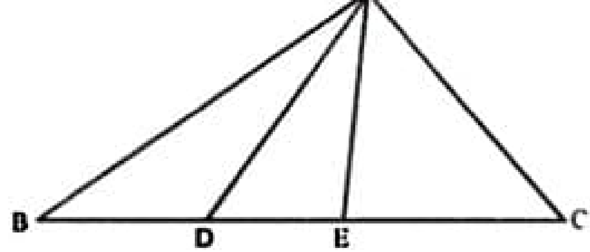


$$\frac{\text{Area of } \Delta ABE}{\text{Area of } \Delta ADC} = \frac{BE}{DC}, \quad \frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ABC} = \frac{BD}{BC}$$

and so on.

EXERCISE 15 (B)

1. In the following figure, point D divides AB in the ratio 3 : 5. Find :

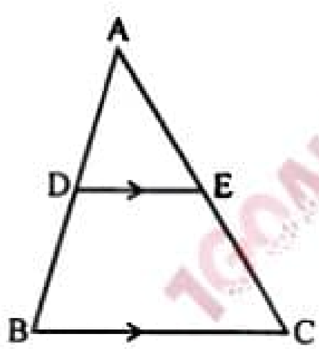
(*i*) $\frac{AE}{EC}$ (*ii*) $\frac{AD}{AB}$

(*iii*) $\frac{AE}{AC}$

Also, if :

(*iv*) DE = 2.4 cm, find the length of BC.

(*v*) BC = 4.8 cm, find the length of DE.



Solution—

In ΔABC, D divides AB in the ratio 3 : 5

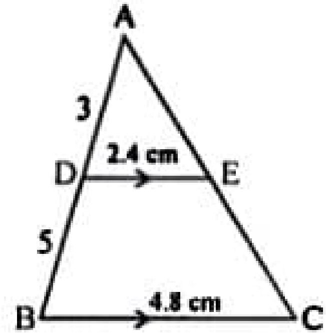
To find :

(*i*) $\frac{AE}{EC}$ (*ii*) $\frac{AD}{AB}$

(*iii*) $\frac{AE}{AC}$

Also, if :

(*iv*) DE = 2.4 cm, find the length of BC.



In ΔADE and ΔABC, we have

$\angle A = \angle A$ (common)

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

$\therefore \Delta ADE \sim \Delta ABC$ (By AAA axiom)

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad ...(i)$$

(corresponding sides are proportional)

Since, DE ∥ BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} = \frac{3}{5} \quad ...(ii)$$

(*i*) From (*ii*), we get $\frac{AE}{EC} = \frac{3}{5}$

(*ii*) $\frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{3}{3+5} = \frac{3}{8}$

(*iii*) $\frac{AE}{AC} = \frac{AE}{AE + EC} = \frac{3}{3+5} = \frac{3}{8}$

(*iv*) From (*i*), $\frac{DE}{BC} = \frac{AE}{AC}$

$\Rightarrow BC = DE \times \frac{AC}{AE} = 2.4 \times \frac{8}{3}$

$\Rightarrow BC = 6.4$ cm

(*v*) From (*i*), $\frac{DE}{BC} = \frac{AE}{AC}$

$\Rightarrow DE = BC \times \frac{AE}{AC} = 4.8 \times \frac{3}{8} = 1.8$ cm

Proof—

Statement

1. Area of Δ ABC $= \frac{1}{2}$ BC $\times$ AM

Area of $\Delta = \frac{1}{2}$ base $\times$ altitude

Area of Δ DEF $= \frac{1}{2}$ EF $\times$ DN

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{\frac{1}{2} BC \times AM}{\frac{1}{2} EF \times DN}$$

$$= \frac{BC}{EF} \times \frac{AM}{DN}$$

....(i)

2. In Δ ABM and Δ DEN,

(i) $\angle B = \angle E$ [Given]

(ii) $\angle AMB = \angle DNE$ [Each angle being 90°]

$\therefore$ $\Delta ABM \sim \Delta DEN$ [By AA postulate]

$$\Rightarrow \frac{AM}{DN} = \frac{AB}{DE} \quad ....(ii)$$

[Corresponding sides of similar triangles are in proportion]

3. Since, $\Delta ABC \sim \Delta DEF$ [Given]

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (Corresponding sides of similar triangles are in proportion) ....(iii)

$\therefore$ $\frac{AM}{DN} = \frac{BC}{EF}$ (From (ii) and (iii))

Substituting $\frac{AM}{DN} = \frac{BC}{EF}$ in (i), we get

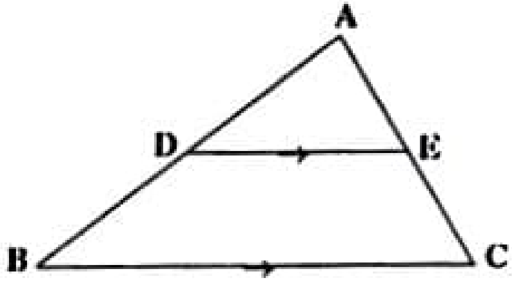
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad ....(iv)$$

Now from (iii) and (iv), we get:

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Q.E.D. **Some Results**

1. A line drawn parallel to any side of a triangle, divides the other two sides proportionally. (Basic Proportionality Theorem)



In the given figure, DE // BC then $\frac{AD}{BD} = \frac{AE}{CE}$

Conversely— If a line divides two sides of a triangle proportionally, the line is parallel to the third side.

i.e. if $\frac{AD}{BD} = \frac{AE}{CE}$ then DE // BC.

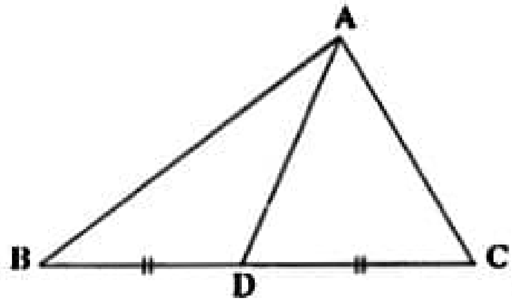
2. In the same figure, given above

$\Delta ADE \sim \Delta ABC$ [By AAA postulate]

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

3. Median divides the triangle into two tringles of equal area.

In the given figure, AD is median



$\Rightarrow$ Area of Δ ABD = Area of Δ ACD

$= \frac{1}{2} \times$ Area of Δ ABC.

4. If many triangles have the common vertex and their bases are along the same straight line, the ratio between their areas is equal to the ratio between the lengths of their bases.

In the given figure, all the triangles have the common vertex at point, A and bases of all the triangles are along the same straight line BC.

$\therefore \quad \frac{PQ}{QR} = \frac{QM}{PQ} = \frac{PM}{PR}$ ....(i)

$\Rightarrow PQ^2 = QR \times QM = 8 \times 3.5 = 28$

$\therefore \quad PQ = \sqrt{28}$ ....(ii)

In Δ PQR, $\angle P = 90^\circ$ and PM $\perp$ QR

$\therefore \quad PM^2 = QM \times MR$

$= 3.5 \times 4.5 \quad (\because MR = QR - QM)$

$\therefore \quad PM = \sqrt{3.5 \times 4.5}$ ....(iii)

From (i) $\frac{PQ}{QR} = \frac{PM}{PR} = \frac{\sqrt{28}}{8} = \frac{\sqrt{3.5 \times 4.5}}{PR^2}$

Squaring both sides,

$$\frac{28}{64} = \frac{3.5 \times 4.5}{PR}$$

$$PR^2 = \frac{3.5 \times 4.5 \times 64}{28} = \frac{35 \times 45 \times 64}{10 \times 10 \times 28} = \frac{10080}{2800}$$

$\Rightarrow PR^2 = 36 = (6)^2$

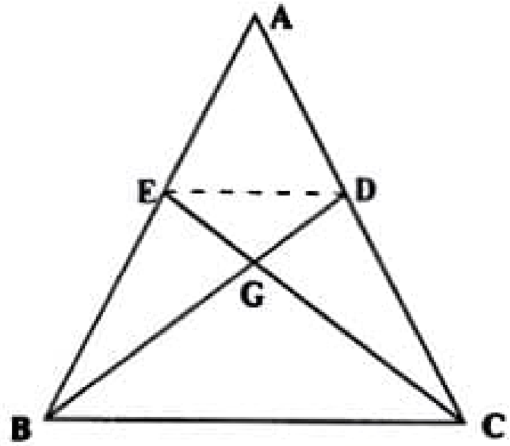
$\therefore$ PR = 6 cm. **Ans.**

28. In the figure given below, the medians BD and CE of a triangle ABC meet at G.

Prove that—

(i) Δ EGD ~ Δ CGB

(ii) BG = 2 GD from (i) above.



Solution—

Given—In Δ ABC, BD and CE are the medians of sides AC and AB respectively which intersect each at G.

To Prove— (i) Δ EGD ~ Δ CGB (ii) BG = 2 GD.

Proof— $\because$ D and E are the mid points of AC and AB respectively.

$\therefore$ ED $\parallel$ BC and ED = $\frac{1}{2}$ BC

Or $\frac{ED}{BC} = \frac{1}{2}$ ....(i)

Now in Δ EGD and Δ CGB,

$\angle$ EGD = $\angle$ BGC (vertically opposite angle)

$\angle$ EDG = $\angle$ GBC (Alternate angles)

$\therefore$ Δ EGD ~ Δ CGB (AA postualte)

$\therefore \quad \frac{GD}{BG} = \frac{ED}{BC} = \frac{1}{2}$ [From (i)

$\therefore$ BG = 2 GD Q.E.D

THEOREM 1

[The areas of two similar triangles are proportional to the squares on their corresponding sides.]

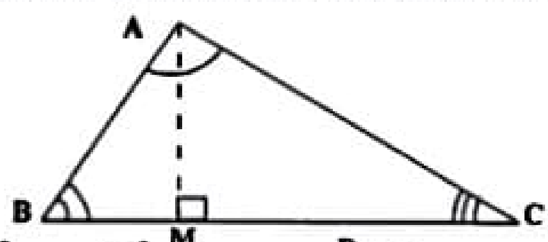
Given— Δ ABC ~ Δ DEF

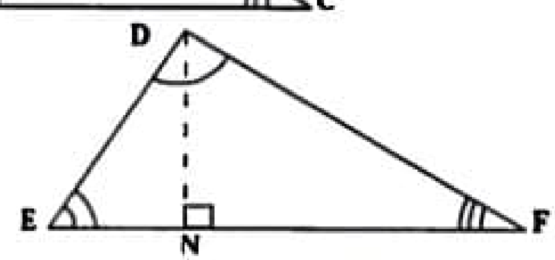
such that $\angle$ BAC = $\angle$ EDF

$\angle$ B = $\angle$ E and $\angle$ C = $\angle$ F.

To Prove—

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$





Construction—

Draw AM $\perp$ BC and DN $\perp$ EF.

∠ ELA = ∠ BLC (vertically opposite angles)

∠ DEM or ∠ AEL = ∠ LBC (proved)

∴ Δ ELA ~ Δ BLC (AA postulate)

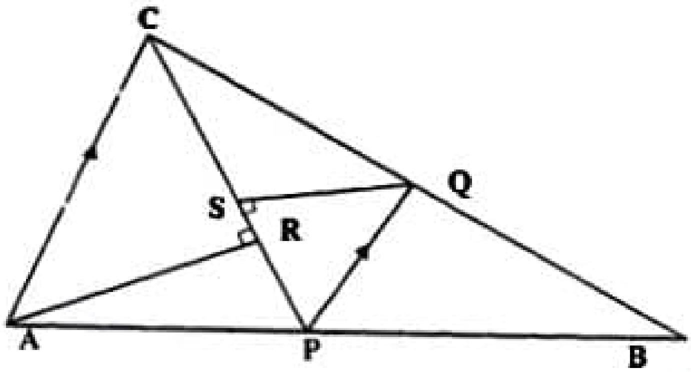
$$\therefore \frac{EA}{BC} = \frac{EL}{LB} \Rightarrow \frac{2\,BC}{BC} = \frac{EL}{LB}$$

$$\Rightarrow \frac{EL}{LB} = 2 \Rightarrow EL = 2\,LB.$$

∴ EL = 2 BL.

Q.E.D.

26. In the figure given below P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.



(i) Calculate the ratio PQ : AC, giving reason for your answer.

(ii) In triangle ARC, ∠ ARC = 90° and in triangle PQS, ∠ PSQ = 90°. Given QS = 6 cm, calculate the length of AR. [1999]

Solution—

Given— In Δ ABC, P is a point on AB such that AP : PB = 4:3 and PQ ∥ AC is drawn meeting BC in Q. CP is joined and QS ⊥ CP and AR ⊥ CP

To Find—

(i) Calculate the ratio between PQ : AC giving reason.

(ii) In Δ ARC ∠ARC= 90° and in Δ PQS, ∠ PSQ = 90°, if QS = 6 cm, calculate AR.

Solution—

(i) In Δ ABC, PQ ∥ AC.

$$\therefore \frac{BQ}{BC} = \frac{BP}{AB} = \frac{PQ}{AC}$$

$$= \frac{PQ}{AC} = \frac{BP}{AB} = \frac{BP}{BP + AP} = \frac{3}{3+4} = \frac{3}{7}$$

∴ PQ : AC = 3 : 7

(ii) Now in Δ ARC and Δ PSQ.

∠ ARC = ∠ PSQ (each 90°)

∠ ACR = ∠ QPS (Alternate angles)

∴ Δ ARC ~ Δ PSQ (AA Postualte)

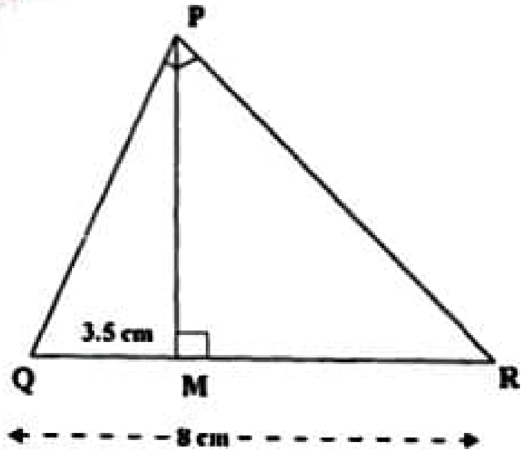
$$\therefore \frac{AC}{PQ} = \frac{AR}{QS}$$

But $\frac{AC}{PQ} = \frac{7}{3}$ and QS = 6 cm

$$\therefore \frac{7}{3} = \frac{AR}{6}$$

$$\Rightarrow AR = \frac{7}{3} \times 6 = 14 \text{ cm Ans.}$$

27. In the right angled triangle QPR, PM is an altitude.



Given that QR = 8 cm and MQ = 3.5 cm. Calculate the value of PR. [2000]

Given— In right angled Δ QPR, ∠ P = 90° PM ⊥ QR, QR = 8 cm, MQ = 3.5 cm

Calculate— PR

Solution—

In Δ PQM and Δ QPR,

∠ PMQ = ∠ QPR (each = 90°)

∠ Q = ∠ Q (common)

∴ Δ PQM ~ Δ QPR (AA postulate)

∴ There are three pairs of similar triangles.

(i) Δ AEB ~ Δ DEC (ii) Δ ABC ~ Δ EFC

(iii) Δ BCD ~ Δ EBF

(ii) ∵ Δ AEB ~ Δ DEC

$$\therefore \quad \frac{AE}{EC} = \frac{BE}{ED} = \frac{AB}{DC}$$

But AB = 67.5 cm, DC = 40.5 cm and AE = 52.5 cm.

$$\therefore \quad \frac{52.5}{EC} = \frac{67.5}{40.5} = EC \times 67.5 = 52.5 \times 40.5$$

$$EC = \frac{52.5 \times 40.5}{67.5} = 31.5 \text{ cm Ans.}$$

In Δ ABC, EF ∥ AB

$$\therefore \quad \frac{AC}{EC} = \frac{AB}{EF}$$

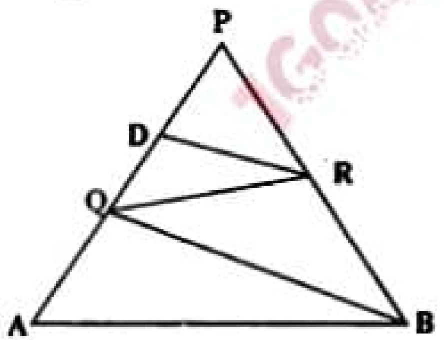
$$\frac{AE + EC}{EC} = \frac{AB}{EF} \qquad = \frac{52.5 + 31.5}{31.5} = \frac{67.5}{EF}$$

$$\Rightarrow \frac{84}{31.5} = \frac{67.5}{EF} \Rightarrow 84 \times EF = 67.5 \times 31.5$$

$$\therefore \quad EF = \frac{67.5 \times 31.5}{84} = \frac{675 \times 315}{10 \times 84 \times 10}$$

$$= \frac{405}{16} = 25\frac{5}{16} \text{ cm Ans.}$$

24. In the given figure, QR is parallel to AB and DR is parallel to QB.



Prove that— PQ² = PD × PA.

Solution—

Given— In the figure QR ∥ AB and DR ∥ QB.

To Prove— $PQ^2 = PD \times PA$

Proof— In Δ PQB,

DR ∥ QB (given)

$$\therefore \quad \frac{PD}{PQ} = \frac{PR}{PB} \qquad \text{....(i)}$$

In Δ PAB,

QR ∥ AB (given)

$$\therefore \quad \frac{PQ}{PA} = \frac{PR}{PB} \qquad \text{....(ii)}$$

from (i) and (ii)

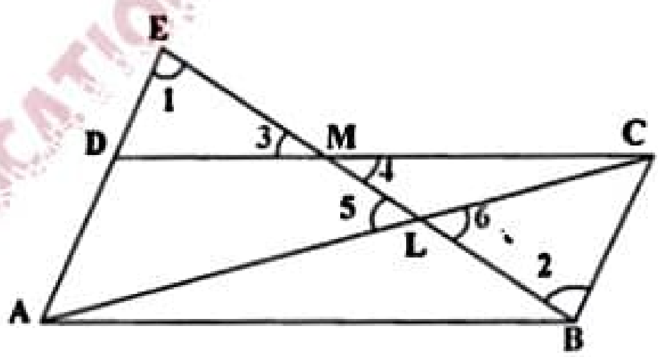
$$\frac{PD}{PQ} = \frac{PQ}{PA} \Rightarrow PQ^2 = PD \times PA$$

Q.E.D.

25. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E.

Prove that : EL = 2 BL.

Solution—



Given— In llgm. ABCD, M is the mid-point of CD. AC is the diagonal. BM is joined and produced meeting AD produced in E and intersecting AC in L.

To Prove— EL = 2 BL.

Proof— In Δ EDM, and Δ MBC,

DM = MC (M is mid-point of DC)

∠ EMD = ∠ CMD (vertically opposite angles)

∠ EDM = ∠ MCB (Alternate angles)

∴ ΔEDM ≅ ΔMBC (ASA postulate of congruency)

∴ ED = CB = AD (c. p. c. t.)

EA = 2 AD = 2 BC

AB = BC (opposite sides of ∥ gm)

∴ ∠ DEM = ∠ MBC (c. p. c. t.)

Now in Δ ELA and Δ BLC,

Prove that—

(i) $\frac{AD}{DG} = \frac{CF}{FG}$ (ii) Δ DFG ~ Δ ACG.

Solution—

Proof— In Δ GAB and Δ GDE,

$\angle$ GAB = $\angle$ GDE (corresponding angles)

($\because$ DE ∥ AB)

$\angle$ AGB = $\angle$ DGE (common)

$\therefore$ Δ GAB ~ Δ GDE (AA postulate)

$\therefore \frac{GA}{GD} = \frac{GB}{GE}$ ....(i)

Similarly in Δ GBC and Δ GEF,

$\angle$ GBC = $\angle$ GEF (corresponding angles)

($\because$ EF ∥ BC)

$\angle$ BGC = $\angle$ EGF (common)

$\therefore$ Δ GBC ~ Δ GEF (AA postulate)

$\therefore \frac{GB}{GE} = \frac{GC}{GF}$ .....(ii)

From (i) and (ii)

$\frac{GA}{GD} = \frac{GC}{GF}$ .....(iii)

Subtracting 1 from each side,

$\frac{GA}{GD} - 1 = \frac{GC}{GF} - 1$

$\Rightarrow \frac{GA - GD}{GD} = \frac{GC - GF}{GF}$

$\Rightarrow \frac{AD}{DG} = \frac{CF}{FG}$

(iii) Now in Δ DFG and Δ ACG,

$\because \frac{GD}{GA} = \frac{GF}{GC}$ [Applying invertendo of (iii)]

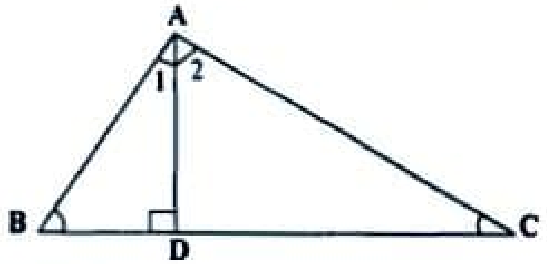
and $\angle$ DGF = $\angle$ AGC (common)

$\therefore$ Δ DFG ~ Δ ACG (SAS psotulate)

Q.E.D.

22. In Δ ABC, AD is perpendicular to side BC and AD^2 = BD × DC.

Show that angle BAC = 90°



Solution—

Given— In Δ ABC, AD ⊥ BC and AD^2 = BD × DC

To Prove— $\angle$ BAC = 90°

Proof—

$\therefore AD^2 = BD \times DC \Rightarrow \frac{AD}{DC} = \frac{BD}{AD}$

Now in Δ ABD and Δ ACD,

$\frac{AD}{DC} = \frac{BD}{AD}$ (Given)

$\angle$ ADB = $\angle$ ADC (each = 90°)

$\therefore$ Δ ADB ~ Δ ACD (SAS postulate)

$\therefore$ $\angle$ B = $\angle$ DAC ....(i)

and $\angle$ BAD = $\angle$ C or $\angle$ C = $\angle$ BAD ....(ii)

Adding (i) and (ii)

$\angle$ B + $\angle$ C = $\angle$ DAC + $\angle$ BAD = $\angle$ BAC = $\angle$ A

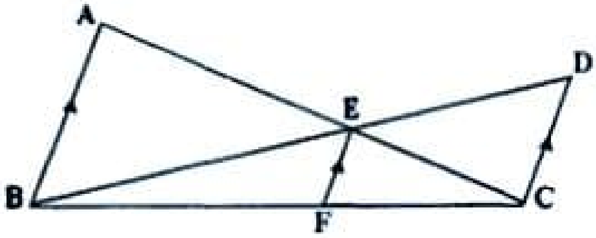
But $\angle$ A + $\angle$ B + $\angle$ C = 180° (Angles of a Δ)

$\therefore$ $\angle$ A + $\angle$ A = 180° = 2 $\angle$ A = 180°

$\therefore \angle A = \frac{180^0}{2} = 90^0$ or $\angle$ BAC = 90°

Q.E.D.

23. In the given figure AB // EF // DC; AB = 67.5 cm. DC = 40.5 cm and AE = 52.5 cm.



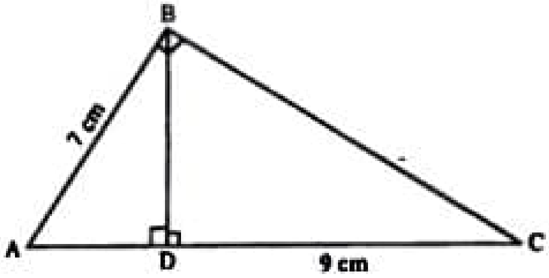
(i) Name the three pairs of similar triangles.

(ii) Find the lengths of EC and EF.

Solution—

(i) In the figure

$\because$ AB ∥ EF ∥ DC



$\angle ADB = \angle ABC$ (Each = 90^0)

$\angle A = \angle A$ (Common)

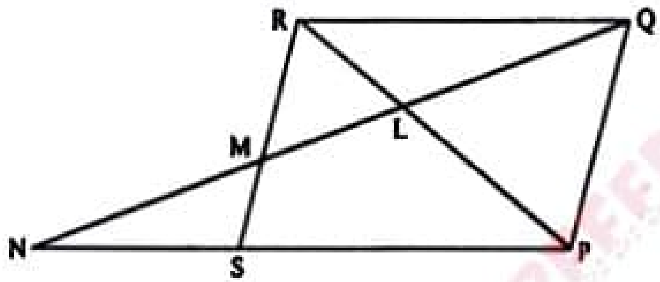
$\therefore$ $\Delta ABD \sim \Delta ABC$ (AA Postulate)

$$\therefore \frac{AB}{AC} = \frac{AD}{AB} \Rightarrow \frac{AB^2}{AC} = AD$$

$$\Rightarrow AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5\frac{4}{9} \text{ Ans.}$$

($\because$ AB = 7cm, AC = 9cm)

20. In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM. **[1997]**

Solution— In Δ LNP and Δ RLQ

$\angle LNP = \angle LQR$ (Alternate angles)

$\angle NLP = \angle QLR$ (Vertically opposite angles)

$\therefore$ $\Delta LNP \sim \Delta RLQ$ (AA Postulate)

$$\therefore \frac{PN}{QR} = \frac{LP}{RL} \Rightarrow \frac{PN}{10} = \frac{3}{2}$$

($\therefore$ LP : RL = 3 : 2)

$$\therefore PN = \frac{10 \times 3}{2} = 15 \text{ cm.}$$

Similarly, we can prove that

Δ LMR and Δ LPQ are similar.

$$\therefore \frac{RM}{QP} = \frac{RL}{LP}$$

$$\therefore \frac{RM}{16} = \frac{2}{3} \Rightarrow RM = \frac{16 \times 2}{3} = \frac{32}{3} \text{ cm,}$$

$$\therefore RM = 10\frac{2}{3} \text{ cm, Ans.}$$

21. In quadrilateral ABCD, diagonals AC and BD intersect at point E. Such that

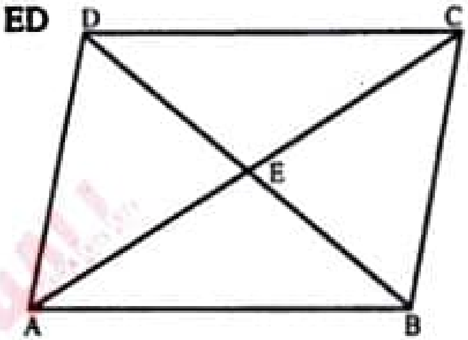
AE : EC = BE : ED.

Show that ABCD is a parallelogram.

Solution—

Given : In quadrilateral ABCD, diagonals AC and BD intersect each other at E and

AE : EC = BE : ED



To Prove : ABCD is a parallelogram.

Proof— In AE : EC = BE : ED

$$\frac{AE}{EC} = \frac{BE}{ED} \Rightarrow \frac{AE}{BE} = \frac{EC}{ED}$$

In Δ AEB and Δ CED

$$\frac{AE}{BE} = \frac{EC}{ED} \quad \text{(given)}$$

$\angle AEB = \angle CED$ (Vertically opposite angles)

$\therefore$ $\Delta AEB \sim \Delta CED$ (SAS axiom)

$\therefore$ $\angle EAB = \angle ECB$

$\angle EBA = \angle CDE$

But, these are pairs of alternate angles

$\therefore$ AB ‖ CD ...(i)

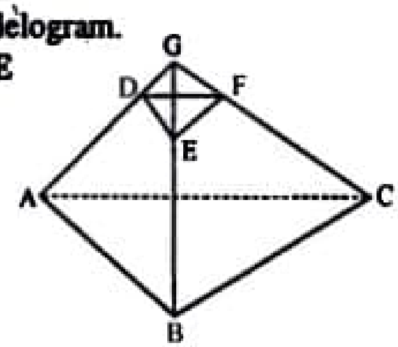
Similarly we can prove that

AD ‖ BC ...(ii)

$\therefore$ from (i) and (ii)

ABCD is a parallelogram.

P.Q. Given : AB // DE and BC // EF.



$\therefore \quad \frac{PQ}{PR} = \frac{PM}{PQ}$

$\Rightarrow PQ^2 = PM \times PR$ ....(i)

Again in Δ QRM and Δ PQR,

$\angle QMR = \angle Q$ (each = 90^0)

$\angle R = \angle R$ (common)

$\therefore$ Δ QRM ~ Δ PQR (AA postulate)

$\therefore \quad \frac{QR}{PR} = \frac{MR}{QR} \Rightarrow QR^2 = PR \times MR$ ....(ii)

Adding (*i*) and (*ii*)

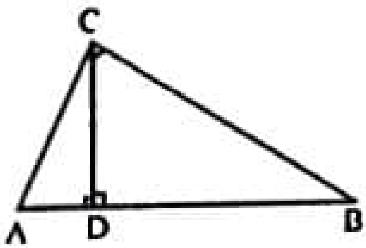
$PQ^2 + QR^2 = PM \times PR + PR \times MR$

$= PR\,(PM + MR) = PR\,(PR) = PR^2$

P.Q. In Δ ABC, right-angled at C, CD ⊥ AB.

Prove : $CD^2 = AD \times DB$

Solution—



Given— In Δ ABC, ∠ C = 90^0 and CD ⊥ AB

To Prove— $CD^2 = AD \times DB$

Proof— ∠ ACD + ∠ A = 90^0 (∵ CD ⊥ AB)

But ∠ A + ∠ B = 90^0 (∵ ∠ C = 90°)

∴ ∠ ACD + ∠ A = ∠ A + ∠ B

∠ ACD = ∠ B

Similarly, we can prove that

∠ BCD = ∠ A

Now in Δ ACD and Δ BCD

∠ A = ∠ BCD (Proved)

∠ ACD = ∠ B (Proved)

∴ Δ ACD ~ Δ BCD (AA Postulate)

$\therefore \quad \frac{CD}{DB} = \frac{AD}{CD}$

$CD^2 = AD \times DB.$

19. In Δ ABC, ∠ B = 90^0 and BD ⊥ AC.

(*i*) If CD = 10 cm and BD = 8 cm; find AD.

∴ ∠A + ∠C = 90° ....(*i*)

and in Δ BDC

∠D = 90°

∴ ∠CBD + ∠C = 90° ....(*ii*)

From (*i*) and (*ii*)

∠A + ∠C = ∠CBD + ∠C

∠A = ∠CBD

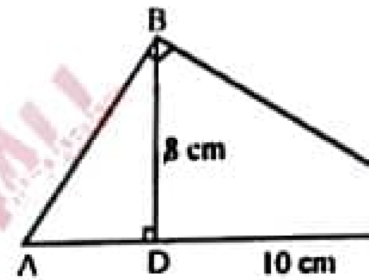
Similarly

∠C = ∠ABD

Now in Δ ABD and Δ CBD,

(*i*) ∠ A = ∠ CBD and ∠ ABD = ∠ C

∴ Δ ABD ~ Δ CBD (AA Postulate)



$\therefore \quad \frac{BD}{CD} = \frac{AD}{BD} = \frac{AB}{AC}$

$\Rightarrow BD^2 = AD \times CD$

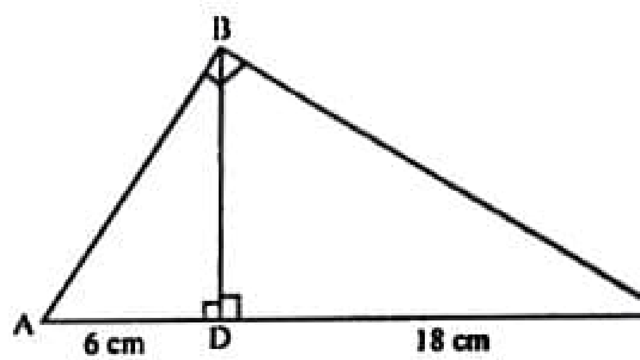
But CD = 10 cm, BD = 8 cm.

$\therefore \quad (8)^2 = AD \times 10 \Rightarrow 10\,AD = 64 \Rightarrow$ AI

= 6.4 cm.

(*ii*) From (*i*) $BD^2 = AD \times CD$

But AC =18 cm, AD = 6 cm



$\therefore \quad BD^2 = 6 \times 12 = 72$

(∵ CD = AC − AD = 18 − 6 = 12 cm

$\therefore \quad BD = \sqrt{72} = \sqrt{36 \times 2}$ cm

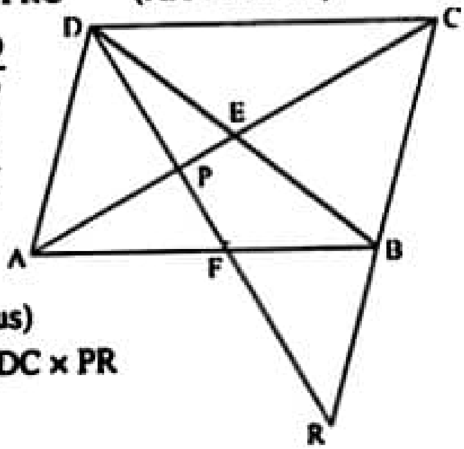
Prove that : DP × CR = DC × PR.

***Solution*— Proof—** In Δ APD and Δ PRC

∠ DPA = ∠ CPR (Vertically opposite angles)

∠ PAD = ∠ PCR (Alternate angles)

∴ Δ APD ~ Δ PRC (AA Postulate)



$\therefore \quad \frac{DP}{PR} = \frac{AD}{CR}$

$\Rightarrow \frac{DP}{PR} = \frac{DC}{CR}$

∵ AD = DC.

(Sides of rhombus)

⇒ DP × CR = DC × PR

Hence Proved.

17. Given : FB = FD, AE ⊥ FD and FC ⊥ AD.

Prove : $\frac{\textbf{FB}}{\textbf{AD}} = \frac{\textbf{BC}}{\textbf{ED}}$

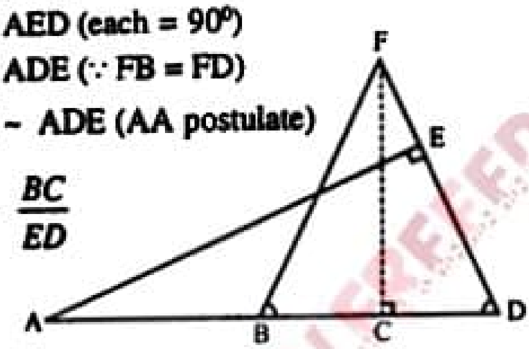
***Solution*— Proof—** In Δ FBC and Δ ADE

∠ FCB = ∠ AED (each = 90°)

∠ FBC = ∠ ADE (∵ FB = FD)

∴ Δ FBC ~ ADE (AA postulate)

$\therefore \quad \frac{FB}{AD} = \frac{BC}{ED}$



P.Q. In Δ ABC, ∠ B = 2 ∠ C and the bisector of angle B meets CA at point D. Prove that :

(*i*) Δ ABC and Δ ABD are similar ,

(*ii*) DC : AD = BC : AB.

***Solution*—**

Given— In Δ ABC, ∠ B = 2 ∠ C.

BD is the bisector of ∠ B

which meets AC at D

To Prove—

(*i*) Δ ABC ~ Δ ABD (*ii*) DC : AD = BC : AB

Proof—

∵ BD is the bisector of ∠ B

$\therefore \quad \angle ABD = \frac{1}{2} \angle B = \angle C$

Now in Δ ABC and Δ ABD,

∠ A = ∠ A (Common)

∠ C = ∠ ABD (Proved)

∴ Δ ABC ~ Δ ABD (AA Postulate)

$\therefore \quad \frac{BC}{BD} = \frac{AB}{AD}$

$\Rightarrow \frac{BC}{AB} = \frac{BD}{AD}$ (By Alternendo)

$\Rightarrow \frac{BC}{AB} = \frac{DC}{AD}$

∵ BD = DC (Opposite sides to equal angles)

∴ BC : AB = DC : AD or DC : AD = BC : AB

18. In Δ PQR, ∠ Q = 90° and QM is perpendicular to PR. Prove that :

(*i*) PQ² = PM × PR (*ii*) QR² = PR × MR

(*iii*) PQ² + QR² = PR²

***Solution*— Given—** In Δ PQR, ∠ Q = 90°

QM ⊥ PR.

To Prove—

(*i*) PQ² = PM × PR (*ii*) QR² = PR × MR

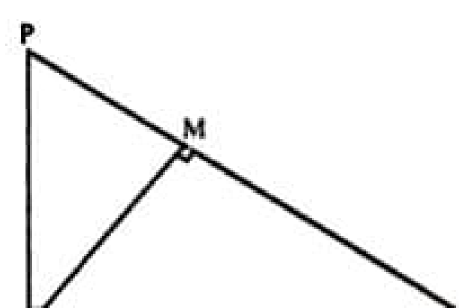
(*iii*) PQ² + QR² = PR²

Proof— In Δ PQM and Δ PQR,

∠ QMP = ∠ PQR (each = 90°)

∠ P = ∠ P (Common)

∴ Δ PQM ~ Δ PQR (AA postulate)



***Solution*—**

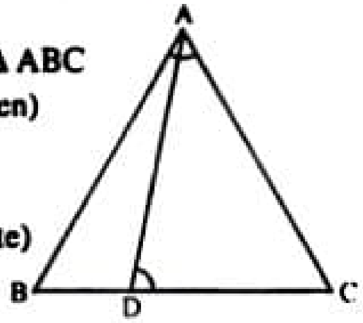
Proof : In Δ ADC and Δ ABC

$\angle ADC = \angle BAC$ (Given)

$\angle C = \angle C$

$\therefore$ $\Delta ADC \sim \Delta ABC$

(AA postulate)



$\therefore \dfrac{CA}{CB} = \dfrac{CD}{CA}$

$\Rightarrow CA \times CA = CB \times CD$

$\Rightarrow CA^2 = CB \times CD$ **Hence Proved.**

14. In the given figure, ΔABC and ΔAMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

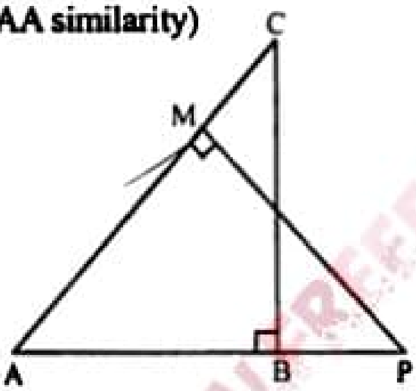
(*i*) Prove $\Delta ABC \sim \Delta AMP$ (*ii*) Find AB and BC.

***Solution*—(*i*)** In ΔABC and ΔAMP,

$\angle A = \angle A$ (Common)

$\angle ABC = \angle AMP$ (Each = 90°)

$\Rightarrow \Delta ABC \sim \Delta AMP$ (AA similarity)



(*ii*) From (*i*), $\dfrac{AC}{AP} = \dfrac{BC}{PM}$

(Sides are proportional)

$\Rightarrow \dfrac{10}{15} = \dfrac{BC}{12} \Rightarrow BC = \dfrac{12 \times 10}{15} = 8$ cm

From right triangle ABC, we have

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow 10^2 = AB^2 + 8^2 \Rightarrow 100 = AB^2 + 64$

$\Rightarrow AB^2 = 100 - 64 = 36 \Rightarrow AB = 6$ cm

Hence, AB = 6 cm, BC = 8 cm

P.Q. E and F are the points in sides DC and AB respectively of parallelogram ABCD. If diagonal AC and segment EF intersect at G ; prove that AG × EG = FG × CG.

***Solution*— Proof :** In Δ AGF and Δ EGC,

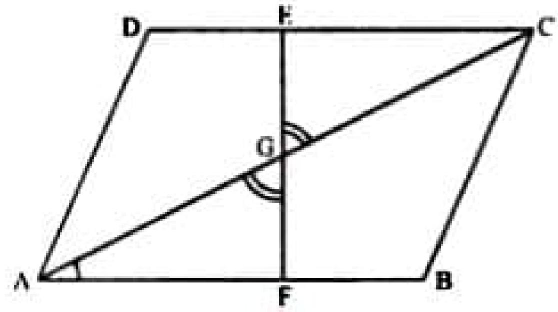
$\angle GAF = \angle GCE$ (Alternate angles)

$\angle AGF = \angle CGE$ (Vertically opposite angles)

$\therefore$ $\Delta AGF \sim \Delta EGC$ (AA postulate)

$\therefore \dfrac{AG}{CG} = \dfrac{FG}{EG}$

$\Rightarrow AG \times EG = FG \times CG.$ **Hence Proved.**



15. Given : RS and PT are altitudes of Δ PQR prove that :

(i) $\Delta PQT \sim \Delta QRS$, (ii) PQ × QS = RQ × QT.

***Solution*— Proof :** In Δ PQT and Δ QRS,

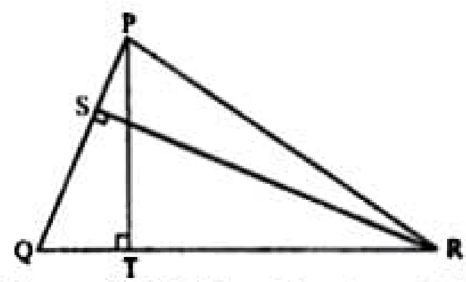
$\angle PTQ = \angle RSQ$ (Each = 90°)

$\angle Q = \angle Q$ (Common)

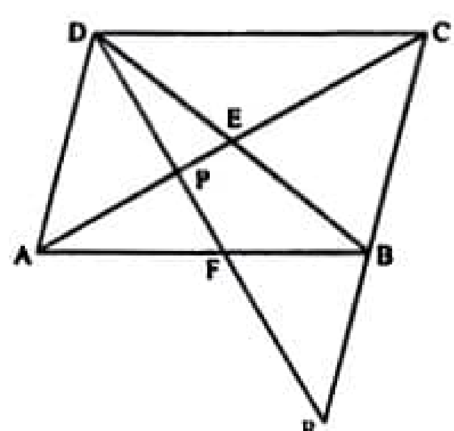
$\therefore$ $\Delta PQT \sim \Delta QRS$ (AA postulate)

$\therefore \dfrac{PQ}{RQ} = \dfrac{QT}{QS} \Rightarrow PQ \times QS = RQ \times QT.$

Hence Proved.



16. Given : ABCD is a rhombus, DPR and CBR are straight lines.



1. State, true or false :

(i) Two similar polygons are necessarily congruent.

(ii) Two congruent polygons are necessarily similar.

(iii) All equiangular triangles are similar.

(iv) All isosceles triangles are similar.

(v) Two isosceles-right triangles are similar.

(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.

(vii) The diagonals of a trapezium, divide each other into proportional segments.

Solution—

(i) False. (ii) True. (iii) True. (iv) False. (v) True. (vi) True. (vii) True.

P.Q. In triangle ABC, DE is parallel to BC; where D and E are the points on AB and AC respectively. Prove that $\Delta ADE \sim \Delta ABC$. Also, find the length of DE, if AD = 12 cm BD = 24 cm, and BC = 8 cm.

Solution—

In Δ ABC, DE ∥ BC.

$\therefore \angle ADE = \angle ABC$ (Corresponding angles)

and $\angle AED = \angle ACB$

Now, in Δ ADE and Δ ABC.

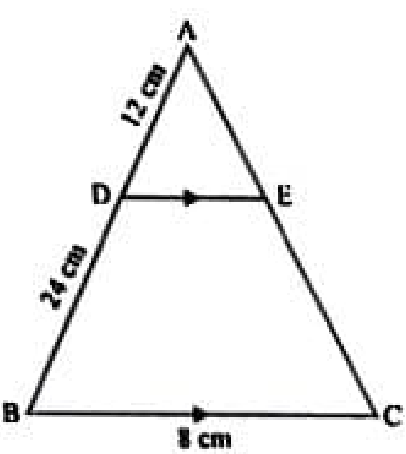
$\angle ADE = \angle ABC$ (Proved)

$\angle AED = \angle ACB$ (Proved)

$\angle A = \angle A$ (Common)

$\therefore \Delta ADE \sim \Delta ABC$ (AAA Postulate)

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$



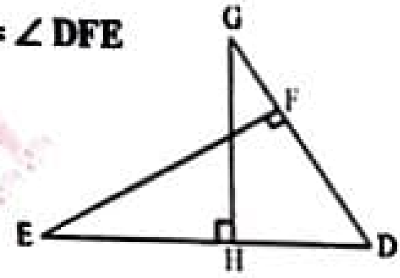
$$\Rightarrow \frac{AD}{AD+DB} = \frac{DE}{BC} \Rightarrow \frac{2}{12+24} = \frac{DE}{8}$$

[$\therefore$ AD = 12 cm, DB = 24 cm and BC = 8 cm]

$$\Rightarrow \frac{12}{36} = \frac{DE}{8}$$

$$\therefore \quad DE = \frac{12\times 8}{36} = \frac{8}{3} = 2\frac{2}{3} \text{ cm Ans.}$$

**12. Given – $\angle GHE = \angle DFE = 90°$,
DH = 8, DF = 12,
DG = 3x – 1 and
DE = 4x + 2.**



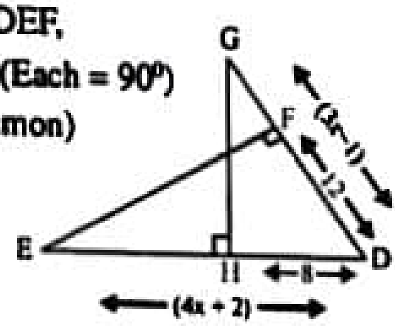
Find; the lengths of segments DG and DE.

Solution—

In Δ DGH and Δ DEF,

$\angle DHG = \angle DFE$ (Each = 90°)

$\angle D = \angle D$ (Common)



$\therefore \Delta DGH \sim \Delta DEF$ (AA postulate)

$$\therefore \quad \frac{DG}{DE} = \frac{DH}{DF}$$

$$\therefore \quad \frac{3x-1}{4x+2} = \frac{8}{12}$$

$\Rightarrow 12(3x-1) = 8(4x+2)$

$\Rightarrow 36x - 12 = 32x + 16$

$\Rightarrow 36x - 32x = 16 + 12 \Rightarrow 4x = 28$

$$\therefore \quad x = \frac{28}{4} = 7$$

$\therefore$ DG = 3x – 1 = 3 × 7 – 1 = 21 – 1 = 20

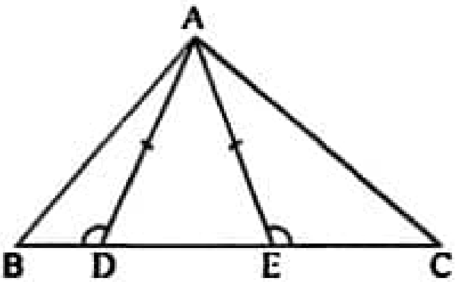
and DE = 4x + 2 = 4 × 7 + 2 = 28 + 2 = 30 Ans.

13. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that $CA^2 = CB \times CD$.

$\Rightarrow ME = \frac{15 \times 6}{24} = \frac{15}{4} = 3.75$ cm

and $DM = \frac{15 \times 24}{24} = 15$ cm

8. In the given figure, AD = AE and $AD^2 = BD \times EC$.
Prove that : triangles ABD and CAE are similar.



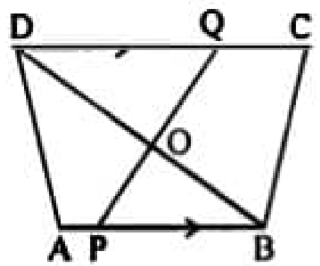
Sol. In the given figure,
AD = AE
$AD^2 = BD \times EC$
To prove : $\Delta ABD \sim \Delta CAE$
Proof : In ΔADC, AD = AE
$\therefore \angle ADE = \angle AED$
(Angles opposite to equal sides)
But $\angle ADE + \angle ADB = \angle AED + \angle AEC = 180°$
$\therefore \angle ADB = \angle AEC$
$AD^2 = BD \times EC$

$\frac{AD}{BD} = \frac{EC}{AD}$

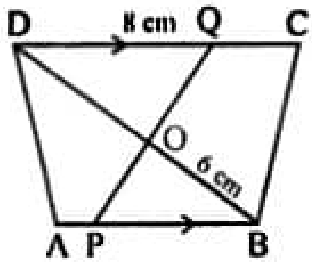
$\Rightarrow \frac{AE}{BD} = \frac{EC}{AD}$ ($\because$ AD = AE)

and $\angle ADB = \angle AEC$
$\therefore \Delta ABD \sim \Delta CAE$ (SAS axiom)

9. In the given figure, AB // DC, BO = 6 cm and DQ = 8 cm; find: BP × DO.



Sol. In the given figure,
AB ∥ DC,
BO = 6 cm, DQ = 8 cm
Find BP × DO
In ΔODQ and ΔOPB
$\angle DOQ = \angle POB$
(Vertically opposite angles)
$\angle DQO = \angle OPB$ (Alternate angles)
$\therefore \Delta ODQ \sim \Delta OPB$



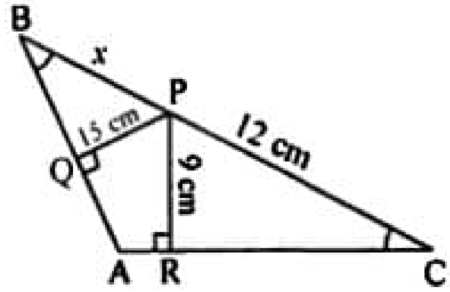
$\therefore \frac{DQ}{PB} = \frac{QO}{OP} = \frac{DO}{OB}$

$\Rightarrow \frac{8}{PB} = \frac{DO}{6}$

By cross multiplication,
$PB \times DO = 8 \times 6 = 48\ cm^2$

10. Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and PR = 9 cm; find the length of PB.

Sol. In ΔABC, $\angle ABC$ is an obtused angle,
AB = AC
P is a point on BC such that PC = 12 cm
PQ and PR are perpendiculars to the sides AB and AC respectively.
PQ = 15 cm and PR = 9 cm



To find the length of PB
In ΔPBQ and ΔPRC
$\angle B = \angle C$ (Opposite angles of equal sides)
$\angle Q = \angle P$ (Each 90°)
$\therefore \Delta PBQ \sim \Delta PRC$ (AA axiom)

$\therefore \frac{PB}{PC} = \frac{PQ}{PR}$

(corresponding sides are proportional)

$\Rightarrow \frac{x}{12} = \frac{15}{9} \Rightarrow x = \frac{15 \times 12}{9} = 20$

$\therefore$ PB = 20 cm

To prove :

(*i*) CB : BA = CP : PA

(*ii*) AB × BC = BP × CA

Proof :

(*i*) In ΔABC,

BP is the bisector of ∠ABC

$\therefore \dfrac{CB}{BA} = \dfrac{CP}{PA}$

⇒ CB : BA = CP : PA

(*ii*) In ΔABC and ΔABP

∠A = ∠A (Common)

∠C = ∠ABP

$\{\angle C = \frac{1}{2}\angle B$ and BP is angle bisector}

∴ ΔABC ~ ΔABP (AA axiom)

$\therefore \dfrac{AB}{AP} = \dfrac{AC}{AB} = \dfrac{BC}{BP}$

$\dfrac{AC}{AB} = \dfrac{BC}{BP}$

By cross multiplication,

AB × BC = BP × CA

Hence proved.

6. In ΔABC; BM ⊥ AC and CN ⊥ AB; show that:

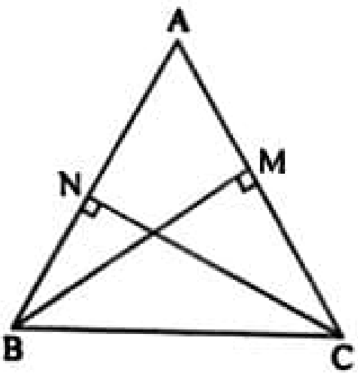
$\dfrac{AB}{AC} = \dfrac{BM}{CN} = \dfrac{AM}{AN}$

Sol. In ΔABC,

BM ⊥ AC and CN ⊥ AB

To prove :

$\dfrac{AB}{AC} = \dfrac{BM}{CN} = \dfrac{AM}{AN}$



Proof : In ΔAMB and ΔANC

∠A = ∠A (Common)

∠M = ∠N (each 90°)

∴ ΔAMB ~ ΔANC (AA axiom)

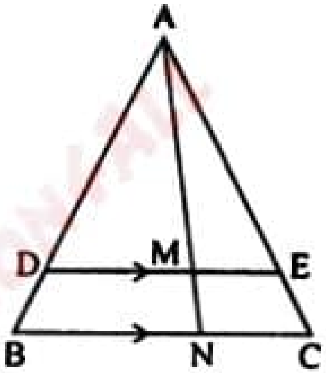
$\therefore \dfrac{AB}{AC} = \dfrac{BM}{CN} = \dfrac{AM}{AN}$

(Corresponding sides are proportional)

7. In the given figure, DE//BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.

(*i*) Write all possible pairs of similar triangles.

(*ii*) Find lengths of ME and DM.



Sol. In the given figure,

DE ∥ BC

AE = 15 cm, EC = 9 cm

NC = 6 cm and BN = 24 cm

(*i*) Write all the possible pairs of similar triangles.

(*ii*) Find lengths of ME and DM

Proof :

(*i*) In ΔABC

DE ∥ BC

∴ Pairs of similar triangles are

(a) ΔADE ~ ΔABC

(b) ΔADM ~ ΔABN

(c) ΔAME ~ ΔANC

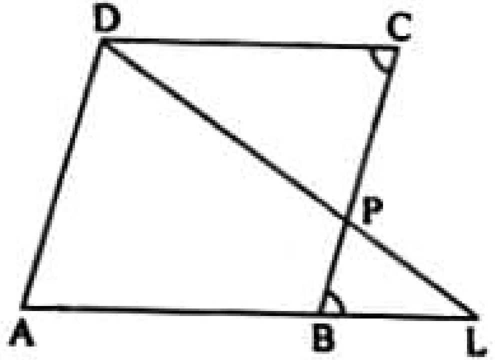
(*ii*) ∵ ΔAME ~ ΔANC

and ΔADM ~ ΔABN

$\therefore \dfrac{AE}{AC} = \dfrac{ME}{NC} = \dfrac{DM}{BN}$

(corresponding sides are proportional)

$\Rightarrow \dfrac{15}{15+9} = \dfrac{ME}{6} = \dfrac{DM}{24}$



To prove :

(i) DP : PL = DC : BL.

(ii) DL : DP = AL : DC.

Proof :

(i) In ΔBPL and ΔCPD

$\angle BPL = \angle CPD$ (Vertically opposite angles)

$\angle PBL = \angle PCD$ (Alternate angles)

$\therefore \Delta BPL \sim \Delta CPD$ (AA axiom)

$$\therefore \frac{PL}{DP} = \frac{BL}{DC}$$

$$\Rightarrow \frac{DP}{PL} = \frac{DC}{BL} \Rightarrow DC : PL = DC : BL$$

(ii) In ΔALD,

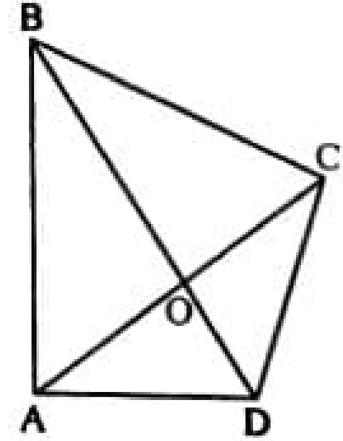
$PB \parallel AD$ $(\because BC \parallel AD)$

$$\therefore \frac{DL}{DP} = \frac{AL}{AB}$$

$$\Rightarrow \frac{DL}{DP} = \frac{AL}{DC} \quad (\because AB = DC)$$

$\therefore DL : DP = AL : DC$

Hence proved.



Proof :

(i) In ΔAOB and ΔCOD

$\angle AOB = \angle COD$

(Vertically opposite angles)

$$\frac{AO}{OC} = \frac{BO}{OD} = \frac{2}{1} \quad \left\{\begin{matrix}\because AO = 2OC \text{ and} \\ BO = 2OD\end{matrix}\right\}$$

$\therefore \Delta AOB \sim \Delta COD$ (SAS axiom)

(ii) $$\frac{AO}{OC} = \frac{BO}{OD}$$

(Corresponding sides are proportional)

$\Rightarrow AO \times OD = BO \times OC$

$\Rightarrow OA \times OD = OB \times OC$

Hence proved.

5. In ΔABC, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that :

(i) CB : BA = CP : PA

(ii) AB × BC = BP × CA

Sol. In ΔABC,

$\angle ABC = 2\angle ACB$

Bisector of $\angle ABC$ meets AC in P.

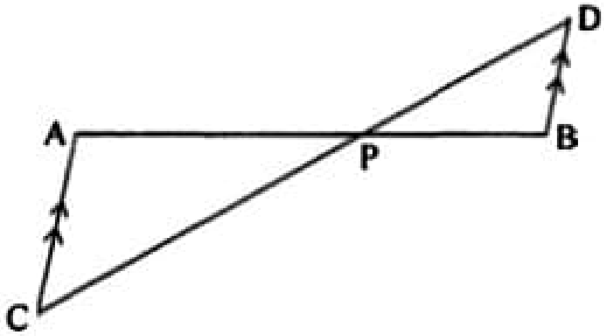
15 SIMILARITY (With Applications to Maps and Models)

EXERCISE 15 (A)

1. In the figure, given below, straight lines AB and CD intersect at P; and AC // BD. Prove that:

(*i*) ΔAPC and ΔBPD are similar.

(*ii*) If BD = 2.4 cm AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm; find the lengths of PA and PC.



Sol. Two line segments AB and CD intersect each other at P.

AC ∥ BD

To prove:

(*i*) ΔAPC ~ ΔBPD

(*ii*) If BD = 2.4 cm, AC = 3.6 cm, PD = 4.0 cm and PB = 3.2, find length of PA and PC

Proof:

(*i*) In ΔAPC and ΔAPD

∠APC = ∠BPD (Vertically opp. angles)

∠PAC = ∠PBD (Alternate angles)

∴ ΔAPC ~ ΔAPD (AA axiom)

(*ii*) $\therefore \frac{PA}{PB} = \frac{PC}{PD} = \frac{AC}{BD}$

(Corresponding sides of similar triangles are proportional)

$$\frac{PA}{3.2} = \frac{PC}{4} = \frac{3.6}{2.4}$$

$$\text{Now } \frac{PA}{3.2} = \frac{3.6}{2.4}$$

$$\Rightarrow PA = \frac{3.2 \times 3.6}{2.4} = 4.8 \text{ cm}$$

$$\text{and } \frac{PC}{PD} = \frac{AC}{BD} \Rightarrow \frac{PC}{4} = \frac{3.6}{2.4}$$

$$PC \times 2.4 = 3.6 \times 4 \Rightarrow PC = \frac{3.4 \times 4}{2.4} = 6 \text{ cm}$$

2. In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that:

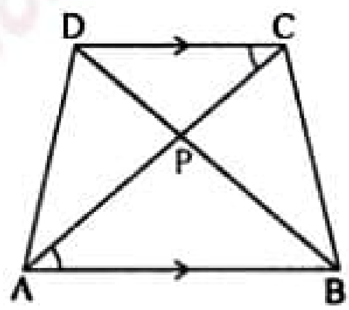
(*i*) ΔAPB is similar to ΔCPD.

(*ii*) PA × PD = PB × PC.

Sol. In trapezium ABCD

AB ∥ DC

Diagonals AC and BD intersect each other at P.



To prove:

(*i*) ΔAPB ~ ΔCPD.

(*ii*) PA × PD = PB × PC.

Proof: In ΔAPB and ΔCPD

∠APB = ∠CPD (Vertically opposite angles)

∠PAB = ∠PCD (Alternate angles)

∴ ΔAPB ~ ΔCPD (AA axiom)

$$\therefore \frac{PA}{PC} = \frac{PB}{PD}$$

⇒ PA × PD = PB × PC

Hence proved.

3. P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that:

(*i*) DP : PL = DC : BL. (*ii*) DL : DP = AL : DC.

Sol. P is a point on side BC of a parallelogram ABCD. DP is produced to meet AB produced at L.