

EXERCISE - 5.3

1. Simplify and express the result in exponential form:

$$\begin{aligned} \text{i), } \left(\frac{-2}{3}\right)^8 \times \left[\left(\frac{-2}{3}\right)^2\right]^3 &= \left(\frac{-2}{3}\right)^8 \times \left(\frac{-2}{3}\right)^6 \\ &= \left(\frac{-2}{3}\right)^{8+6} = \left(\frac{-2}{3}\right)^{14} \end{aligned}$$

$$\begin{aligned} \text{ii), } (-5)^4 \div (-5)^6 &= (-5)^{4-6} \\ &= (-5)^{-2} = \frac{1}{(-5)^2} \\ &= \left(-\frac{1}{5}\right)^2 \end{aligned}$$

$$\begin{aligned} \text{iii), } \left(\frac{4}{7}\right)^9 \times \left[\left(\frac{4}{7}\right)^2\right]^4 &= \left(\frac{4}{7}\right)^9 \times \left(\frac{4}{7}\right)^{2 \times 4} \\ &= \left(\frac{4}{7}\right)^9 \times \left(\frac{4}{7}\right)^8 = \left(\frac{4}{7}\right)^{9+8} \\ &= \left(\frac{4}{7}\right)^{17} \end{aligned}$$

$$\text{iv), } \left(\frac{11}{7}\right)^6 \times \left(\frac{3}{7}\right)^6 = \left(\frac{11}{7} \times \frac{3}{7}\right)^6 = \left(\frac{33}{49}\right)^6$$

$$\begin{aligned} \text{v), } \left[\left(\frac{17}{4}\right)^2\right]^4 \div \left[\left(\frac{17}{4}\right)^3\right]^4 &= \left(\frac{17}{4}\right)^{2 \times 4} \div \left(\frac{17}{4}\right)^{3 \times 4} \\ &= \left(\frac{17}{4}\right)^8 \div \left(\frac{17}{4}\right)^{12} = \left(\frac{17}{4}\right)^{8-12} = \left(\frac{17}{4}\right)^{-4} \\ &= \left(\frac{4}{17}\right)^4 \end{aligned}$$

$$\begin{aligned} \text{(vi), } 8^7 \times 8^2 \times (8^3)^2 &= 8^7 \times 8^2 \times 8^{3 \times 2} \\ &= 8^7 \times 8^2 \times 8^6 \\ &= 8^{7+2+6} = 8^{15} \end{aligned}$$

$$\text{(vii), } \left[\left(\frac{-2}{5} \right)^3 \right]^4 = \left(\frac{-2}{5} \right)^{3 \times 4} = \left(\frac{-2}{5} \right)^{12}$$

$$\begin{aligned} \text{(viii), } \left(\frac{3}{13} \right)^{16} \times \left(\frac{-9}{4} \right)^{16} &= \left(\frac{3}{13} \times \frac{-9}{4} \right)^{16} \\ &= \left(\frac{-27}{52} \right)^{16} \end{aligned}$$

$$\begin{aligned} \text{(ix), } (7^2)^3 \times \left[\left(\frac{1}{7} \right)^2 \right]^3 &= 7^{2 \times 3} \times \left(\frac{1}{7} \right)^{2 \times 3} \\ &= 7^6 \times \left(\frac{1}{7} \right)^6 = \left(7 \times \frac{1}{7} \right)^6 \\ &= \left(\frac{7}{7} \times \frac{1}{7} \right)^6 = \left(\frac{1}{1} \right)^6 = 1^6 = 1 = 7^0 \end{aligned}$$

$$\begin{aligned} \text{(x), } \left(\frac{-7}{19} \right)^{18} \div \left(\frac{-7}{19} \right)^{28} &= \left(\frac{-7}{19} \right)^{18-28} \\ &= \left(\frac{-7}{19} \right)^{-10} = \left(\frac{-19}{7} \right)^{10} \end{aligned}$$

$$\begin{aligned} \text{(xi), } \left(\frac{5}{3} \right)^4 \times \left(\frac{5}{3} \right)^4 \div \left(\frac{5}{3} \right)^2 &= \left(\frac{5}{3} \right)^{4+4} \div \left(\frac{5}{3} \right)^2 = \left(\frac{5}{3} \right)^{8-2} = \left(\frac{5}{3} \right)^6 \end{aligned}$$

2. Evaluate:

$$\text{i), } (3^0 - 2^0) \times 5^0 = (1 - 1) \times 1$$

$$= 0 \times 1$$

$$\text{ii), } [(-5)^2]^3 \div [(-5)^2]^4 = (-5)^{2 \times 3} \div (-5)^{2 \times 4}$$

$$= (-5)^6 \div (-5)^8$$

$$= (-5)^{6-8} = (-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$$

$$\text{iii), } \left(\frac{3}{7}\right)^3 \times \left(\frac{49}{9}\right)^3 = \left(\frac{1}{3} \times \frac{7}{9}\right)^3$$

$$= \left(\frac{7}{3}\right)^3 = \frac{343}{27}$$

$$\text{iv), } \left(\frac{2}{9}\right)^5 \div \left[\left(\frac{2}{9}\right)^2\right]^3 = \left(\frac{2}{9}\right)^5 \div \left(\frac{2}{9}\right)^{2 \times 3}$$

$$= \left(\frac{2}{9}\right)^5 \div \left(\frac{2}{9}\right)^6 = \left(\frac{2}{9}\right)^{5-6}$$

$$= \left(\frac{2}{9}\right)^{-1} = \frac{9}{2}$$

$$\text{v), } \left(\frac{1}{36}\right)^4 \times (12)^4 = \left(\frac{1}{36} \times 12\right)^4$$

$$= \left(\frac{1}{3}\right)^4 = \frac{1}{3^4} = \frac{1}{81}$$

$$\begin{aligned}
 \text{(vi), } & \left(\frac{221}{82}\right)^{18} \div \left[\left(\frac{221}{82}\right)^3\right]^6 \\
 & = \left(\frac{221}{82}\right)^{18} \div \left(\frac{221}{82}\right)^{3 \times 6} \\
 & = \left(\frac{221}{82}\right)^{18-18} = \left(\frac{221}{82}\right)^0 = 1
 \end{aligned}$$

3. Find the value of:

$$\begin{aligned}
 \text{i), } & \frac{5^8 \times 2^4 \times 3^5}{5^2 \times 2^6 \times 3^7} = \frac{5^{8-2}}{2^{6-4} \times 3^{7-5}} \\
 & = \frac{5^6}{2^2 \times 3^2} \\
 & = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{2 \times 2 \times 3 \times 3} \\
 & = \frac{25 \times 25 \times 25}{4 \times 9} \\
 & = \frac{15,625}{36}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii), } & \frac{\left(\frac{3}{2}\right)^3 \times \left(\frac{2}{5}\right)^4}{\frac{9}{4} \times \left(\frac{2}{5}\right)^3} = \frac{\left(\frac{3}{2}\right)^3 \times \left(\frac{2}{5}\right)^{4-3}}{\left(\frac{3}{2}\right)^2} \\
 & = \left(\frac{3}{2}\right)^{3-2} \times \left(\frac{2}{5}\right)^1 \\
 & = \frac{3}{2} \times \frac{2}{5} = \frac{3}{5}
 \end{aligned}$$

$$\text{iii), } \frac{8^0 \times 7^0 \times \left[\left(\frac{16}{7} \right)^3 \right]^0}{\left(\frac{4}{3} \right)^0 + \left(\frac{3}{4} \right)^0} = \frac{1 \times 1 \times 1}{1 + 1}$$

$$= \frac{1}{2}$$

$$\text{iv), } \left[\left(\frac{3}{4} \right)^9 \times \left(\frac{3}{4} \right)^7 \right] \div \left[\left(\frac{3}{4} \right)^6 \right]^2$$

$$= \left(\frac{3}{4} \right)^{9+7} \div \left(\frac{3}{4} \right)^{6 \times 2}$$

$$= \left(\frac{3}{4} \right)^{16} \div \left(\frac{3}{4} \right)^{12}$$

$$= \left(\frac{3}{4} \right)^{16-12} = \left(\frac{3}{4} \right)^4$$

$$= \frac{3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4}$$

$$= \frac{81}{256}$$

$$\text{v), } \frac{\left(\frac{2}{3} \right)^4 \times \left(\frac{3}{4} \right)^4}{\left(\frac{6}{7} \right)^5 \times \left(\frac{7}{12} \right)^3} = \frac{\left(\frac{2}{3} \times \frac{3}{4} \right)^4}{\left(\frac{6}{7} \times \frac{7}{12} \right)^3}$$

$$= \frac{\left(\frac{1}{2} \right)^4}{\left(\frac{1}{2} \right)^3} = \left(\frac{1}{2} \right)^{4-3} = \frac{1}{2}$$

(vi),

$$\frac{\left(\frac{3}{4}\right)^2 \times \left(\frac{5}{7}\right)^2}{\left(\frac{5}{7}\right)^3 \times \left(\frac{4}{3}\right)^2} = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{7}\right)^{3-2} \times \left(\frac{3}{4}\right)^{-2}}$$

$$= \frac{\left(\frac{3}{4}\right)^{2+2}}{\left(\frac{5}{7}\right)^1} = \frac{\left(\frac{3}{4}\right)^4}{\frac{5}{7}}$$

$$= \left(\frac{3}{4}\right)^4 \times \frac{7}{5}$$

$$= \frac{3 \times 3 \times 3 \times 3 \times 7}{4 \times 4 \times 4 \times 4 \times 5}$$

$$= \frac{9 \times 9 \times 7}{16 \times 16 \times 5}$$

$$= \frac{567}{1280}$$

4. Simplify:

$$i) \left[\left(\frac{-8}{19}\right)^7 \div \left(\frac{-8}{19}\right)^5 \right] \times \left(\frac{-19}{8}\right)^2$$

$$= \left(\frac{-8}{19}\right)^{7-5} \times \left(\frac{-8}{19}\right)^{-2}$$

$$= \left(\frac{-8}{19}\right)^2 \times \left(\frac{-8}{19}\right)^{-2}$$

$$= \left(\frac{-8}{19}\right)^{2-2} = \left(\frac{-8}{19}\right)^0 = 1$$

$$\text{ii, } \left[\left(\frac{-7}{4} \right)^2 \right]^3 \times \left[\left(\frac{16}{35} \right)^3 \right]^2$$

$$= \left(\frac{-7}{4} \right)^{2 \times 3} \times \left(\frac{16}{35} \right)^{3 \times 2}$$

$$= \left(\frac{-7}{4} \right)^6 \times \left(\frac{16}{35} \right)^6$$

$$= \left(\frac{-7}{4} \times \frac{16}{35} \right)^6$$

$$= \left(\frac{-4}{5} \right)^6$$

$$= \frac{(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4)}{5 \times 5 \times 5 \times 5 \times 5 \times 5}$$

$$= \frac{64 \times 64}{125 \times 125}$$

$$= \frac{4096}{15,625}$$

iii,

$$\left(\frac{-5}{2} \right)^3 \times \frac{32}{125} = \frac{(-5)^3}{2^3} \times \frac{32}{125}$$

$$= \frac{-125}{8} \times \frac{32}{125}$$

$$= \frac{-4}{1} = -4$$

$$\begin{aligned}
 \text{iv), } \frac{\left(\frac{4}{3}\right)^3 \times \left(\frac{-27}{125}\right)}{\left(\frac{-3}{5}\right)^2 \times \left(\frac{16}{9}\right)} &= \frac{\left(\frac{4}{3}\right)^3 \times \left(\frac{-3}{5}\right)^3}{\left(\frac{-3}{5}\right)^2 \times \left(\frac{4}{3}\right)^2} \\
 &= \frac{\left(\frac{4}{3} \times \frac{-3}{5}\right)^3}{\left(\frac{-3}{5} \times \frac{4}{3}\right)^2} \\
 &= \frac{\left(\frac{-4}{5}\right)^3}{\left(\frac{-4}{5}\right)^2} = \frac{-64}{125} \div \frac{+16}{25} \\
 &= \frac{-64}{125} \times \frac{25}{16} \\
 &= \frac{-4}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{v), } \left[\frac{3^3}{16} \times \left(\frac{4}{3}\right)^3 \right] \div \left(\frac{6}{5}\right)^2 \\
 \left[\frac{3^3 \times 4^3}{4^2 \times 3^3} \right] \div \frac{6^2}{5^2} \\
 = 4^{3-2} \div \frac{36}{25} \\
 = \frac{4}{1} \times \frac{25}{36} = \frac{25}{9}
 \end{aligned}$$

$$(vi) \left[\left(\frac{-1}{4} \right)^2 \right]^3 \div \left[\left(\frac{-1}{4} \right)^3 \right]^2$$

$$= \left(\frac{-1}{4} \right)^6 \div \left(\frac{-1}{4} \right)^6$$

$$= \left(\frac{-1}{4} \right)^{6-6} = \left(\frac{-1}{4} \right)^0 = 1$$

5. If $\frac{p}{q} = \left[\left(\frac{2}{3} \right)^2 \right]^5 \div \left[\left(\frac{2}{3} \right)^3 \right]^3$ find $\left(\frac{q}{p} \right)$

$$\frac{p}{q} = \left(\frac{2}{3} \right)^{2 \times 5} \div \left(\frac{2}{3} \right)^{3 \times 3}$$

$$= \left(\frac{2}{3} \right)^{10} \div \left(\frac{2}{3} \right)^9$$

$$= \left(\frac{2}{3} \right)^{10-9} = \frac{2}{3}$$

Then, $\frac{q}{p} = \frac{3}{2}$

$$\therefore \left(\frac{q}{p} \right)^3 = \left(\frac{3}{2} \right)^3 = \frac{27}{8}$$

6. $\frac{p}{q} = \left(\frac{-3}{4} \right)^{16} \div \left[\left(\frac{-3}{4} \right)^4 \right]^4$

$$= \left(\frac{-3}{4} \right)^{16} \div \left(\frac{-3}{4} \right)^{4 \times 4}$$

$$= \left(\frac{-3}{4} \right)^{16} \div \left(\frac{-3}{4} \right)^{16} = \left(\frac{-3}{4} \right)^{16-16} = \left(\frac{-3}{4} \right)^0 = 1$$

$$\begin{aligned} \therefore \left(\frac{p}{q}\right)^2 + \left(\frac{p}{q}\right)^3 &= (1)^2 + (1)^3 \\ &= 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned} (7) \quad \frac{p}{q} &= \left[\left(\frac{3}{5}\right)^3\right]^5 \times \left[\left(\frac{10}{9}\right)^5\right]^3 \\ &= \left(\frac{3}{5}\right)^{3 \times 5} \times \left(\frac{10}{9}\right)^{5 \times 3} \\ &= \left(\frac{3}{5}\right)^{15} \times \left(\frac{10}{9}\right)^{15} \\ &= \left(\frac{3}{5} \times \frac{10}{9}\right)^{15} \\ &= \left(\frac{2}{3}\right)^{15} = \frac{2^{15}}{3^{15}} \end{aligned}$$

$$\therefore \frac{q}{p} = \frac{3^{15}}{2^{15}}$$

$$\begin{aligned} \therefore \frac{p}{q} + \frac{q}{p} &= \frac{2^{15}}{3^{15}} + \frac{3^{15}}{2^{15}} \\ &= \frac{(2^{15} \times 2^{15}) + (3^{15} \times 3^{15})}{3^{15} \times 2^{15}} \\ &= \frac{2^{15+15} + 3^{15+15}}{(2 \times 3)^{15}} \end{aligned}$$

$$\therefore \left(\frac{p}{q}\right)^2 + \left(\frac{p}{q}\right)^3 = (1)^2 + (1)^3$$

$$= 1 + 1 = 2$$

$$(7) \quad \frac{p}{q} = \left[\left(\frac{3}{5}\right)^3\right]^5 \times \left[\left(\frac{10}{9}\right)^5\right]^3$$

$$\Rightarrow \left(\frac{3}{5}\right)^{3 \times 5} \times \left(\frac{10}{9}\right)^{5 \times 3}$$

$$= \left(\frac{3}{5}\right)^{15} \times \left(\frac{10}{9}\right)^{15}$$

$$= \left(\frac{3}{5} \times \frac{10}{9}\right)^{15}$$

$$= \left(\frac{2}{3}\right)^{15} = \frac{2^{15}}{3^{15}}$$

$$\therefore \frac{q}{p} = \frac{3^{15}}{2^{15}}$$

$$\therefore \frac{p}{q} + \frac{q}{p} = \frac{2^{15}}{3^{15}} + \frac{3^{15}}{2^{15}}$$

$$= \frac{(2^{15} \times 2^{15}) + (3^{15} \times 3^{15})}{3^{15} \times 2^{15}}$$

$$= \frac{2^{15+15} + 3^{15+15}}{(3 \times 2)^{15}}$$

$$= \frac{2^{30} + 3^{30}}{6^{15}} = \frac{4^{15} + 9^{15}}{6^{15}}$$

8. Simplify and express the following as ¹⁵ a power of 4:

$$\begin{aligned} \text{i), } 18^2 \times \left(\frac{1}{18}\right)^2 &= \left(\frac{18^1}{1} \times \frac{1}{18}\right)^2 \\ &= (1)^2 = 1 = 4^0 \end{aligned}$$

$$\begin{aligned} \text{ii), } [(-19)^2 \times (-19)^4] \div (-19)^6 \\ &= (-19)^{2+4} \div (-19)^6 \\ &= (-19)^{6-6} = (-19)^0 = 1 = 4^0 \end{aligned}$$

$$\begin{aligned} \text{iii), } \left(\frac{3}{4}\right)^2 \times \left(\frac{16}{3}\right)^2 \times 4^3 &= \left(\frac{3^1}{4} \times \frac{16}{3}\right)^2 \times 4^3 \\ &= 4^2 \times 4^3 = 4^{2+3} = 4^5 \end{aligned}$$

$$\begin{aligned} \text{iv), } \left(\frac{5}{10}\right)^8 \div \left[\left(\frac{5}{10}\right)^2\right]^4 &= \left(\frac{5}{10}\right)^8 \div \left(\frac{5}{10}\right)^{2 \times 4} \\ &= \left(\frac{5}{10}\right)^8 \div \left(\frac{5}{10}\right)^8 \\ &= \left(\frac{5}{10}\right)^{8-8} = \left(\frac{5}{10}\right)^0 = 1 = 4^0 \end{aligned}$$

$$\begin{aligned} \text{v), } \left(\frac{-16}{15}\right)^3 \times \left(\frac{15}{8}\right)^3 \times \left(\frac{-1}{2}\right) \\ &= \left(\frac{-16}{15} \times \frac{15}{8}\right)^3 \times \left(\frac{-1}{2}\right) \end{aligned}$$

$$= \left(\frac{-2}{1}\right)^3 \times \left(\frac{-1}{2}\right)$$

$$= \frac{-8}{1} \times \left(\frac{-1}{2}\right)$$

$$= \frac{(-8) \times (-1)}{1 \times 2}$$

$$= \frac{8}{2} = 4 = 4^1$$

$$\text{vi), } (10^0 + 7^0) \div (2^0 + 3^0)$$

$$= (1+1) \div (1+1)$$

$$= 2 \div 2$$

$$= \frac{2}{2} = 1 = 4^0$$

EXERCISE - 5.4

1. Express the following in Scientific notation:

$$\begin{aligned} \text{i), } 892,000,000,000 &= 8.92 \times 100000000000 \\ &= 8.92 \times 10^{11} \end{aligned}$$

$$\begin{aligned} \text{ii), } 26,480,000,000,000 &= 2.648 \times 10000000000000 \\ &= 2.648 \times 10^{13} \end{aligned}$$

$$\begin{aligned} \text{iii), } 84,973 &= 8.4973 \times 10000 \\ &= 8.4973 \times 10^4 \end{aligned}$$

$$\begin{aligned} \text{iv), } 80,04,000,000,000 &= 8.004 \times 100000000000 \\ &= 8.004 \times 10^{12} \end{aligned}$$

$$\begin{aligned} \text{v), } 764,000,000,000 &= 7.64 \times 100000000000 \\ &= 7.64 \times 10^{11} \end{aligned}$$

$$\begin{aligned} \text{vi), } 3,18,65,00,000 &= 3.1865 \times 1000000000 \\ &= 3.1865 \times 10^9 \end{aligned}$$

2. Express in usual form:

$$\begin{aligned} \text{i), } 2.38 \times 10^8 &= 2.38 \times 100000000 \\ &= 238,000,000 \end{aligned}$$

$$\begin{aligned} \text{ii), } 3.964 \times 10^7 &= 3.964 \times 10000000 \\ &= 39,640,000 \end{aligned}$$

$$\begin{aligned} \text{iii), } 9.6524 \times 10^5 &= 9.6524 \times 100000 \\ &= 965,240 \end{aligned}$$

$$\begin{aligned} \text{iv), } 4.111 \times 10^6 &= 4.111 \times 1000000 \\ &= 4,111,000 \end{aligned}$$

$$\begin{aligned} \text{v), } 2.394 \times 10^4 &= 2.394 \times 10000 \\ &= 23,940 \end{aligned}$$

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3. Express the numbers appearing in the following statements in Scientific notation:

$$\begin{aligned} \text{i), } 384,000,000 \text{ m} &= 3.84 \times 100000000 \text{ m} \\ &= 3.84 \times 10^8 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{ii), } 300,000,000 \text{ m/s} &= 3 \times 100000000 \text{ m/s} \\ &= 3 \times 10^8 \end{aligned}$$

$$\begin{aligned} \text{iii), } 1,27,56,000 \text{ m} &= 1.2756 \times 10000000 \text{ m} \\ &= 1.2756 \times 10^7 \text{ m} \end{aligned}$$

$$\text{iv, } 1,400,000,000 \text{ m} = 1.4 \times 1000000000 \text{ m} \\ = 1.4 \times 10^9 \text{ m}$$

$$\text{v, } 12,000,000,000 = 1.2 \times 10000000000 \\ = 1.2 \times 10^{10}$$

$$\text{vi, } 1,027,000,000 = 1.027 \times 1000000000 \\ = 1.027 \times 10^9$$

Miscellaneous Exercise

1. Fill in the blanks with $>$, $<$ or $=$ sign:

$$\text{i, } 2^3 \times 2^2 = 2^{3+2} = 2^5$$

$$3^2 \times 3^3 = 3^{2+3} = 3^5$$

$$2 < 3 \Rightarrow 2^5 < 3^5$$

$$\Rightarrow 2^3 \times 2^2 < 3^2 \times 3^3$$

$$\text{ii, } \left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^3 = \left(\frac{4}{7}\right)^{2+3} = \left(\frac{4}{7}\right)^5 > 0$$

$$\left(\frac{-4}{7}\right)^2 \times \left(\frac{-4}{7}\right)^3 = \left(\frac{-4}{7}\right)^{2+3} = \left(\frac{-4}{7}\right)^5 < 0$$

$$\therefore \left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^3 > \left(\frac{-4}{7}\right)^2 \times \left(\frac{-4}{7}\right)^3$$

$$\text{iii, } \left(\frac{-3}{4}\right)^2 = \frac{(-3)^2}{4^2} = \frac{(-3)^2}{16}$$

$$\frac{(-3)^2}{16} < \frac{(-3)^2}{4}$$

$$\Rightarrow \left(\frac{-3}{4}\right)^2 < \frac{(-3)^2}{4}$$