into two equal halves.

(iii) Prove that, in a right angled triangle, a perpendicular drawn from midpoint of hypotenuse to other side of the triangle bisects the side into two equal halves.

7. In a ΔPQR, M and N are two midpoints present on the sides $\overline{PQ}$ and $\overline{QR}$. If S is a point lying on $\overline{PR}$, Prove that $\overline{MN}$ bisects $\overline{QS}$.

8. In a Trapezium ABCD, $\overline{AB} \parallel \overline{CD}$, if P is point of intersection of diagonal $\overline{AC}$ and $\overline{BD}$, prove that

(i) AP : PC = BP : PD

(ii) CP : AC = DP : BD

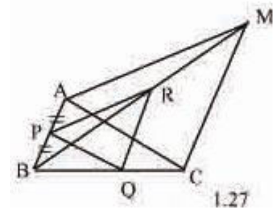
9. In a Trapezium ABCD, $\overline{AB}$: $\overline{DC}$ and P is the midpoint on $\overline{AD}$. If the $\overline{PQ}$ is parallel to $\overline{AB}$ bisects $\overline{BC}$ at point Q, prove that Q is the midpoint of $\overline{BC}$.

10. In a quadrilateral ABCD, the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are P,Q,R and S respectively.

(i) Prove that PQRS is a parallelogram

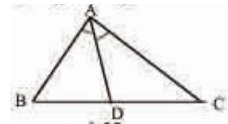
(ii) If the diagonals of the above give quadrilateral ABCD are perpendicular to each other then prove PQRS is a rectangle.

11. In fig – 1.27, the sides $\overline{BA}$ and $\overline{CM}$ of the ΔABC are equal. If P is the midpoint of $\overline{AB}$ and $\overline{PQ} \parallel \overline{AC}$ so as $\overline{QR}$ to $\overline{CM}$. Prove that $\overline{PR} \parallel \overline{AM}$.



EXERCISE 1 (b)

1. In fig 1.33, D is a point on side $\overline{BC}$ of the ΔABC such that $\overline{AD}$ is the bisector of $\angle BAC$. Choose the correct ratio to fill in the blank given below :



1.33

The ratio of the area of the ΔABD and ΔADC is ___________
(AB:DC, BD:AC, AB:AC, AD:BC)

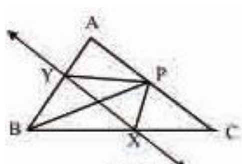
2. In a ΔABC, the bisector of $\angle ABC$ intercept $\overline{AC}$ at point D. If AB=4 cm, BC=6cm and AC=5 cm. Find AD and CD.

3. $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are the three sides of the ΔABC marked as c, a, and b respectively. The bisector of $\angle ACB$ intersect $\overline{AB}$ at point M, prove that

(i) $AM = \frac{bc}{a+b}$ (ii) $BM = \frac{ca}{a+b}$

4.

(i) In fig 1.34, $\overline{BP}$ is the median on $\overline{AC}$ of ΔABC. The internal bisector of $\angle BPC$ and $\angle BPA$ intercept BC and AB at points X and Y respectively. Prove that $\overleftrightarrow{XY} \parallel \overline{AC}$.



1.34

(ii) In the fig 1.34, the bisectors of $\angle APB$ and $\angle BPC$ intercept $\overline{AB}$ and $\overline{BC}$ at points Y and X respectively. If $\overleftrightarrow{XY} \parallel$ AC, show that P is the midpoint of $\overline{AC}$.

5. In fig 1.34 $\overleftrightarrow{BP}$ is the median of ΔABC. PY the bisector of $\angle ABP$ intercept $\overline{AB}$ and Y. YX is drawn parallel to AC at point Y such that it intercept BC at X. Prove that $\overrightarrow{PX}$ is the bisector of $\angle BPC$.

6. In ΔABC the bisector of $\angle BAC$ intercepts $\overline{BC}$ at point P and bisector of $\angle ABC$ intercepts $\overline{AP}$ at Q. Prove that $\frac{AQ}{QP} = \frac{AB+AC}{BC}$.

7. In a parallelogram, the bisector of $\angle BAD$ intercept by hypotenuse $\overline{BD}$ at point K and the bisector of $\angle ABC$ intercept hypotenuse $\overline{AC}$ at point L. Prove that $\overleftrightarrow{LK} \parallel \overline{AB}$.

8. In a quadrilateral ABCD the bisectors of $\angle DAB$ intercept each other on the hypotenuse $\overline{BD}$. Prove that the bisectors of $\angle ABC$ and $\angle ADC$ intercept each other on hypotenuse $\overline{AC}$.

9. In a ΔABC the bisector of $\angle B$ intercept $\overline{AC}$ at point C and the bisector of $\angle C$ intercept $\overline{AB}$ at point F. Prove that ΔABC is a isosceles triangle.

10. In a ΔABC the bisectors of $\angle A$, $\angle B$ and $\angle C$ intercept $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ at points D, E and F respectively. Prove that $\frac{BD}{DC}, \frac{CE}{EA}, \frac{AF}{FB} = 1$.

EXERCISE 1 (c)
PART A

1. Choose the right answer

i. In a Δ ABC and Δ DEF, if $m\angle A = m\angle D$, $m\angle B = m\angle E$, AB = 3cm, BC = 5 and DE = 7.5 find EF ________ cm. (10, 10.5, 12, 12.5).

ii. In a ΔABC where AB = 5 cm, BC = 7 cm, CA = 8 cm; cm ΔPQR where PQ = 10 cm, QR = 14 cm. Find PR = ----- cm, where ΔABC and ΔPQR have similar angles. (12, 16, 20, 24)

iii. In a ΔABC and ΔPQR, $\angle B \cong \angle Q$. In Δ ABC, AB = 8 cm and BC = 12cm. and in ΔPQR, PQ = 12 cm and QR = 18cm. If the area of the Δ ABC = 48 cm^2, then the area of ΔPQR is ___ cm^2.(84, 96, 104, 108)

iv. In a ΔABC, a line segment AB bisects the $\angle$ABC at the point P. If AB=12 cm and BC = 9 cm, find AP:AC __________________ (4:3, 3:4, 7:4, 4:7)

v. The ratio areas of two isosceles triangles is 16:25. Find the ratio of lengths of the corresponding sides of the above triangles is ________ (4:5, 2:5, 5:4, 5:2)

vi. In fig 1.47, if $m\angle A=50°$ and $m\angle BDC = 100°$ and ΔDBC ~ ΔCBA, find $m\angle$ACD.....(60°,70°,80°,90°)

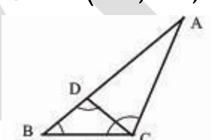


Fig 1.47

vii. In fig 1.48, if the area of ΔABE is equal to the area of ΔACD, ΔBOC ~ _(ΔADE, ΔDOB, ΔEOD, ΔOEC)

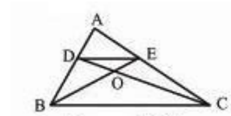


Fig 1.48

viii. In fig 1.49, AE and BD are drawn on sides of AC and BC of a ΔABC from the vertices opposite to it, such that ΔBEM ~ Δ______________ (BEA, ABD, BDC, AEC)

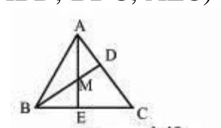


Fig 1.49

ix. In fig 1.50, D is a point lying the line segment BC. If $\angle ADC \cong \angle BAC$, CB.CD=_____ ($AC^2$, AB^2, AD. AB, AD. AC)

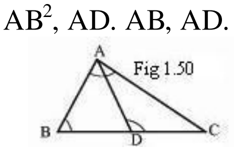


Fig 1.50

x. In a triangle ABC, the line segment BC bisects $\angle$BAC meets at point M. If AB:AC = 2:3 and BC = 15 cm. find BM = ___ cm. (6, 9, 10, 12)

PART B

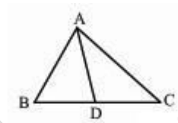
2.

i. In a ΔABC, AB = 2.5 cm BC = 2 cm, AC = 3.5 cm and in ΔPQR, PQ = 5 cm QR = 4cm PR = 7 cm. If $m\angle A = x^0$, $m\angle Q = y^0$, find $m\angle B$, $m\angle C$, $m\angle P$. $m\angle R$.

ii. In a ΔABC and ΔDEF, $\angle B \cong \angle E$, AB = 4 cm BC = 6 cm, EF = 9 cm and DE = 6 cm.If the area of ΔABC is 20 cm^2 , find the area of ΔDEF.

iii. In two similar triangles, the area of the first triangle is 9 times more than the second, find the ratio between the similar sides of the two triangle.

iv. In fig 1.51 $\angle BAC \cong \angle ADC$, AC=12 cm and BC=15 cm. If the area of $\angle ADC$ is 32 cm^2, find the area of ΔABC.



1.51

v. In a ΔABC, AB = 5 cm, BC = 7 cm. CA = 9 cm. If $\Delta PQR \sim \Delta ABC$ and the circumference of ΔPQR is 63 cm. find PQ, QR and PR.

vi. If $\Delta ABC \sim \Delta PQR$; AB = 5 cm, BC = 12 cm, AC = 13 cm, and QR = 8 cm., find the area of ΔPQR.

vii. $\Delta ABC \sim \Delta PQR$. If the circumference of the ΔABC is 60cm and its area is 81 cm^2 and the circumference of ΔPQR is 80 cm, find its area.

3. Prove that in two similar triangles,

a) the lengths of the corresponding heights of the triangle is proportional to the lengths of the corresponding sides.

b) the lengths of the bisectors of corresponding angles is proportional to lengths of the sides of the two similar triangles.

c) the corresponding medians are proportional to the corresponding sides.

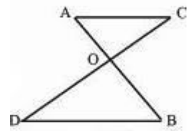
4. If the perimeter of the two similar triangles are equal, prove that the triangles are equivalent.

5. If the areas of the two similar triangles are equal, prove that the triangles are equivalent.

6. Prove that the ratios of areas of two similar triangles are equal to
   i) squares of their corresponding heights
   ii) squares of the lengths of bisectors of the angles
   iii) squares of the lengths of medians
   iv) squares of their perimeters

7. In a ΔABC, P and Q are the two points lying on side AB and BC such that the areas of the ΔBQP and ΔCPQ are equal. Prove that $\frac{PQ}{BC} = \frac{AP}{BC}$

8. In Fig 1.52, O is the point of intersection of $\overline{AB}$ and $\overline{CD}$
(i) if AO . OD = BO . OC, prove that ΔAOC~ΔBOD.
(ii) if CO . OD = AO . OB, prove that ΔAOC ~ ΔDOB

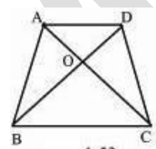


1.52

(iii) Find where $\overline{AC}$ II $\overline{DB}$ in above two cases

9. In a ABCD trapezium, $\overline{AB}$ II $\overline{DC}$ and the diagonals $\overline{AC}$ and $\overline{BD}$ intersect each other at point O. AO = 3 cm and OC = 5cm. If the area of ΔAOB is 36 sq cm, find the area of the ΔCOD.

10. In fig 1.53, ΔABC and ΔDBC is lying on the same base $\overline{BC}$. $\overline{AC}$ and $\overline{BD}$ intersect at the point O. Prove that $\frac{Area\ of\ \Delta ABD}{Area\ of\ \Delta BCD} = \frac{AO}{OC}$.

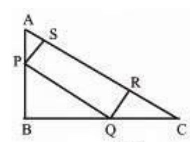


1.53

PART C

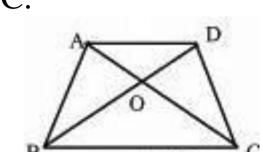
11. Prove that, if the lines drawn from the mid points of a triangle are joined, four triangles are formed are congruent and similar to original triangle. Once again, prove that the area of each triangles formed is equal to the one fourth area of original triangle.

12. In fig 1.54, ∠ABC is a right angle in ΔABC. If PQRS is a rectangle, prove that ΔAPS ~ ΔQCR ~ ΔPQB ~ ΔACB



1.54

13. In Fig 1.55, if $\overline{AB}$ II $\overline{DC}$ and ΔADO ~ ΔBCO, prove that AD = BC.



1.55

14. In a trapezium ABCD, $\overline{AD}$ II $\overline{BC}$. If ∠ABD ≅ ∠DCB, prove that BD^2 = AD . BC.

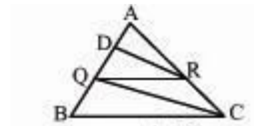
15. In a ΔABC, X and Y are the two points lying on $\overline{AB}$ and $\overline{BC}$ such that $\overline{XY}$ II $\overline{BC}$. Prove that median AD of ΔABC bisects $\overline{XY}$.

16. In a ΔABC, AD is the median and E is midpoint of it. If $\overrightarrow{BE}$ is a ray which intersect AC, prove that BE = 3EX.

17. In a ΔABC, if $\overline{AD} \perp \overline{BC}$ and AD^2 = DC . BD, prove that (i) ∠BAC is a right angle and (ii) areas of ΔABD and ΔCAD is proportional to AB^2 and AC^2

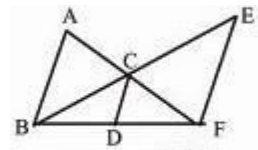
18. In ΔABC and ΔDEF, m∠A = m ∠D, m∠B = m∠E. X and Y are the mid points of $\overline{BC}$ and $\overline{EF}$ respectively. Prove that (i) ΔAXC ~ ΔDYF (ii) ΔAXB ~ ∠DYE.

19. In Fig 1.56, ΔABC, Q is a point lying on $\overline{AB}$ and $\overline{QR} \parallel \overline{BC}$ forming A-R-C, $\overline{DR} \parallel \overline{QC}$ forming A-D-B. Prove that $AQ^2 = AD \times AB$.



1.56

20. In Fig 1.56, $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ and $\overline{AF}$ and $\overline{BE}$ intersect each other at point C. Prove that $EF \times BD = DF \times AB$.

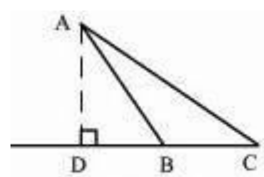


1.57

21. Prove that the ratio of radius of incircles drawn in two similar triangles is equal to ratio of lengths of corresponding sides of the triangles.

22. If A-P-B and A-Q-B and $\frac{AP}{PB} = \frac{AQ}{QB}$, prove that P and Q are not same (different).

23. In fig 1.58, $\angle ABC$ is an obtuse angle in the given ΔABC. A line drawn from A meets at point D of the ray extended from $\overleftarrow{BC}$. If $AD^2 = BD \cdot DC$, prove that $\angle BAD$ and $\angle CAD$ are corresponding angles.



1.58

24. In ΔABC, X and Y are the two points lying on $\overline{AB}$ and $\overline{BC}$. If the area of the trapezium XBCY is 8 times of the area of ΔAXY, find AX : BX.

25. ABCD is a parallelogram where $\overrightarrow{AG}$ intersect $\overline{BD}$, $\overline{CD}$ and $\overrightarrow{BC}$ at points E, F and G. Prove that AE : EG = AF : AG.

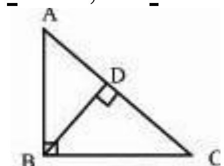
<u>**EXERCISE 1 (d)**</u>

PART A

1. Choose the correct answer

(i) In ΔABC of the give figure 1.62, where $m\angle ABC = 90^o$ and $\overline{BD} \perp \overline{AC}$.

Find $m\angle ABD$ =________ [$m\angle BAD$, $m\angle DBC$, $m\angle DCB$, $2m\angle BAD$]



1.62

(ii)In ΔABC of the give figure 1.62, where $m\angle ABC$ is right angle and $\overline{BD} \perp \overline{AC}$, find

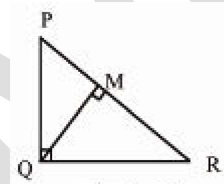
(a) AB^2 = AD x ..... [BC, CD, AC, BD]

(b) BC^2 = AC x ..... [DC, AD, BD, AB]

(c) BD^2 = DC x ..... [AC, BC, AB, AD]

PART B

2. In ΔPQR of the give figure 1.63, where $m\angle PQR = 90^o$ and $\overline{QM} \perp \overline{PR}$. If



1.63

(i) QM = 12cm, PM = 6 cm, find PR.

(ii) PQ = 6 cm and PM = 3 cm, find PR

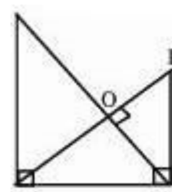
(iii) QR = 12 cm and MR = 9 cm find PM

(iv) PQ = 12 cm and RM = 7 cm find PM

(v) PQ = 8 cm and QR = 15 cm find QM and MR

3. In fig 1.64, $m\angle ABC = m\angle DCB = 90^o$ and O is the point of intersection of line AC and BD and $\overline{AC} \perp \overline{BD}$. OC = 6 cm and OD = 4 cm,

(i)Find BO (ii) Find OA (iii) Find BC (iv) Find AB (v) Find CD



1.64

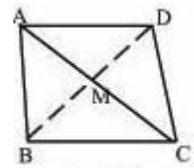
PART C

4. In a Δ ABC, $\angle ABC$ is a right angle and $\overline{BD} \perp \overline{AC}$, AD = p units and BD = q units. Prove that BC = $\frac{q(p+q)}{\sqrt{p2+q2}}$ (ii) AB = $\frac{p(p+q)}{\sqrt{p2+q2}}$

5. In a Δ ABC, if $\angle ABC = 90^o$ and $\overline{BD} \perp \overline{AC}$, Prove that $AB^2 : BC^2$ = AD : DC.

6. In a Δ ABC, $\angle ABC$ is a right angle and BC^2 = AC . BD. Prove that $\overline{BD}$ bisects $\angle ABC$.

7. In the given Fig 1.65, ABCD is a quadrilateral where $m\angle ABC = m\angle ADC = 90^o$ and AB = AD. If M is the point of intersection of the diagonals, prove that $AM \times MC = DM^2$.



1.65

8. In a Δ ABC, if $\angle ABC = 90^o$ and $\overline{BD} \perp \overline{AC}$ and a bisector of $\angle ABC$, intersect $\overline{AC}$ at a point E. Prove that $AE^2 : EC^2 = AD : DC$

9. In a Δ ABC, if $\angle BAC = 90^o$ and $\overline{AD} \perp \overline{BC}$, prove that $\angle ADC = \frac{AB \ x \ AC^3}{2BC^2}$.

10. In a Δ ABC, $\angle ABC$ is right angle and $BD \perp AC$, a bisector of $\angle BAC$, intersect $\overline{BD}$ at a point E. Prove that $BE^2 : DE^2 = AC : AD$.

LESSON 2 : CIRCLE

EXERCISE 2(a)

PART A

1. Write True /False for the following given statements.

1. A circle is a collection of those points on an arc lying on a plane and that are at a given constant distance from a given point in the plane.
2. Every point on a circle is the one end of a radius.
3. A circle has infinite number of diameters.
4. Centre is the only point on the circle which lies on all the diameters.
5. A chord divides a circle internally into two halves, each of them are covert sets.
6. If a diameter of circle bisects the chord, they are perpendicular to each other.
7. The every circumcenter is its interior point.
8. The centre is the only interior point of the circle which has equidistance from rest of the points lie on the circle.
9. If a ray intersects a circle at one point, then the point of intersection is the interior point of the circle.
10. If $\overline{AB}$ and $\overline{AC}$ are two congruent chords of a circle, the radius at B bisects the $\angle ABC$
11. A point cannot be a centre of two or more than two circles.
12. A line segment always intersects a circle at two points only.

2. Choose the correct answer

1. The point of intersection of two unequal chords is ________________
   a. Interior point of a circle c. Exterior point of a circle
   b. A Point on the circle. d. Point on the circle or interior of a circle
2. If P is the exterior point of a circle, __________ pairs of points lie equidistant from the point P.
   a) 1 b) 2 c) 8 d) infinite
3. A line segment can be radius of __________ number of circles.
   a) 1 b) 2 c) 4 d) infinite
4. A line segment can be chords of __________ number of circles.
   a) 1 b) 2 c) 4 d) infinite
5. The distance between one end of a chord and the centre of a circle is 5 cm, while the distance between midpoint of the chord and centre of circle is 3 cm. The length of the chord is _____ cm.
   a) 8 b) 12 c) 16 d) 20

PART B

3. The radius $\overline{OP}$ of a circle bisect a 16 cm long chord at point D. If the radius of the circle is 10 cm, find the length of the $\overline{DP}$.

4. The centre of the circle is O. If D is the centre of a chord, prove that $\overline{OD}$ bisects $\angle ABC$.

5. $\overline{AB}$ and $\overline{AC}$ are the congruent chords of a circle whose centre is O. Prove that $\overline{OA}$ bisects $\angle BAC$.

6. $\overline{AB}$ and $\overline{AC}$ are the parallel chords of a circle whose centre is O. If P and Q are the midpoints of the parallel chords $\overline{AB}$ and $\overline{AC}$, prove that O is the centre lying on $\overleftrightarrow{PQ}$.

7. In an equilateral triangle prove that sides of the triangle are equidistant from its centroid.

8. Prove that the diameter of a circle is the largest chord. (Hint : If the distance between the chord and the centre of the circle is $d \geq 0$ and the radius is r , then the length of the chord $2\sqrt{r^2 - d^2} \leq$ 2r = D (D=Diameter).

9. If centre of a circle is located at one side of the two parallel chords, prove that the chords are not congruent.

10. $\overline{AB}$ and $\overline{CD}$ are the parallel chords of a circle. AB=CD=8 cm. If radius r=5 cm, find the distance between the two chords.

PART C

11. Find the lengths of the parallel chords $\overline{AB}$ and $\overline{CD}$ which is at a distance of 4 cm from the centre of the circle of radius 10 cm.

12. A triangle Δ ABC is inscribed in a circle. If AB = AC prove that the ray bisecting the $\angle BAC$ passes through the centre of the circle.

13. Two chords of a circle is bisected by a diameter, prove that the chords are parallel.

14. If two chords of a circle bisect each other, prove that the point of intersection is the centre of the circle. (Use Method of Contradiction)

15. Two chords of a circle $\overline{AB}$ and $\overline{BC}$ makes an angle of 90^0 at B. If O is the centre of circle, prove that A O and C are collinear.

16. Prove that the midpoint of the hypotenuse of a right angle triangle is the centre of the circum circle.

17. $\overline{PQ}$ is the chord of a circle. The perpendiculars drawn from P and Q intersect the circle at point at point R and S. Prove that PQRS is a rectangle.

18. In the fig. 2.17, A and B are the centre of two intersection circles and P and Q are the point of intersection of the circles. Prove that

(i) $\overleftrightarrow{AB}$ bisects the chord $\overline{PQ}$

(ii) $\overleftrightarrow{AB}$ is $\perp \overline{PQ}$

(Hint : If the point of intersection of AB and PQ is C, compare the mid points of ΔACP, ΔACQ and ΔAPB, ΔAQB)

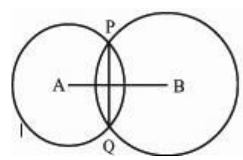


fig 2.17

19. In the fig 2.18, P and Q are the point of intersections of two circles. The perpendicular drawn from point P on $\overline{PQ}$ intersects the circles at A and B. Similarly the perpendicular drawn from point Q on $\overline{PQ}$ intersects the circles at C and D. Prove that AB=CD.

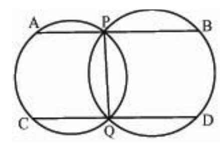
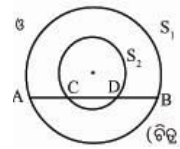


fig 2.18

20. Two circles with centres A and B intersect each other at point P and Q. A line is drawn on P parallel to $\overline{AB}$ intersects the circles at M and N. Prove that MN=2AB.
(Hint : Show that AB=CD by drawing the perpendiculars $\overline{AC}$ and $\overline{BD}$ on $\overline{MN}$)

21. If a line intersects the two concentric circles S1 and S2 at points A, C, D and B is shown in fig 2.19. Prove that AC=DB.



(ଚିତ୍ର fig 2.19

22. Point P is an exterior point of the circle from where two secants are produced intersect the circle at A, B and C, D such that it makes P-A-B and P-C-D. If AB=CD, prove that PA=PC and $\overline{AC} \parallel \overline{BD}$.

23. Two congruent chords of a circle ABC with centre O, intersect each other at an internal point P. If B and C are lying one side of $\overline{OP}$ prove that PA=PC and $\overline{AC} \parallel \overline{BD}$.
(Hint : Draw $\overline{OE} \perp \overline{AB}$ and $\overline{OF} \perp \overline{CD}$ and join O, P)

EXERCISE 2(b)
PART A

1. Write True ∠False for the following given statements.

1. The subset of a circle is called an arc.
2. The interior point of an arc is not the interior point corresponding circle.
3. If P and Q are the end points two arcs of a circle, they are supplementary arcs.
4. When we produce the end points of an arc to the centre of the circle, the angle subtend by an arc is central angle.
5. The sum of the degree measures of two arcs cannot be more than 360^0.
6. Circle is not a convex set.
7. If two arcs of a circle having common end points then they are adjacent arcs.
8. If two congruent chords are adjacent to two corresponding arcs, then a major arc is formed when both the arcs are joined.
9. Two congruent chords of a circle intersect each other perpendicularly at interior point P. $\overline{OQ}$ and $\overline{OR}$ are drawn from centre O. The point O, Q, P and R are the vertices of the square.
10. Degree measure of $\widehat{BPC}$ is 30^0. If A is the concyclic point on the circle, then ∠A of ΔABC is always 15^0.
11. An arc is collection of infinite points.
12. A cyclic rhombus is a square.

2. Fill in the blanks

1. The degree measure of a major arc is more than ________________
2. The degree measure of the central angle of a circum circled hexagon formed when angle subtended by its side to the centre is __________________
3. ABCD is a cyclic quadrilateral where m∠A=50^0 and m∠B=120^0, ___________ is the difference between m∠C and m∠D.
4. Two congruent chords of a circle ABC with centre O, intersect each other at an internal point P. If B and C are lying one side of $\overline{OP}$, $\widehat{AD}$ and _________ are congruent.
5. If the length of a chord of a circle is equal to its radius then the degree measure of smaller arc is _______________
6. C and D are the two points lying on one side the $\overline{AB}$. m∠ACB= m∠ADB=20^0. If O is the centre of the circum-circled ΔABC, m∠AOB= ____________
7. If m∠ABC=90^0, $\overline{AC}$ is _____________ of the circum-circled ΔABC.
8. ABCD is a cyclic quadrilateral. m∠BAD is half of the _________ arc.
9. The degree measure of a semicircle is _______________
10. The degree measure of an arc of a circle is 90^0. The ratio between the corresponding chord and radius is ________________

PART B

3. Fig 2.50 is a circum-circled acute angle ΔABC. If D, E and F are three concyclic points, answer the following questions :

i. The arc inscribes the ∠B
ii. The arc intersected by ∠B
iii. The major and minor arcs intersected by $\overline{BC}$
iv. The value of ∠A is half of which central angle
v. If AB=BC in a ΔABC, find which two arcs are congruent.
vi. Name two adjacent arcs such that they form $\widehat{BAD}$ when they are joined.
vii. Take a point P on $\widehat{BFC}$ such that m∠BPA=m∠C. How many points are there? Is there any points on ADC and $\widehat{BEA}$.

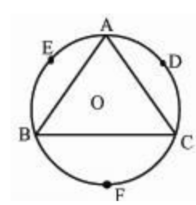


fig 2.50

4. The diagonals of cyclic quadrilateral ABCD shown in Fig 2.51 intersect at centre of the circle. If m$\widehat{AEB}$ = 100, find

i. the degree measure of all the angles of quadrilateral
ii. Relation between $\widehat{AHD}$ and $\widehat{BFC}$
iii. What type of quadrilateral is ABCD

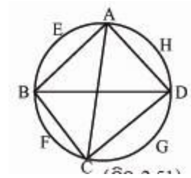


fig 2.51

5. In Fig 2.52, $\overline{AB}$ and $\overline{CD}$ chords interest at internal point P. If m∠PBD=80^0 and m∠CAP=45^0, find

i. the angle measure of ΔBPD
ii. the angle measure of ΔAPC
iii. relation between ΔBPD and ΔAPC

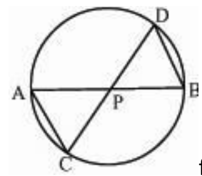


fig 2.52

6. The bisector of ∠A intersect the circum-circled ΔABC at point D. Prove that ΔBDC is isosceles.

7. In fig 2.53, two rays $\overrightarrow{AP}$ and $\overrightarrow{AR}$ are produced from exterior point of a circle intersect the circle at P, Q and R, S forms A-P-Q and A-R-S. Prove that

i. ΔAPR ~ ΔAQS
ii. ΔAPS ~ ΔARQ
iii. If T is the point of intersection of $\overline{PS}$ and $\overline{QR}$,
   a) Prove TP . TS = TR . TQ
   b) m∠PTR = $\frac{1}{2}$(m $\widehat{QS}$ – m $\widehat{PR}$)
iv. If m∠PAR=15^0 and m $\widehat{QXS}$ = 50^0, find m∠ PTR

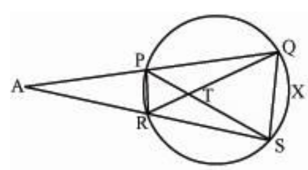


fig 2.53

8. In fig 2.54, the degree measure of arcs $\widehat{AXB}$ and $\widehat{BYC}$ of circle is 80^0 and 140^0 respectively, find

i. m∠BAC
ii. m $\widehat{ABC}$
iii. m $\widehat{ACB}$
iv. relation between two arcs $\widehat{AZC}$ and $\widehat{BYC}$

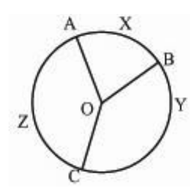


fig 2.54

9. $\overline{AB}$ is the diameter of a circle with centre O. The concyclic points P and Q are situated on the same side of $\overline{AB}$. If the degree measure of arcs having the end points A & P and B & Q is 60^0 and 50^0, find

i. degree measure of minor arc with end points A & Q
ii. degree measure of major arc with end points B & P
iii. degree measure of major arc with end points P & Q

10. $\overline{AB}$ and $\overline{CD}$ are two parallel chords shown in fig 2.55. Prove that i) m $\widehat{AXC}$ = m $\widehat{BYD}$, (ii) AC = BD

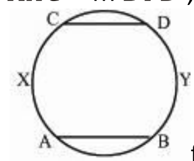


fig 2.55

11. ABCD is a cyclic quadrilateral. If
(i) AC = BD and AB II CD, prove that AD = BC
(ii) AD = BC, prove that AC = BD and AB II CD

12. (i) $\widehat{AXB}$is an arc of a circle. Prove that C is the one and only interior point found in the $\widehat{AXB}$ such that the arcs $\widehat{AC}$ and $\widehat{BC}$ are congruent. (C is also known as the mid point of $\widehat{AXB}$)
(Hint : If the bisector of ∠AOB intersects $\widehat{AXB}$ at point C, then it is an essential point)
(ii) Using the mid point of an arc, prove that $\widehat{AXB}$has infinite points.

13. In Fig 2.56 $\overline{AB}$ is the diameter of a circle with centre O and radius $\overline{OD}$. If AC II OD, prove that $\widehat{BXD}$ and $\widehat{DYC}$ are congruent and D be the mid point of $\widehat{BDC}$. (Hint : to show m∠BOD=2m∠DOC, draw $\overline{OC}$.

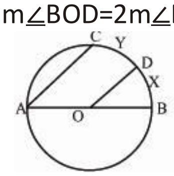


fig 2.56

PART C

14. In Fig 2.57, chord $\overline{CD}$ is parallel to diameter $\overline{AB}$ and CD=OB. Prove that m∠BDC = 2m∠OBD.

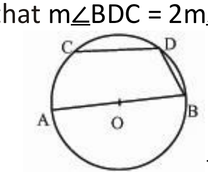


fig 2.57

15. The diagonals $\overline{AC}$ and $\overline{BD}$ of a cyclic quadrilateral intersect each other at point P. O is the centre of the circle and B, C lie opposite to $\overleftrightarrow{OP}$. If AC = BD, prove
(i) AB = CD, (ii) PA = PD (iii) $\overline{BC}$II $\overline{AD}$

16. (i) Prove that the inscribed angle of a minor arc is an obtuse angle.
(ii) Prove that the inscribed angle of a major arc is an acute angle.
(Hint : (i) $\widehat{APB}$ is a minor arc and $\widehat{AQB}$ is a major arc. Draw diameter $\overline{AD}$. m∠APD = 90^0 < m∠APB .)

17. (i) If centre O of the circumcircle of ΔABC is an interior point of triangle, prove that m∠BAC + m∠OBC = 90^0.
(ii) If centre O of the circumcircle of ΔABC is an exterior point of triangle, prove that m∠BAC - m∠OBC = 90^0.

18. If non-parallel sides of a trapezium are congruent, prove that the trapezium is cyclic.

19. Two circles intersect each other at P and Q. If a straight is drawn passing through P as mid-point it touches circles at K and L. Similarly, If a straight is drawn passing through Q as mid-point it touches circles at M and N. If K and L lie on one side of $\overline{PQ}$, prove $\overline{KM}$ II $\overline{LN}$.

20. The bisectors of ∠B and ∠D of a cyclic quadrilateral ABCD meet at point E. If $\overleftrightarrow{DE}$ intersects the circle at point F, prove that $\overline{BE}$ ⊥$\overline{BF}$.

21. Bisectors of angles A, B and C of a ΔABC intersect its circum-circle at X,Y and Z respectively. Prove that the angles of ΔXYZ are $90^0 - \frac{1}{2}\angle A$, $90^0 - \frac{1}{2}\angle B$ and $90^0 - \frac{1}{2}\angle C$.

22. ΔABC is an inscribed equilateral triangle. P is the point on the minor arc corresponding to chord $\overline{BC}$. Prove that PA = PB + PC.
(Hint : Take D on $\overrightarrow{BP}$ such that PC = PD. Compare between ΔBCD and ΔACP)

23. Bisectors of $\angle$A of a ΔABC intersect its circum-circle at P. The end points of the perpendiculars from of point P meet $\overrightarrow{AB}$ and $\overrightarrow{AC}$ at Q and R respectively. Prove that AQ = AR $= \frac{AB + AC}{2}$. (Note : show that ΔABQ $\cong$ PCR=>BQ = CR).

24. Bisectors of $\angle$A of a ΔABC intersect its circum-circle at P. If D is the point of intersection of $\overline{AP}$ and $\overline{AC}$, prove that ΔABD and ΔAPC are similar. Hence prove that AB . AC = BD . DC + AD^2.
(Hint : $\because \Delta ABD \sim \Delta APC \Rightarrow AB \cdot AC = AD \cdot AP$, $AD^2 = AD\,(AP - PD)$)

25. (TELMI'S Corollary) If ABCD is cyclic quadrilateral, prove that AC.BD = AB . CD + BC . AD.
(the product of the lengths of the diagonals of a cyclic quadrilateral is equal to the sum of the products of lengths of its opposite sides)
(Hint : Let m$\angle$ADB > m$\angle$BDC. E, be the point on $\overline{AC}$, so that m$\angle$BDC = m$\angle$ADE. $\Delta ADE \sim \Delta BDC \Rightarrow \frac{AE}{BC} = \frac{AD}{BD}$.
$\Delta ADB \sim \Delta EDC \Rightarrow \frac{CD}{BD} = \frac{EC}{AB}$.

CHAPTER 3
TANGENTS TO A CIRCLE

SECTION A

1. Fill in the blanks

(i) O is centre of a circle, ‘P’ be the external point, PF be the tangent of this circle then $m\angle OTP$=_____

(ii) O is the centre, P be the external point and PX and PY are two tangents of a circle. $m\angle XPY$ is a acute angle, $m\angle XOY$ is _________________ angle.

(iii) O is he centre, P be the external point and PT is a tangent of the circle then
$m\angle TOP + m\angle TPO$ = ________________

(iv) O is a centre, P be an external point and PX and PY are two tangents to the circle then
   (a) XOP angle and ____________ angle are equal.
   (b) YPO angle and ____________ angle are equal.

(v) In a circle, O is the centre and radius is r units. P be a point lying on plane of a circle, ___________ should be present between OP and r so that tangent segment can be drawn from point P to the circle.

(vi) If P is the point lying on a plane surface at distance of 13 cm away from the center of a circle of radius 5 cm., ________________ is the length of the tangent segment PT.

(vii) A circle of radius r cm, centre O and P is the point lying outside the centre of the circle. A tangent segment is drawn from P to O of length ‘t’ cm, then OP = ___________ cm.

(viii) In two externally tangent circles :
   (a) Number of direct common tangents = _______________
   (b) Number of transverse common tangents = ____________

(ix) In two internally tangent circles :
   (a) Number of direct common tangents = _______________
   (b) Number of transverse common tangents = ____________

(x) In the two externally non intercepting circles :
   (a) Number of direct common tangents = _______________
   (b) Number of transverse common tangents = ____________

(xi) In the two non externally non intercepting circles :
   (a) Number of direct common tangents = _______________
   (b) Number of transverse common tangents = ____________

(xii) In ΔABC, AB=AC. P is a point lying on a tangent drawn from a point A lying on the circumference of the circle such that P and B points are found on the opposite side the $\overline{AC}$.
If $m\angle PAC = 70^\circ$, Find $m\angle ABC$= _____________.

(xiii) If the radius of a circle is 8 cm, the distance between two parallel tangents is _______ cm.

(xiv) The distance between the centers of the two externally tangent circles is equal to ________ of the radius of the two circles.

(xv) The distance between the centers of the two internally tangent circles is equal to _________ of the radius of the two circles.

(xvi) _____ number of direct common tangents of a circle can be drawn from a P lying on a line.

2. Without using a negative statement, correct the following given statements.

(i) If L is a transverse line (secant) of a circle of r units, the distance between L and the centre of the circle = r units.

(ii) P be the point lying external to circle of centre O on the same plane. If $\overline{PT}$ is a tangent drawn from point P to the circle, $\angle$POT is the right angle of ΔOPT.

(iii) Radius of a circle is r units. If the length of $\overline{PT}$, the tangent drawn on the circle from a point P lying external to the circle on same plane is t units, and the distance between the centre of the circle O and the point P is d units, then $d^2 + r^2 = t^2$.

(iv) P be the point lying external to circle on the same plane and $\overline{PT}$ is a tangent drawn from point P to the circle. A transversal line drawn from point P touching the circle at point A and B forming P-A-B. Therefore making PT^2=PA x PB.

(v) Two tangents can be drawn to circle from a point Q lying internal to the circle.

(vi) The length of the tangent drawn to the circle from a point P lying external to the circle of fixed radius is remains fixed.

(vii) If the distance between the centres of two tangent circles is equal to the sum of the radii of the same circle, then both are internally tangents.

(viii) The distance between the centres of two internally tangent circles is equal to the difference between their radii.

(ix) In between two circles one lie inside the other. These two circles has a common tangent.

(x) Two circles intersect each other at two points have only one transverse common tangent.

(xi) Vertex of two internally tangent circles is not external to incircle.

(xii) The vertex of the two externally tangent circles is not an internal point to any of the two circles.

3. In a circle, O is the centre point and is radius of 8 cm. and P is an external point lying outside the circle and if PO=17 cm, find the length of the tangent drawn from P to the circle.

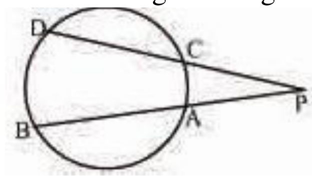
SECTION B

4. The radii of two externally tangent circles are 4.5 cm and 12.5 cm. If a common tangent touches the points P and Q, find the $\overline{PQ}$.

5. A transverse common tangent of two non intersecting circle touch both the circles at point P and Q. If the distance between two centers is 20 cm and radii are 7 cm and 5 cm respectively, find the length of PQ in cm.

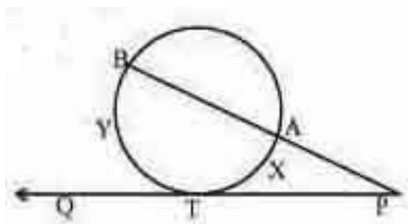
6. In the given fig, 3.28, P is the external point of the circle. From P, P-A-B intersects the circle at point A and B. P-C-D is drawn from point P intersect at C and D.

(i) Prove that PA x PB = PC x PD using theory related to tangent to circles.
(ii) PA=10 cm PB=16 cm and PD=20 cm find CD.
(iii) PA=8 cm and AB=10 cm. Find the length at tangent drawn from P.



3.28

7. In the given fig. 3.29, P is the external point. A intersector from P intersect the circle at point A and B as P-A-B. A tangent ray drawn from the point P touches the circle at point T.



3.29

(i) If m $\widehat{AXT} = 60^{o}$ and m $\widehat{BYT} = 130^{o}$, find m∠ATP, m∠APT, m∠ATB, and m∠BTQ.
(ii) If m∠BTQ=2M m∠ATP, Prove (a) BT=TP (b) TA=AP.
(iii) If PA=8cm and PT=12 cm, find AB.
(iv) If PT=2AP and PB=24cm, find PT.

8. (a) If two circles are externally tangents to each other, prove that the transverse common tangents of the circles drawn from any point to the circle are congruent.

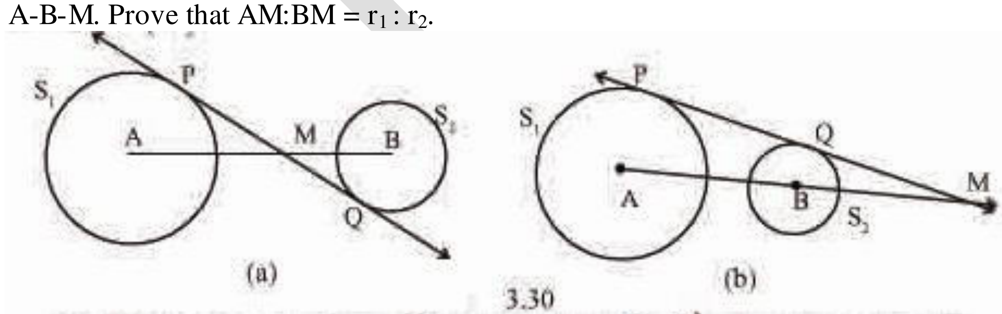
(b) If two circles are internally tangents to each other, prove that the transverse common tangents of the circles drawn from any point to the circle are congruent.

9. A and B are the intersection points of two circles intersecting each other. P be the point on $\overleftrightarrow{AB}$ as A-B-P. Prove that the two tangents drawn from the point P to the circle are congruent.

10. In the given fig 3.30 let r_1 and r_2 be the radius of circle S_1 and S_2 with centre A and B respectively.

(a) In the fig 3.30 (a) A transverse common tangent of the two circle intersect $\overline{AB}$ at point M. Prove that AM:MB = $r_1 : r_2$.

(b) In fig 3.30 (b) A direct common tangent of the two circles intersect $\overline{AB}$ at point M as A-B-M. Prove that AM:BM = $r_1 : r_2$.

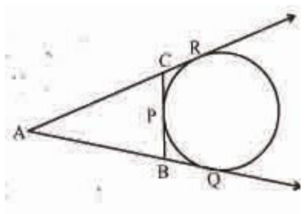


11. In a circle, the chords $\overline{PQ}$ and $\overline{PR}$ are congruent. Prove that the tangents drawn to the circle at point P parallel with $\overline{QR}$.

12. Among the two concentric circles, a chord $\overline{AB}$ is drawn from one of the circles intersects the other circle at point P, prove that AB is bisected at point P.

13. Prove that the line joining the contact point of the two tangents is diameter of the circle.

14. In fig 3.31, a ΔABC, the side $\overline{BC}$ intersects rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ touching the circle PQR at points P, Q and R respectively. Prove AQ = ½(AB + BC + AC)



3.31

15. If a circle touches all the sides of a parallelogram, prove that the parallelogram is a Rhombus.

SECTION C

16. O is the centre of circle and P is the external point of the circle. $\overline{PA}$ and $\overline{PB}$ are the two tangents from the point to the that circle, prove that ΔABP is equilateral.

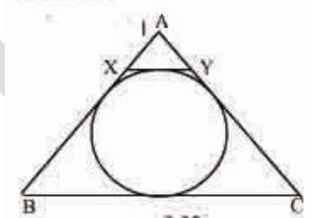
17. O is the centre of the circle. P is an external point of the circle. T is the point of contact of the tangent $\overline{PT}$. the midpoint of $\overrightarrow{OP}$ is Q (point over the circle) then prove that QT=QP.

18. P is an external point of a circle and the point of contact of the tangent $\overrightarrow{PT}$ is T. A line segment tends to the point P intersect the circle at A & B as P-A-B. C is the point between A & B of $\overline{AB}$. Prove that

(a) (i) $\overrightarrow{TC}$ is the intersector of $\angle$ATB
(ii) PC = PT

(b) If PC = PT, then $\angle$ATB is bisected $\overrightarrow{TC}$

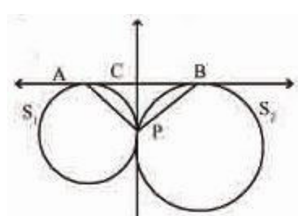
19. X and Y are the points lying over the sides $\overline{AB}$ and $\overline{AC}$ of the ΔABC so that $\overline{XY}$ touches the incircle of ΔABC (3.32). Prove AX+XY +YA=AB+AC-BC.



3.32

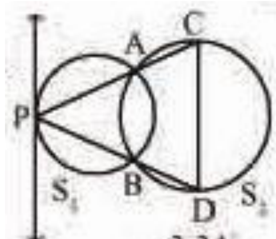
20. Two circles S_1 and S_2 touch each other externally at point P. A direct common tangent touches the circles S_1 and S_2 at point A and B respectively. (3.33). If a common tangent passes through point P intersect $\overleftrightarrow{AB}$ at C then prove that

(a) AC=BC (b) m$\angle$APB=90°



3.33

21. The circles S_1 and S_2 intersect each a point A and B (3.34). $\overrightarrow{PA}$ and $\overrightarrow{PB}$ drawn at point P intersects the circle S_2 at the point C and B respectively. Prove that the tangents drawn at the point and the circle S_1 is parallel to $\overline{CD}$.



3.34

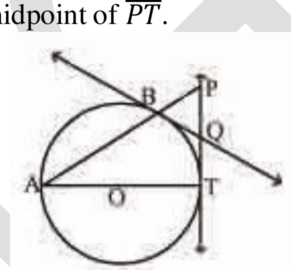
22. The radius of the two non intersector circles are r_1 and r_2 unit where $r_1 > r_2$. The distance between the two circles is d_1 units

(a) If the points of contact of direct common tangents are A and B, prove that $AB^2=d^2-(r_1 - r_2)^2$.

(c) The point of contact of a transverse common tangent to the both the circles are C and D, prove that $CD^2 = d^2- (r_1 + r_2)^2$.

23. P is an external point of a circle. The point of contacts are Q and R of the common tangents which tends to point P. The minor arc intercepted by the chord $\overline{QR}$ and S is its midpoint, prove that $\angle$PQR is bisected by $\overrightarrow{QS}$.

24. In fig 3.35, $\overline{AT}$ is the diameter of a circle, B is another point on that circle. The AB and the tangent drawn at P intersect each other at P. The tangent $\overleftrightarrow{TP}$ drawn on the circle at point B intersects the circle at Q. Prove that Q is the midpoint of $\overline{PT}$.

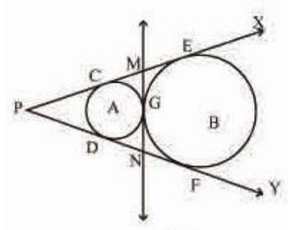


3.35

25. $\overline{AB}$ is the diameter of a circle. C is a point on the tangent and $\overline{CA}$ intersects the circle at D. Prove that AB^2= AC x AB.

26. $\overline{AB}$ is the diameter of a circle. C and D are the two points on the tangent to the circle at point B to form C-B-D. If $\overline{CA}$ and $\overline{DA}$ intersects the circle at P and Q, prove that AC x AP = AD x AQ

27. In fig 3.36, two circles S_2 and S_2 touch each other externally at G. P is the starting point of the direct common tangent $\overrightarrow{PX}$ and $\overrightarrow{PY}$. $\overrightarrow{PX}$ intersects the circle S_2 and S_2 at the point C and E and PY intersects the circles S_2 and S_2 at D and F respectively



3.36

(a) Prove that

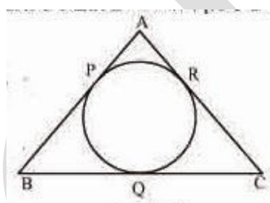
(i) P, A, G, B lie on a straight line

(ii) CE = DF

(b) Direct common tangents to both of the circle intersects $\overrightarrow{PX}$ and $\overrightarrow{PY}$ at the point M and N respectively, prove that (i) PM = PN, (ii) MG = NG.

28. The point of contact of two internally touching circles is P. A straight line insects one circle at points A and B and the other circle at C and D. Prove that $\angle APC$ and $\angle BPD$ are congruent.
(A-C-D and A-D-C is provable in all cases)

29. The incircle drawn in ΔABC touches the side $\overline{AB}, \overline{BC}$ and $\overline{AC}$ at points P, Q and R respectively. (Fig 3.37) BQ = 8 cm and CQ = 6 cm and the perimeter of the ΔABC is 36 cm. Find AB and AC.

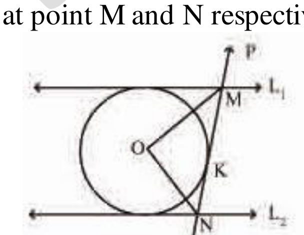


3.37

30. O is the centre of a circle and circumscribed quadrilateral is ABCD. Prove that $\angle AOB$ and $\angle COD$ are supplementary.

31. AB is the chord of a circle. The tangent to the circle at a point P is parallel to the chord AB. Prove that $\widehat{APB}$ is bisected at the point P.

32. In fig 3.38, O is the centre of the circle, L_1 and L_2 are two tangents and $L_1 \parallel L_2$. $\overleftrightarrow{PQ}$ is tangent drawn at point K intercepts L_1 and L_2 at point M and N respectively. Prove that $\angle MON$ is a right angle.



3.38

Exercise – 4(a)
Part A

1. Chose the correct answer from given options.

(a) $\sin 80^0 =$	[$\sin 10^0$, $\sin 20^0$, $\cos 10^0$, $\cos 20^0$]
(b) $\cos 65^0 =$	[$\sin 25^0$, $\sin 35^0$, $\cos 25^0$, $\cos 35^0$]
(c) $\sin 180^0 =$	[$1, -1, 0, \pm 1$]
(d) $\cos 90^0 =$	[$1, -1, 0, \pm 1$]
(e) $\cos 110^0 + \sin 20^0 =$	[$2\cos 110^0$, $2\sin 20^0$, 0, 1]
(f) $\sin 75^0 - \cos 15^0 =$	[$\frac{\sqrt{3}}{2}, \frac{1}{2}, 0, 1$]
(g) $\sin 0^0 =$	[$\cos 0^0$, $\sin 90^0$, $\sin 180^0$, $\cos 180^0$]
(h) $\sin 15^0 + \cos 105^0 =$	[$0, 1, -1, \pm 1$]
(i) $\cos 121^0 + \sin 149^0 =$	[$1, -1, 0, \pm 1$]
(j) $\tan 102^0 - \cot 168^0 =$	[$0, -1, 1, \pm 1$]

2. Find the trigonometric ratio ($0^o<\theta<90^o$) for $90^o+\theta$ or $90^o - \theta$ or $180^o - \theta$.
(i) $\sin 111^0$ (ii) $\cos 122^0$ (iii) $\tan 99^0$ (iv) $\cot 101^0$
(v) $\sin 91^0$ (vi) $\operatorname{cosec} 93^0$ (vii) $\cos 128^0$ (viii) $\operatorname{cosec} 132^0$ (ix) $\cot 131^0$

3. Find the trigonometric ratio for the following identities between 0^o to 45^o.
(i) $\cos 85^o + \cot 85^o$ (ii) $\sin 75^o + \tan 75^o$ (iii) $\cot 65^o + \tan 49^o$

4. Evaluate the following :
i) $\dfrac{\sin 18^0}{\cos 72^0}$ ii) $\dfrac{\tan 26^0}{\cot 64^0}$ iii) $\dfrac{\sin 116^0}{\cos 26^0}$ iv) $\dfrac{\operatorname{cosec} 74^0}{\operatorname{cosec} 106^0}$ v) $\dfrac{\sin 28^0}{\cos 118^0}$

Part B

5. Simplify :
(i) $\operatorname{cosec} 31^0 - \sec 59^0$
(ii) $\sin(50^0 + \theta) - \cos(40^0 - \theta)$
(iii) $\dfrac{\cos^2 20^0 + \cos^2 70^0}{\sin^2 59^0 + \sin^2 31^0}$
(iv) $\tan(55^0 - \theta) - \cot(35^0 + \theta)$
(v) $\cos 1^0 . \cos 2^0 .... \cos 180^0$
(vi) $\left(\dfrac{\sin 27^0}{\cos 63^0}\right)^2 + \left(\dfrac{\cos 63^0}{\sin 27^0}\right)^2$
(vii) $\cot 112^0 . \cot 158^0$
(viii) $\cos^2(90^0 + \alpha) + \cos^2(180^0 - \alpha)$
(ix) $\sec^2(105^0 + \alpha) - \tan^2(75^0 - \alpha)$
(x) $\sin^2(110^0 + \alpha) + \cos^2(70^0 - \alpha)$

6. Evaluate

(i) $\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ$ (ii) $\dfrac{\sin 51^\circ + \sin 156^\circ}{\cos 39^\circ + \cos 66^\circ}$

(iii) $\dfrac{\cos 68^\circ + \sin 131^\circ}{\sin 22^\circ + \cos 41^\circ}$ (iv) $\dfrac{\sin 162^\circ + \cos 153^\circ}{\cos 72^\circ - \cos 27^\circ}$

(v) $\dfrac{\cos 38^\circ + \sin 120^\circ}{2\sin 52^\circ + \sqrt{3}}$ (vi) $\dfrac{2\cos 67^\circ}{\sin 23^\circ} - \dfrac{\tan 40^\circ}{\cot 50^\circ} - \sin 90^\circ$

(vii) $\dfrac{\sec 61^\circ + \operatorname{cosec} 120^\circ}{\sqrt{3}\operatorname{cosec} 29^\circ + 2}$

7. Prove that

(i) $\cos(90^\circ - \theta) \,.\, \operatorname{cosec}(180^\circ - \theta) = 1$

(ii) $\dfrac{\cos 29^\circ + \sin 159^\circ}{\sin 61^\circ + \cos 69^\circ} = 1$ (iii) $\sin^2 70^\circ + \cos^2 110^\circ = 1$

(iv) $\sin^2 110^\circ + \sin^2 20^\circ = 1$ (v) $\sec^2\theta + \operatorname{cosec}^2(180^\circ - \theta) = \sec^2\theta \,.\, \operatorname{cosec}^2\theta$

(vi) $2\sin\theta \,.\, \sec(90^\circ + \theta) \,.\, \sin 30^\circ \,.\, \tan 135^\circ = 1$

8. Prove that

(i) $\cos^2 135^\circ - 2\sin^2 180^\circ + 3\cot^2 150^\circ - 4\,\text{tna}^2 120^\circ = \dfrac{-5}{2}$

(ii) $\tan 30^\circ \,.\, \tan 135^\circ \,.\, \tan 150^\circ \,.\, \tan 45^\circ = 1$

(iii) $\dfrac{\sec^2 180^\circ + \tan 150^\circ}{\operatorname{cosec}^2 90^\circ + \cot 120^\circ} = 1$

(iv) $\sin^2 135^\circ + \cos^2 120^\circ - \sin^2 120^\circ + \tan^2 150^\circ = \dfrac{1}{3}$

Part C

9. Find the value of

(i) $\tan 10^\circ \times \tan 20^\circ \times \tan 30^\circ \times \ldots \times \tan 70^\circ \times \tan 80^\circ$

(ii) $\cot 12^\circ \,.\, \cot 38^\circ \,.\, \cot 52^\circ \,.\, \cot 60^\circ \,.\, \cot 78^\circ$

(iii) $\tan 5^\circ \,.\, \tan 15^\circ \,.\, \tan 45^\circ \,.\, \tan 75^\circ \,.\, \tan 85^\circ$

10. Prove that

(i) $\sin 120^\circ + \tan 150^\circ \,.\, \cos 135^\circ = \dfrac{3+\sqrt{2}}{2\sqrt{3}}$

(ii) $\dfrac{\sec^2 180^\circ + \tan 150^\circ}{\operatorname{cosec}^2 90^\circ + \cot 120^\circ} = 2 - \sqrt{3}$

(iii) $\dfrac{\sec^2 180^\circ + \tan 45^\circ}{\operatorname{cosec}^2 90^\circ - \cot 120^\circ} = 3 - \sqrt{3}$

11. Simplify

(i) $\sin(180^\circ - \theta) \cdot \cos(90^\circ + \theta) + \sin(90^\circ + \theta) \cdot \cos(180^\circ - \theta)$

(ii) $\dfrac{\cos(90^\circ - A) \cdot \sec(180^\circ - A)}{\sin(90^\circ + A) \cdot \tan(90^\circ + A)} \cdot \dfrac{\sin(180^\circ - A)}{\operatorname{cosec}(90^\circ + A)}$

12. In a Δ ABC, $m\angle B = 90^\circ$, prove that $\sin^2 A + \sin^2 C = 1$
13. In a Δ ABC, prove $\cos(A+B) + \sin C = \sin(A + B) - \cos C$.
14. If A and B are two complementary angles, evaluate $\sin A \cdot \cos B + \cos A \cdot \sin B$
15. If ABCD is a cyclic quadrilateral, find the value of $\tan A + \tan C$.

16. Prove

$$\frac{\sin^2 135^\circ + \cos^2 120^\circ - \sin^2 150^\circ + \tan^2 150^\circ}{\sin^2 120^\circ - \cos^2 150^\circ + \tan^2 120^\circ + \tan^2 135^\circ - \cos 180^\circ} = \frac{5}{18}$$

17. Prove

$$\frac{5\sin^2 150^\circ + \cos^2 45^\circ + 4\tan^2 120^\circ}{2\sin 30^\circ \cdot \cos 60^\circ - \tan 135^\circ} = \frac{55}{6}$$

Exercise – 4(b)

Part A

1. Fill in the blanks

i) $\sin(A-B) = \frac{\sin A}{\ldots\ldots} - \frac{\cos A}{\ldots\ldots}$

ii) $\cos(\theta+\alpha) + \cos(\alpha-\theta) = \ldots\ldots\ldots\ldots$

iii) $\cos(60^0 - A) + \ldots\ldots\ldots\ldots = \cos A$

iv) $\sin(30^0 + A) + \sin(30^0 - A) = \ldots\ldots$

v) $2\sin A . \sin B = \ldots\ldots\ldots - \cos(A+B)$

vi) $\tan(45^0+\theta) . \tan(45^0-\theta) = \ldots\ldots\ldots\ldots\ldots\ldots$

Part B

2. Prove that

i) $\frac{\sin(A-B)}{\cos A.\cos B} = \tan A - \tan B$

ii) $\frac{\cos(A+B)}{\cos A.\cos B} = 1 - \tan A . \tan B$

iii) $\frac{\cos(A-B)}{\cos A.\sin B} = \cot B + \tan A$

iv) $\frac{\sin\alpha}{\sin\beta} - \frac{\cos\alpha}{\cos\beta} = \frac{\sin(\alpha-\beta)}{\sin\beta.\cos\beta}$

v) $\frac{\cos\alpha}{\sin\beta} - \frac{\sin\alpha}{\cos\beta} = \frac{\cos(\alpha+\beta)}{\sin\beta.\cos\beta}$

3. Prove

i) $\cos(A+45^0) = \frac{1}{\sqrt{2}}(\cos A - \sin A)$

ii) $\sin(45^0-\theta) = -\frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)$

iii) $\tan(45^0+\theta) = \frac{1+\tan\theta}{1-\tan\theta}$

iv) $\cot(45^0+\theta) = \frac{\cot\theta+1}{\cot\theta-1}$

4. Prove

i) $\cos(45^0-A) . \cos(45^0-B) - \sin(45^0-A) . \sin(45^0-B) = \sin(A+B)$

ii) $\sin(40^0+A) . \cos(20^0-A) + \cos(40^0+A) . \sin(20^0-A) = \frac{\sqrt{3}}{2}$

iii) $\cos(65^0+\theta) . \cos(35^0+\theta) + \sin(65^0+\theta) . \sin(35^0+\theta) = \frac{\sqrt{3}}{2}$

iv) $\cos n\theta . \cos\theta + \sin n\theta . \sin n\theta = \sin(n-1)\theta$

v) $\tan(60^0-A) = \frac{\sqrt{3}\cos A - \sin A}{\cos A + \sqrt{3}\sin A}$

Part C

5. Prove

(i) $\tan 62^0 = \dfrac{\cos 17^0 + \sin 17^0}{\cos 17^0 - \sin 17^0}$ (ii) $\dfrac{\cos 25^0 + \sin 25^0}{\cos 25^0 - \sin 25^0}$

(iii) $\tan 7A \,.\, \tan 4A \,.\, \tan 3A = \tan 7A - \tan 4A - \tan 3A$

(iv) $\tan (x + y) - \tan x - \tan y = \tan (x + y) \,.\, \tan x \,.\, \tan y$

(v) $(1 + \tan 15^0)(1 + \tan 30^0) = 2$

(vi) $(\cot 10^0 - 1)(\cot 35^0 - 1) = 2$

(vii) $\dfrac{1}{\cot A + \tan A} - \dfrac{1}{\tan A + \cot B} = \tan(A - B)$

(viii) $\sqrt{3} + \cot 50^0 + \tan 80^0 = \sqrt{3}\ \cot 50^0 \,.\, \tan 80^0$

6. Find the value of $\cos 75^o$ and $\sin 15^o$.

7. (i)If $\cos \alpha = \dfrac{8}{17}$ and $\sin \beta = \dfrac{5}{13}$, find the value of $\sin (\alpha - \beta)$.

(ii)If $\tan A = \frac{1}{2}$ and $\cot B = 3$ then show A + B.

(iii) if $\tan\beta = \dfrac{1 - \tan \alpha}{1 + \tan \alpha}$, show $\tan(\alpha + \beta)$

8. If $A+B+C=90^o$, prove that

(i) $\cot A + \cot B + \cot C = \cot A \,.\, \cot B \,.\, \cot C$

(ii) $\tan A \,.\, \tan B + \tan B \,.\, \tan C + \tan C \,.\, \tan A = 1$

9. (i) If $A + B + C = 180^o$ and $\sin C = 1$, prove $\tan A \,.\, \tan B = 1$

(ii) If $A + B + C = 180^o$, prove $\cot .\cot B + \cot B . \cot C + \cot C \,.\, \cot A = 1$

(iii) If $A + B + C = 180^o$ and $\cos A = \cos B \,.\, \cos C$, then prove

(a) $\tan A = \tan B + \tan C$

(b) $\tan B . \tan C = 2$

10. Show that

(i) $\sin (A+B) .\sin (A - B) = \sin^2 A - \sin^2 B$

(ii) $\cos (A + B) \,.\, \cos (A - B) = \cos^2 A - \sin^2 B$

11. Prove

(i) $\sin 50^0 + \sin 40^0 = \sqrt{2}\ \sin 85^0$

(ii) $\cos 50^0 + \cos 40^0 = \sqrt{2}\ \cos 5^0$

(iii) $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$

12. Evaluate

(i) $\sin (A + B) = \dfrac{1}{\sqrt{2}}$, $\cos (A - B) = \dfrac{1}{\sqrt{2}}$

(ii) $\cos (A + B) = -\dfrac{1}{2}$, $\sin (A - B) = \dfrac{1}{2}$

(iii) $\tan (A - B) = \dfrac{1}{\sqrt{3}} = \cot (A + B)$,

(iv) $\tan (A + B) = -1$, $\mathrm{cosec}\ (A - B) = \sqrt{2}$

Exercise – 4(c)

Part A

1. The angle of elevation of the top of a tree at a distance of 120 m from its foot on a horizontal plane is found to be 30^{o}. Find the height of the tower.

2. The angle of depression of the top of 27 m high light house to a ship is about 30^{o}. Find the distance between the light house and the ship.

3. An observer having height of 2 m saw a pillar standing at a distance of 24 m and the angle of elevation is 30^{o}. Find the height of the pillar.

4. A ladder is placed along a wall such that its upper end is resting against a vertical wall. The foot of the ladder is 3m from wall and ladder is making an angle of 60^{o}. Find the length/height of the ladder.

5. A 1.5 m tall observer stands at a point, distance of 12 m from a building. He observed the angle of elevation of the building from the point is 60^{o}. Find the height of the building.

6. The length of the shadow of a tree was 15 m when the angle of elevation of Sun rays was 60^{o}. Find the height of the tree.

Part B

7. If angle of elevation and depression of pillar standing on a horizontal plane from the 300 m high cliff is 30^{o} and 60^{o} respectively, find the height of the pillar.

8. The length of a shadow of a pillar was increase to 24 m when the angle of elevation of the Sun's rays decreases from 60^{o} to $30^{o.}$ Determine the height of the pillar.

9. Two vertical poles are erected at difference of 40 m on the same horizontal plane. The height of one pole is two times more than the other. When the foot points of both poles are added by drawing a line segment, they form a complementary angle at the centre of the line segment. Determine the height of the two poles.

10. The angle of depression of a thing on the floor from the top of the tree is 60^{o}. If we come down by 1.5 m from top, then the angle of depression of the thing on floor reduces to 30^{o}. Find the height of the tree.

11. The angle of elevation and depression of a temple on horizontal plane from the top of the 10 m tall pillar is 45^o and 30^o. Determine the height of the temple.

12. Two building are standing opposite to each other on either side of the road which is 10 m wide such that the window of the opposite building forms right angle with other building. If the angle of elevation of window from foot of building is 30^o, then find the height of the building.

13. A man standing on the bank of river sees the angle of elevation of fort erected on the opposite side of the bank is 60^o. He moved away from the bank in a straight line from the point where he was standing by 60 and the angle of elevation of the fort became 45^o. Find the breadth of the river.

14. Two poles are erected at a distance of 12 m on a plain surface. The height of one pole is two times more than the other. The angle of elevation forms complementary angle if it is seen from the midpoint of the line segment joining the foot of the poles. Determine the heights of two poles.

15. The angle of elevation of the top of the fort seen from the two different points of foot of the fort on a horizontal plane is 30^o and 45^o respectively. If the height of the fort is 30 m, then find the distance between the two points.

16. The height of a building is 12m. The angle of elevation and depression of a pillar standing on the horizontal plane is 60^o and 30^o respectively from the top of the building. Find the height of the pillar and distance between the building and the pillar.

MENSURATION

Exercise 5 (a)

1. (a) Find the circumference of circle for the following radii

(i) 10 cm, (ii) 2.8 cm, (iii) 14 cm, (iv) 4.2 cm $\left(\pi \simeq \frac{22}{7}\right)$

(b) Find the radius of the circle for the following circumferences

(i) 34.9 cm (ii) 1047 cm (iii) 25.128 cm (iv) 15.705 cm $(\pi \simeq 3.141)$

2. In a circle, if the length of an arc is L, radius is 'r' and degree measure of the arc is θ, then solve the following

(a) if $r = 56$ cm, $\theta = 45^\circ$ find L ?

(b) if $L = 110$ m, $\theta = 75^\circ$ find r ?

(c) if $2r = 9$ dam, $L = 22$dam find θ?

3. Answer the following questions. $\left(\pi \simeq \frac{22}{7}\right)$

(a) If the radius of a circle is 10.5 cm and length of an arc is 11 cm, find the degree measure of the arc.

(b) If degree measure of an arc is 72° and radius of the circle is 21 cm, find the length of the arc.

(c) Find the radius of the circle if the length of an arc is 11 cm and degree measure of the arc is 10° .

(d) Find the radius of circle in terms of π, when radius of the circle is X units , length of the arc is Y units and the degree measure is Z°.

(e) If a square of length 'a' units is inscribed in a circle of radius 'r' units, find the relation between 'a' and 'r'.

4. The diameter of the earth from equator is 12530 Km. find the circumference of the circular equator. $\left(\pi \simeq \frac{22}{7}\right)$

5. How many circles of radius 5 cm can be formed from a wire of length 44 meters. $\left(\pi \simeq \frac{22}{7}\right)$

6. The outer and inner circumference of the road is 396 and 352 meters respectively. Find the width/breadth of the road.

7. The difference between circumferences of two circles is 44 meters whereas the sum of their radii is 77 m. Find the circumference of each circle.

8. The ratio of the radii of two concentric circles is 3 : 4. The sum of their circumferences is 308 cm. Find the width of the circles.

9. The outer and inner circumference of a annular (circular) path is 300 and 200 meters respectively. Find the width of the path.

10. How many rounds a person has to move on a circle of radius 7 m. so that he covers a distance of 11 Km.

11. Each wheel of a cycles revolves 80 times per minute. If the outer diameter of the wheel is 42 cm., find the speed of the cycle in Km.

12. The ratio of the circumference of the large wheel and small wheel of a vehicle is 4:1, the small wheel makes 15 revolutions more than the larger wheel to cover a distance of 440 meters. Find the circumference of each wheel.

13. If the total cost of fencing a semicircular land is Rs 216 at the rate of 75 Paise per meter, find the diameter of the semicircular land.

14. If a horse takes 10 min 12 sec to complete one revolution and reach the centre of the circle then what will be the time taken by it to complete only one revolution.

15. A person takes 45 sec less time to cover the distance of the diameter of the circular path than covering one completer revolution. If the speed of the person is 80 m/s, find the diameter of the circle.

16. The area of the equilateral triangle drawn by a wire is $1936\sqrt{3}$ sq.m. What will be the diameter of the circle if drawn having same circumference as the perimeter of the equilateral triangle.

17. If a circle is inscribed in a square of side 20 cm., find the circumference of the circle.

18. Find the circumferences of inscribed and circumscribed circles of an equilateral triangle of sides 42 cm.

19.
(a) Find the degree measure of the sector, if the perimeter of the sector is 21 cm and radius is 64cm.
(b) Find the radius of a sector whose degree measure is 40° and perimeter is 26.98 cm.

20. If the centre (degree measure) of a sector is 90° and radius is 5 cm., find the perimeter of the sector.

21. The degree measure of an arc of a circle is 40° and the degree measure of an arc of another circle of equal length is 60°. Find the ratio of lengths of the radius of two circles.

22. The tip of the minute hand of a clock draws an arc of $7\frac{1}{3}$ cm of length in 5 minutes. Find the length of the minute hand.

23. The circumference of a circle is three times more than another circle. If the degree measure of a 10 cm long arc of first circles is 30°, find the circumference of the second circle.

24. If the circumference of a circle is 6.282 and it is inscribed in a equilateral triangle, find the length of the side of the triangle.

25. The degree measure of a sector is 60°. A circle is inscribed touching the two radii and the arc of the sector. Prove that the ratio of the circumference of the circle and perimeter of the sector is 11:16.

Exercise 5 (b)

1. Find the area of a circle whose
   (i) radius is 31.5 cm (ii) diameter is 112 cm
   (iii) circumference 286 cm (iv) semi-circumference 44 m

2. (i) Find the diameter of a circle if its area is 154 square meteres.
   (ii) Find the circumference of a circle if its area is 7546 sq m.

3. Find the area of a sector of a circle having
   (i) degree measure = 120^{o} and radius = 28 cm
   (ii) area of the same circle = 7546 sq m and degree measure = 105^{o}
   (iii) circumference = 396 m and length of an arc = 36 m.
   (iv) length of an arc = 66 m and degree measure of arc = 70^{o}

4. Find the radius of the sector whose
   (i) area = 1848 sq m and degree measure = 120^{o}
   (ii) area = 48.4 sq decameter and length of an arc = 121 meter

5. Find the degree measure of a sector
   (i) radius = 36 m and area = 792 sq m
   (ii) area = 924 sq cm and area of the same circle = 2464 sq cm.
   (iii) area = 231 sq m and length of the arc = 22 m

6. Find the difference between the areas of the circles if degree measure of the two concentric circles is same –
   (i) and the difference between the lengths of the arcs is 25 m and the sum of two radii is 80 m
   (ii) and the sum of the lengths of the arcs is 50 cm and difference in their radii is 24 cm.

7. The area of a circle is X sq units. Find
   (i) the length of the hypotenuse of right angled triangle inscribed in the circle
   (ii) the length of the side of the inscribed square
   (iii) the length of the side of the inscribed equilateral triangle

8. The radii of two circles are 42 cm and 56 cm respectively. If the area of the third circle is same as the sum of the areas of the first two circles, find the length of the radius of the third circle.

9. The area of a square is equal to the area of a circle. Find the ratio of their perimeter.

10. The radius of a circle is 5 cm. Find the radius of a circle of 9 times greater than the area of previous circle.

11. Find the radius of a circle, if unit measure of circumference of a circle and the area enclosed by it in sq. units is same.

12. The area of a square is C sq. units. Find the radius of inscribed and circumscribed circle of it.

13. Prove that if area of circle is equal to area of an equilateral Δ, then the ratio of the radius of circle and length of the side of triangle is √√¾π : 1.

14. If the circumference of a semi circular region is 252 cm. Find its area.

15. If the circumference of a semicircle is 44 m more than its diameter, find the area of semicircular region.

16. The area of the semicircular land is 2772 sq. m., find the total expenditure for fencing the land if the cost of fencing is 37 paise per meter.

17. If the diameter of inner and outer circle of a circular path is 56 cm and 42 cm respectively. Find the area of the path.

18. A road is constructed along the periphery of a circular garden of diameter 32 cm. If the area of the road is 352 sq. meters, find the breadth/width of the road.

19. The sum of the circumferences of two circles is 220 cm. and difference of area is 770 sq. cm. Find the radii of the circles.

20. The area of a square made by an iron wire is 484 sq cm. If the same wire shaped into a circle, find the area of that circle.

21. The ratio of radii of two circles is 4 : 5. If the area of the first circle is 352 sq. cm, find the area of other circle.

22. If the length of the side of an equilateral triangle inscribed in a circle is 14√3, find the area of the circle.

23. The area of the circle which is inscribed in an equilateral triangle is 154 sq. m. Find the perimeter of the triangle.

24. In a circle, the length of an arc of one sector is three times of the length of an arc of second sector. If the area of first sector is 9 sq cm., find the area of second sector.

25. The total cost of fencing along the sector shaped area is Rs 75 at the rate of Rs 1.50 per meter. If the degree measure of the sector is 90^{o}, find its radius.

26. Three circles of radius 7 cm each touch each other. Find the area of the place surrounded by outer surface of the circles, up to two decimal place only. $(\sqrt{3} \simeq 1.73)$, $(\pi \simeq 3.14)$

27. If an area of a circle is equal to the area of a circular annulus of inner radius 12 cm and outer radius 13 cm., find the radius of the circle.

28. In a circle, the degree measure of a sectorial arc $\overset{\frown}{AXB}$ drawn is 60^{o}. If the area of a circle touching radii $\overline{OA}$, $\overline{OB}$ and $\overset{\frown}{AXB}$ is 9π, find

(i) the radius of first circle

(ii) find the ratio of the sector OAXB and the circle which is inscribed in it.

29. In a circle of radius 8 cm

(i) find the area of a minor segment formed by the interception of a cord of length 8 cm

(ii) find the area of the minor segment formed by the interception of a cord of length $8\sqrt{2}$ cm.

30. Find the area of the segment creates an angle of 60^{o} at the centre of a circle of radius 20 cm.

($\sqrt{3} \simeq 1.73$), ($\pi \simeq 3.14$)

31. Find the area of the segment creates an angle of 120^{o} at the centre of a circle of radius 10 cm.

($\sqrt{3} \simeq 1.73$), ($\pi \simeq 3.14$)

Exercise 5 (c)

1. In a regular triangular base prism, the length of sides is taken as a, b, and c, height as h, lateral surface area as L and total surface area as W, solve the following questions.

(a) a = 10 cm., b = 6 cm., c = 8 cm., h = 20 cm. find L and W.

(b) a = 5 m., b = 5 m., c = 6 m, h = 8 m., find, L and W.

(c) a = b = 15 m., c = 24 m., h = 18 m. find, L and W.

2. If the height of the prism is h, lateral surface area is L and total surface area is W, then solve the following questions

(a) if the base of the prism is right angled isosceles and the length of the hypotenuse is 40 m and height is 50 m, find L and W.

(b) the length of the side of a regular hexagonal base is 6 dm. , height h=20 dm, find L and W.

(c) the base of the triangle is equilateral where the length of side = 16 cm and height h=25 cm. Find L and W.

3. The length of the sides of the triangular shaped base of a prism is 13 cm., 14 cm, and 15 cm. respectively. The lateral surface area of the prism is 840 sq cm. Find the height and total surface area of the prism.

4. The right prism shaped pillar whose base is an equilateral triangle. The cost of covering the later surface of the prism is Rs 18.90 at the rate of 15 paise per sq. cm. by paper. The height of the pillar is $8\sqrt{3}$ cm. Find the length of the side of base of prism.

5. The lengths of the sides of the triangular base shaped prism are 12m, 16m and 20m respectively and its height is 18m. Find the total surface area of the prism.

6. The lateral surface area of a prism is 2100 sq cm. and height is 30 cm. Its base is right angles triangle whose length of the biggest side is 29 cm. Find the length of other two sides.

7. The base of the prism is an isosceles triangle, whose length is 50 cm and height is 1.2 cm. Find the total surface area of the prism.

9. The lengths of the sides of a triangular shaped base of a prism are 13 cm, 14 cm and 15 cm respectively. The lateral surface area of it is 1050 sq cm. Find the height and total surface area of that prism.

10. A wood stick is a simple prism whose base is like an equilateral triangle. The cost of covering its lateral sides is Rs 18.90 at the rate of 15 paise per sq cm. The height of the wooden sick is $8\sqrt{3}$ cm. Find the length of the sides of the base.

SURFACE OF THE CYLINDER

11. Answer the following questions if the radius of the cylinder is r, diameter is d, and height is h
(i) if d=16 cm, h=21 cm, find the curved surface area.
(ii) if the curved surface area is 1189 sq m, find h.
(iii) if area of the base is 1386 sq m and h = 36 cm, find the total surface area.

12. The length of a roller is 1.6 m and height is 70 cm. How many times the roller revolves on a 26.4 acres of land to make it plain?

13. A roller is required to be rolled 90 times on a land of 1540 sq m land. If the length of the roller is equal to its diameter, find the length of the roller.

14. The total cost of painting the curved surface area of a cylinder shaped pillar is Rs 792 at the rate of 60 paise per sq m. If the area of its base is 154 sq m, find its height.

15. The outer radius of a hollow cylinder whose both ends are open is 5 meters. Its height is 14 m and the total surface area is 748 sq m. Find the inner radius of the hollow cylinder.

16. The length of an iron pipe is 84 cm. The width of it is 2 cm. The outer radius is 8 cm. Find its total surface area.

17. If the length of an iron pipe is 100 cm and the width of the iron is 4 cm. If its total surface area is 9152 sq cm., find the inner radius and outer radius of the base.

Exercise 5 (d)

1. The total surface area of a right prism is 2520 sq m. The lengths of the sides of its triangular shaped base are 20 m, 21m, and 29m respectively. Find its volume.

2. If the base of a right prism is equivalent to right angled isosceles triangle of hypotenuse $8\sqrt{3}$ cm long hypotenuse and height of 14 cm, find the volume.

3. The volume of a right prism is 2520 m^3 (cubic meter). Its base is a right angled triangle whose adjoining length of two sides are 7 m and 24 m. Find the height and lateral surface area of the prism.

4. If the base of the 15 cm high prism is right angled triangle where the length of its hypotenuse is 10 cm and volume is 360 m^3. Find the lengths of the other two sides of the base.

5. The lateral surface area of a right prism is $8/9^{th}$ of the total surface area of it. If the lateral surface are of the prism is 96 sq m and volume is 48 m^3, find its height.

6. The perimeter of the base of the right prism is 56 meters. If the lateral surface area of the prism is 1680 sq m and volume is 2520 m^3, find the area of the base of the prism.

7. The volume of the right prism is $84\sqrt{3}$ m^3. Its height is 7 cm and base is an equilateral triangle. Find the lengths of the sides of its base.

8. The height of the right prism is 336 cm. The lengths of it base are 21 cm, 72 cm and 75 cm respectively. If the volume of this prism is equal to the volume of second prism of height 288 cm whose base is right angled triangle of hypotenuse $42\sqrt{2}$ cm, find the lengths of the sides of the base of the prism.

9. The base of the prism of height $8\sqrt{3}$ m is equilateral triangle. If the volume of the prism is 864 m^3, find the total surface area of the prism.

VOLUME OF CYLINDER

10. If a well of diameter 4 m and depth of 9 m is dug and the mud is stored in a dome of cylindrical shape whose diameter is 12 m, find the height of the dome.

11. The cost of constructing a cylindrical pillar is Rs 352 at the rate of Rs 8 per 100 cubic dm. If the diameter of the base of the cylindrical pillar is 20 dm., find its height.

12. The volume of a cylinder of 28 m high is equal to the volume of a cube of sides 5½ long. Find the diameter of base of the cylinder.

13. The volume of cylinder is 9504 cm. and the curved surface area is 1584 sq cm. Find its height.

14. The height of the cylinder is twice the diameter of its base. If its volume is 539 cubic dm., find the total surface of the cylinder.

15. The total surface area of the right circular solid cylinder is 704¼ sq cm and curved surface area is 528 sq cm. Find its volume.

16. The ratio of height and diameter of a cylinder of circular base is 3 : 2. Its total surface area is 1232 sq cm. Find its volume.

17. The volume of the metal used by a hollow cylinder whose both sides are closed is 4928 cm^3 and the difference of surface area of both sides is 352 cm^2. If the height of the cylinder is 28 cm, find its internal and external radius.

Exercise 5 (e)

1. Height of a cone shaped cap is h and slant height l is given. Find the quantity of cloth used for making the each cap and area of its base –

(i) h = 3.5 cm, l = 9.1 cm, (ii) h = 5.6 cm, l = 11.9 cm, (iii) h = 3.5 cm, l = 12.5 cm.

2. Slant height l and radius of the base r of a cone shaped tent is given. Find the inner volume of the tent and quantity of cloth used.

(i) r = 10 .5 m. l = 14.5 m. (ii) h = 24 m. l = 25 m.

3.
(i) The volume of a cone is 12936 m^3. If its height is 28 m., find the area of the base and area of the curved surface.

(ii) The volume of a cone is 9240 cubic units. If the radius of its base is 21 units, find the area of the curved surface.

4.
(i) The area of the curved surface 550 sq cm and radius of its base is 7 cm. Find the volume of the cone and total curved surface area of the cone.
(ii) The area of the curved surface 4070 sq cm and its slant height is 37 cm. Find the area of the base and its volume.

5. Determine the volume and curved surface area of a cone whose total curved surface area is 2816 sq cm and radius of its base is 14 cm.

6. Find the volume of a cone whose total curved surface area is 1386 sq cm and curved surface area is 770 sq cm.

7.
(i) if volume of a cone is 12936 cm^3 and ratio of r : h = 3:4, Find curved surface area of it.
(ii) if volume of a cone is 17248 m^3 and ratio of r : l = 4:5, Find curved surface area of it.

8.
(i) The ratio of radii of two cones is 3 : 5 and its height is 1 : 3. Find the ratio of their volumes.
(ii) The ratio of radii of two cones is 2 : 7 and its slant height is 3 : 8. Find the ratio of their curved surface areas.
(iii) The ratio of areas of base of two cones is 1 : 9 and areas of curved surface is 5 : 21. Find the ratio of their slant heights.
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9.
(i) The height of a cone is half of its slant height. If the radius of cone is $5\sqrt{3}$, find its volume.
(ii) The height of a cone is half of its radius. If the slant height of the cone is 50 cm, find its volume.
(iii)The ration of height and diameter of the base of a cone is 2 : 3 and its slant height is 20 cm, find its volume.

10. The length of each side of a cube shaped log is 21 cm. Find the volume and total curved surface area of a major voluminous cone cut from the log.

11. A cone shaped container is made after soldering the radius of two sides of a sectorial shaped tin. The radius of the tin container is 12 cm and the angle between the two radii is 120^{o}, how much water the container can hold?

12. The diameter of the base of a solid cone is 6 cm and its height is 8 cm. It is immersed into a cylinder partially filled with water. If inside diameter of the cylinder is 8 cm, find the rise in water level.

13. The lower portion of a tent is like a cylinder whose radius is 35m and height is 8 m and its upper portion is like a cone of radius 35 m and height is 12m. Find the canvas used to prepare the tent in square meters.

14. The lower portion of a tent is like a right circular base shaped cylinder of height 30 m and the upper portion is like a cone. The radius of its base is 22 m and the height from the ground to the vertex of the tent is 58 m. Find the area of the canvas used in the tent.

15. The radius of the circular edged cone shaped container filled with water is 2.5 cm and depth is 11 cm. How many lead pellets of radius 0.25 cm are to be dropped to make 2/5 part of water over flow from the container.

16. In a right angle Δ, the lengths of two sides adjacent to right angle are 12 cm and 5 cm. If a cone is drawn by dragging the triangle around the larger sides by keeping it constant, find the volume and total curved surface area in terms of π.

Exercise 5 (f)

1. Radius r or diameter d of a sphere is given below, find the curved surface area and volume of it.
(i) r = 21cm (ii) d = 14 cm (iii) r = 10 .5 cm

2. The radius of three metal spheres is given below. If all the three metal sphere are melt together and form a single metal sphere, find the radius of new metal sphere.

(i) 3 cm, 4 cm, 5 cm (ii) 8cm, 6 cm, 1cm. (iii) 17 cm, 14 cm, 7 cm.

3. The ratio of diameters and radius of two spheres is given below. Find the ratio of volume and curved surface area of each

(i) $\frac{d1}{d2} = \frac{3}{4}$ (ii) $\frac{r1}{r2} = \frac{1}{3}$ (iii) $\frac{r1}{r2} = \frac{2}{5}$

4. If the volume of a sphere is $\frac{792}{7}$ cm^3, find its curved surface area.

5.
(i) If the curved surface area of a sphere is 616 sq cm, find the volume of the sphere.
(ii) If the curved surface area of a sphere is 5544 sq cm, find the radius of the sphere.

6. If the curved surface area of a sphere is 19404 sq cm. Find the radius of the cube like hemisphere.

7. A metallic sphere of 9 cm radius is melted, find
(i) How many small spheres of 1 cm radius can be produced
(ii) find the length of the wire having circular cross section is drawn, if diameter of the circular cross section is 1 cm.

8. The inner diameter of the water tank of hemisphere shape is 4.2 m. Determine how much water can the tank hold.

9. If the volume of a cone and cylinder of same base is equal, find the ratio of their heights.

10. The inner radius of a hollow sphere is 3 cm and its outer radius is 6 cm. If the mass of the metal per cubic cm is 8 gm, find its mass.

11. The outer radius of a container of hemisphere shape is 8 cm and its thickness is 1 cm. Find the total curved surface area of the container.

12. A large sized sphere is cut from a solid cube. The volume of the remaining portion of the cube is 12870 cubic cm. Find the length of the side of the cube.

13. The thickness and outer radius of a hemisphere is 1 cm and 10 cm respectively. Find the (i) total curved surface area and (ii) the volume of metal used.

CONSTRUCTION

Exercise - 6(a)

Draw circum-circle of a triangle of which the length of one side and the measure of the angle opposite to it are given.

1. Δ ABC, BC = 6 cm, $m\angle A = 45^{o}$
2. Δ ABC, AC = 7 cm, $m\angle B = 60^{o}$
3. Δ ABC, AB = 6.5 cm, $m\angle C = 90^{o}$
4. Δ ABC, $m\angle A = 120^{o}$, BC = 4.5

5. Draw a Δ ABC, where BC = 7 cm, $m\angle A = 60^{o}$, median AX = 4.5cm.

6. In a Δ ABC, $\angle B$ is a right angle. $\overline{AC}$ = 7 cm and a perpendicular is drawn from point B to $\overline{AC}$. The length of $\overline{BD}$ is 3 cm. Draw a triangle and find the number of points present on the side AC from point B.

7. In a Δ ABC, BC = 8 cm, $m\angle A = 45^{o}$, height of AD 3 cm. Draw a triangle.

8. Draw a Δ ABC where $m\angle B = 60^{o}$, AC = 6.5 cm. the length of the Median $\overline{AX}$=5 cm.

9. In a Δ ABC, $m\angle A = 60^{o}$, BC = 7 cm, $\overline{BE} \perp \overline{AC}$, BE = 6.3 cm, construct a Δ.

10. In a Δ ABC, $m\angle A = 150^{o}$, BC = 5 cm, height of AD = 3 cm. Construct a triangle.

11. In a Δ ABC, $m\angle A = 60^{o}$, b:c = 2:3, BC = 7 cm. Construct a triangle.

12. Construct a parallelogram ABCD where AB = 5.5 cm, the length of diagonal $\overline{BD}$ = 8 cm and $m\angle DAC = 60^{o}$.

Exercise - **6(b)**

1. Construct a circle of radius 3 cm. Draw a tangent at any point on it.

2. Construct a circle of radius 3.5 and draw a tangent at any point on it without taking the help of centre of the circle.

3. Construct a circle of radius 3 cm. O be the centre of it and P is an exterior point on the circle. OP=7 cm. Draw two tangents PA and PB from point P to the circle. Measure the length of the two tangents and the relation between them.

4.Draw AB of length 4 cm. Construct a circle considering AB as diameter of the circle. Draw tangents on circle at point A and B. Conclude how the two tangents are related to each other.

5.

(i) O is the centre of the circle whose diameter is 4 cm. OA and OB are two radii and m$\angle$AOB= 90^{o}. Draw two tangents AX and BY on to circle and extend the tangent such that they meet at a point M. Find out what type of quadrilateral is formed from OAMB.

(ii) Construct a circle of radius 2.5 cm and name the centre as O. Draw OA and OB two radii such that, the m$\angle$AOB = 120^{o}. Draw tangents on point A and B and extent the tangents and name the point of their intersection as P. Draw diagonals OP and AB of quadrilateral OAPB. Find the relation between the two diagonals.

6. Draw a line segment AB of 8 cm. Construct a circle of radius 3 cm taking A as centre. Draw tangents from point B to the circle.

7. Construct a triangle of diameter = 6cm. Place a point P at anywhere outside the circle and draw a line of 4.5 cm from point P to the nearest point of the circle. Draw tangents from point P to the circle.

8. Construct a triangle of radius is equal to 3 cm. P is an external point of the circle and draw tangents from point P such that m$\angle$APB = 60^{o}.

Exercise - **6(c)**

1. Draw a circle of radius 4 cm and inscribe an equilateral triangle inside the circle.

2. Draw a circle of radius 3.5 cm and construct an equilateral triangle circumscribing the circle.

3. Draw a circle of radius 2.5 cm and inscribe a square in the circle.

4. Draw a circle of radius 1.5 cm and construct a square circumscribing the circle.

5. Draw a circle of radius 3.5 cm and inscribe a regular hexagon inside the circle.

6. Draw a circle of radius 3.5 cm and construct a regular hexagon circumscribing the circle.

7. Draw a circle of radius 4 cm and construct a regular hexagon circumscribing the circle.

8. Draw a circle of diameter 7.5 cm and inscribe a right-angled isosceles triangle inside the circle.

9. Draw a circle of diameter 8 cm and construct an isosceles triangle circumscribing the circle.
   (The internal angles of three radii should be 90^{o}, 135^{o}, and 135^{o} respectively.)

10. Draw a circle of diameter 9 cm and inscribe an isosceles triangle ABC having a base BC=7 cm.

11. Draw a circle of radius 3 cm and construct an isosceles triangle with an altitude/height of 7 cm.

12. Draw a circle of radius 4 cm and inscribe an isosceles triangle with an altitude/height of 6 cm.

13. Draw a circle of radius 2.5 cm and construct an isosceles triangle whose vertical angle is 45^{o}.

14. Draw a quadrilateral of length and breadth 7.5 cm and 4 cm respectively. Circumscribe the quadrilateral.

Exercise - **6(d)**

1.(i) Draw a straight line $\overline{AB}$ of 6.5 cm and determine its mid/centre point.

(ii) Draw a straight line $\overline{PQ}$ of 6.5 cm. and divide it into four equal parts.

2. Draw a line segment of 7.2 cm and divide into six equal parts.

3. Draw a straight line $\overline{AB}$ of 6.4 cm and determine the points dividing the line segment in 3:2 ratio internally.

4. Draw a straight line $\overline{BC}$ of 6.5 cm and determine the both the points dividing the line segment in 5:3 ratio internally as well as externally.

5. Draw a straight line $\overline{PQ}$ of 6.5 cm. and divide it into two parts in the ratio of their length 4:3. Draw a quadrilateral having length and breadth equal to the ratio of the two parts of straight line PQ.

6. In a $\triangle$ABC, the length of BC=6.5 cm, the length of the median of $\overline{BY}$ is 6 cm and the length of median of $\overline{CZ}$ is 5.5 cm. Draw the triangle.

Exercise - **6(e)**

1. (i) Draw a $\triangle ABC$, where BC=6 cm, $m\angle AOB = 60^o$ and the length of median $\overline{AD}$ is 4.5 cm.

   (ii) Inscribing a similar triangle like $\triangle ABC$ in a circle of radius 3.5 cm.

2. (i) Draw a $\triangle ABC$, where BC=6 cm, $m\angle B = 60^o$ and the length of altitude $\overline{AD}$ is 4.5 cm.

   (ii) Circumscribe a similar triangle like $\triangle ABC$ on a circle of radius 2.5 cm.

3. Draw $\triangle$ XYZ of any size. Draw a triangle similar to $\triangle XYZ$ where the lengths of the arms of the triangle should be of 2/3.

4. Draw $\triangle$ ABC where BC=5.7 cm., $m\angle B = 60^o$ and the length of median $\overline{BE}$ 4.8 cm. and Circumscribe a triangle similar to drawn triangle in a circle of radius 2.3cm.

5. Draw $\triangle ABC$ where BC=5.3 cm., $m\angle B = 60^o$ and $m\angle C = 45^o$. Inscribe a triangle similar to $\triangle$ ABC in a circle of radius of 2.5 cm.

6. Draw $\triangle$ ABC where BC=7 cm., $m\angle B = 60^o$ and b+c =11.2 cm. Draw the triangle and circumscribe a similar angled triangle in a circle of radius 1.5 cm.

7. Draw $\triangle$ ABC where $m\angle A = 75^o$, AC=9 cm and AB=6 cm. Draw the triangle and inscribe a similar angled triangle in a circle of radius 2 cm.