

$$\begin{array}{r}
 \underline{7.} \quad \begin{array}{r|l}
 & 411 \\
 \hline
 4 & \overline{168921} \\
 +4 & 16 \\
 \hline
 81 & 89 \\
 +1 & 81 \\
 \hline
 821 & 821 \\
 +1 & 821 \\
 \hline
 & 0
 \end{array}
 \end{array}$$

$$\therefore \sqrt{168921} = 411$$

$$\begin{array}{r}
 \underline{8.} \quad \begin{array}{r|l}
 & 156 \\
 \hline
 1 & \overline{24336} \\
 +1 & 1 \\
 \hline
 25 & 143 \\
 +5 & 125 \\
 \hline
 306 & 1836 \\
 +6 & 1836 \\
 \hline
 & 0
 \end{array}
 \end{array}$$

$$\therefore \sqrt{24336} = 156$$

$$\begin{array}{r}
 \underline{9.} \quad \begin{array}{r|l}
 & 140 \\
 \hline
 1 & \overline{19600} \\
 +1 & 1 \\
 \hline
 24 & 96 \\
 +4 & 96 \\
 \hline
 & 00
 \end{array}
 \end{array}$$

$$\therefore \sqrt{19600} = 140$$

$$\begin{array}{r}
 \underline{10.} \quad \begin{array}{r|l}
 & 625 \\
 \hline
 6 & \overline{390625} \\
 +6 & 36 \\
 \hline
 122 & 306 \\
 +2 & 244 \\
 \hline
 1245 & 6225 \\
 +5 & 6225 \\
 \hline
 & 0
 \end{array}
 \end{array}$$

$$\therefore \sqrt{390625} = 625$$

11. Find the least number that should be subtracted from 99880 to make a perfect square.

Sol.

The least number to be subtracted from 99880 is 24.

$$\begin{array}{r}
 \begin{array}{r|l}
 & 316 \\
 \hline
 3 & \overline{99880} \\
 +3 & 9 \\
 \hline
 61 & 98 \\
 +1 & 61 \\
 \hline
 626 & 3780 \\
 +6 & 3756 \\
 \hline
 & 24
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \underline{7.} \quad \begin{array}{r}
 \begin{array}{|l}
 \hline
 4 \quad | \quad 168921 \\
 \hline
 +4 \quad | \quad 16 \\
 \hline
 81 \quad | \quad 89 \\
 +1 \quad | \quad 81 \\
 \hline
 821 \quad | \quad 821 \\
 +1 \quad | \quad 821 \\
 \hline
 \quad \quad | \quad 0 \\
 \hline
 \end{array}
 \end{array}
 \end{array}$$

$$\therefore \sqrt{168921} = 411$$

$$\begin{array}{r}
 \underline{8.} \quad \begin{array}{r}
 \begin{array}{|l}
 \hline
 1 \quad | \quad 24336 \\
 \hline
 +1 \quad | \quad 1 \\
 \hline
 25 \quad | \quad 143 \\
 +5 \quad | \quad 125 \\
 \hline
 306 \quad | \quad 1836 \\
 +6 \quad | \quad 1836 \\
 \hline
 \quad \quad | \quad 0 \\
 \hline
 \end{array}
 \end{array}
 \end{array}$$

$$\therefore \sqrt{24336} = 156$$

$$\begin{array}{r}
 \underline{9.} \quad \begin{array}{r}
 \begin{array}{|l}
 \hline
 1 \quad | \quad 140 \\
 \hline
 +1 \quad | \quad 1 \\
 \hline
 24 \quad | \quad 96 \\
 +4 \quad | \quad 96 \\
 \hline
 \quad \quad | \quad 00 \\
 \hline
 \end{array}
 \end{array}
 \end{array}$$

$$\therefore \sqrt{19600} = 140$$

$$\begin{array}{r}
 \underline{10.} \quad \begin{array}{r}
 \begin{array}{|l}
 \hline
 6 \quad | \quad 625 \\
 \hline
 +6 \quad | \quad 36 \\
 \hline
 122 \quad | \quad 306 \\
 +2 \quad | \quad 244 \\
 \hline
 1245 \quad | \quad 6225 \\
 +5 \quad | \quad 6225 \\
 \hline
 \quad \quad | \quad 0 \\
 \hline
 \end{array}
 \end{array}
 \end{array}$$

$$\therefore \sqrt{390625} = 625$$

11. Find the least number that should be subtracted from 99880 to make a perfect square.

Sol.

The least number to be subtracted from 99880 is 24.

$$\begin{array}{r}
 \begin{array}{|l}
 \hline
 3 \quad | \quad 99880 \\
 \hline
 +3 \quad | \quad 9 \\
 \hline
 61 \quad | \quad 98 \\
 +1 \quad | \quad 61 \\
 \hline
 626 \quad | \quad 3780 \\
 +6 \quad | \quad 3756 \\
 \hline
 \quad \quad | \quad 24 \\
 \hline
 \end{array}
 \end{array}$$

(77)

12. Find the least number that should be subtracted from 423922 to make a perfect square. Also, find the square root of the perfect square.

Sol.

The least number to be subtracted from 423922 is 121.

$$\begin{aligned}
 \text{Required perfect square} &= \\
 &= 423922 - 121 \\
 &= 423801
 \end{aligned}$$

$$\begin{array}{r}
 \begin{array}{|l}
 \hline
 6 \quad | \quad 423922 \\
 \hline
 +6 \quad | \quad 36 \\
 \hline
 125 \quad | \quad 639 \\
 +5 \quad | \quad 625 \\
 \hline
 1301 \quad | \quad 1922 \\
 +1 \quad | \quad 1301 \\
 \hline
 \quad \quad | \quad 121 \\
 \hline
 \end{array}
 \end{array}$$

$$48 - 3 = 45$$

$$45 - 5 = 40$$

$$40 - 7 = 33$$

$$24 - 11 = 13$$

$$13 - 13 = 0$$

Since, we performed subtraction for 7 times,

$$\text{i.e. } \sqrt{49} = 7.$$

(75)



EXERCISE 3.3.

Evaluate (Q 1 to 10):

1. $\sqrt{33856}$

2. $\sqrt{42849}$

3. $\sqrt{4761}$

4. $\sqrt{3249}$

5. $\sqrt{105625}$

6. $\sqrt{511225}$

7. $\sqrt{168921}$

8. $\sqrt{24336}$

9. $\sqrt{19600}$

10. $\sqrt{390625}$

Sol. 1.

$$\begin{array}{r} 184 \\ \hline 1 \overline{) 33856} \\ +1 \\ \hline 28 238 \\ +8 224 \\ \hline 364 1456 \\ +4 1456 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{33856} = 184$$

2.

$$\begin{array}{r} 207 \\ \hline 2 \overline{) 42849} \\ +24 \\ \hline 407 2849 \\ +7 2849 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{42849} = 207$$

3.

$$\begin{array}{r} 69 \\ \hline 6 \overline{) 4761} \\ +6 36 \\ \hline 129 1161 \\ +9 1161 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{4761} = 69$$

4.

$$\begin{array}{r} 57 \\ \hline 5 \overline{) 3249} \\ +5 25 \\ \hline 107 749 \\ +7 749 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{3249} = 57$$

5.

$$\begin{array}{r} 325 \\ \hline 3 \overline{) 105625} \\ +3 9 \\ \hline 62 156 \\ +2 124 \\ \hline 645 3225 \\ +5 3225 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{105625} = 325$$

6.

$$\begin{array}{r} 715 \\ \hline 7 \overline{) 511225} \\ +7 49 \\ \hline 141 212 \\ +1 141 \\ \hline 1425 7125 \\ +5 7125 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{511225} = 715$$

(76)

7.

$$\begin{array}{r} 411 \\ \hline 4 \overline{) 168921} \\ +4 16 \\ \hline 81 89 \\ +1 81 \\ \hline 821 821 \\ +1 821 \\ \hline 0 \end{array}$$

8.

$$\begin{array}{r} 156 \\ \hline 1 \overline{) 24336} \\ +1 \\ \hline 25 143 \\ +5 125 \\ \hline 306 1836 \\ +6 1836 \\ \hline 0 \end{array}$$

(ii) $n = 121$

$$\begin{array}{l|l} 121 - 1 = 120 & 85 - 13 = 72 \\ 120 - 3 = 117 & 72 - 15 = 57 \\ 117 - 5 = 112 & 57 - 17 = 40 \\ 112 - 7 = 105 & 40 - 19 = 21 \\ 105 - 9 = 96 & 21 - 21 = 0 \\ 96 - 11 = 85 & \end{array}$$

Since, we performed subtraction for 11 times.

i.e. $\sqrt{121} = 11$.

(iii) $n = 81$

$$\begin{array}{l|l} 81 - 1 = 80 & 56 - 11 = 45 \\ 80 - 3 = 77 & 45 - 13 = 32 \\ 77 - 5 = 72 & 32 - 15 = 17 \\ 72 - 7 = 65 & 17 - 17 = 0 \\ 65 - 9 = 56 & \end{array}$$

Since, we performed subtraction for 9 times.

i.e. $\sqrt{81} = 9$.

(iv) $n = 49$

$$\begin{array}{l|l} 49 - 1 = 48 & 33 - 9 = 24 \\ 48 - 3 = 45 & 24 - 11 = 13 \\ 45 - 5 = 40 & 13 - 13 = 0 \\ 40 - 7 = 33 & \end{array}$$

Since, we performed subtraction for 7 times,

i.e. $\sqrt{49} = 7$.

6. The students of a class contributed money for a relief camp. Each child contributed as many rupees as the number of children in the class. If the total collection is ₹ 5184, find the strength of the class.

Sol. Let no. of students in the class = x

Then, Contribution of each student = x

$$\therefore x \times x = ₹ 5184 \Rightarrow x^2 = 5184$$

$$\Rightarrow x = \sqrt{5184}$$

$$\Rightarrow x = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

$$\Rightarrow x = 2 \times 2 \times 2 \times 3 \times 3$$

$$\Rightarrow x = 72.$$

$$\begin{array}{r|l} 2 & 5184 \\ \hline 2 & 2592 \\ \hline 2 & 1296 \\ \hline 2 & 648 \\ \hline 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 54 \\ \hline 3 & 18 \\ \hline 3 & 6 \\ \hline 3 & 2 \\ \hline 2 & 1 \end{array}$$

Hence, no. of students in the class = 72.

7. Find the square root of the following by repeated subtraction :

(i) 64

(ii) 121

(iii) 81

(iv) 49

Sol. Subtract 1, 3, 5, 7, ... successively from the given number n .

(i) $n = 64$.

$$64 - 1 = 63$$

$$63 - 3 = 60$$

$$60 - 5 = 55$$

$$55 - 7 = 48$$

$$48 - 9 = 39$$

$$39 - 11 = 28$$

$$28 - 13 = 15$$

$$15 - 15 = 0$$

Since, we performed subtraction for 8 times.

$$\text{i.e. } \sqrt{64} = 8.$$

(iv) Number of digits in the square root $524176 = \frac{6}{2} = 3$

(v) Number of digits in the square root $1413721 = \frac{7+1}{2} = 4$

(vi) Number of digits in the square root $10588516 = \frac{8}{2} = 4$

(vii) Number of digits in the square root $1061912569 = \frac{10}{2} = 5$

5. In an auditorium, the number of rows is equal to the number of the chairs in each row. If the capacity of the auditorium is 2304, find the number of rows in the auditorium.

Sol. let the number of rows in the auditorium be x .

$$\therefore \text{Total number of chairs} = x \times x = x^2$$

but total number of chair (capacity) = 2304,

$$x^2 = 2304$$

$$\Rightarrow x = \sqrt{2304}$$

$$\Rightarrow x = \sqrt{\underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}}_{2304}}$$

$$\Rightarrow x = 2 \times 2 \times 2 \times 2 \times 3$$

$$\Rightarrow x = 48$$

$$\begin{array}{r|l} 2 & 2304 \\ \hline 2 & 1152 \\ \hline 2 & 576 \\ \hline 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Hence, Number of rows in the auditorium = 48.

(iv) $1300 = 2 \times 2 \times 5 \times 5 \times 13$

Since the prime factor 13 is not in a pair we divide 1300 by 13 to get 100, which has prime factors in pairs.

$$100 = 2 \times 2 \times 5 \times 5$$

$$\sqrt{100} = 2 \times 5 = 10.$$

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(v) $24000 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3} \times \underline{5} \times \underline{5} \times \underline{5}$

Since the prime factor $3 \times 5 = 15$ is not in a pair we divide 24000 by 15 to get 1600, which has prime factors in pairs.

$$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

$$\therefore \sqrt{1600} = 2 \times 2 \times 2 \times 5 = 40.$$

4. Without any calculation, find the number of digits in the square root of each of the following numbers:

(i) 961

(ii) 4489

(iii) 63001

(iv) 524176

(v) 1413721

(vi) 10588516 (vii) 1061912569

Sol. We know that, If a perfect square is of n digits, then its square root will have $\frac{n}{2}$ digits if n is even or $\frac{n+1}{2}$ digits if n is odd.

(i) Number of digits in the square root 961 = $\frac{3+1}{2} = 2$

(ii) Number of digits in the square root 4489 = $\frac{4}{2} = 2$

(iii) Number of digits in the square root 63001 = $\frac{5+1}{2} = 3.$

3. For each of the numbers in Q. 2, find the smallest number by which it should be divided so as to get a perfect square. Also, find the square root of the perfect square.

Sol. (i) $1250 = 2 \times 5 \times 5 \times 5 \times 5$

Since the prime factor 2 is not in pair, we divide 1250 by 2 to get 625, which has prime factors in pairs.

$$\therefore 625 = 5 \times 5 \times 5 \times 5$$

$$\Rightarrow \sqrt{625} = 5 \times 5 = 25$$

72/768

(ii) $7350 = 2 \times 3 \times 5 \times 5 \times 7 \times 7$

Since the prime factor $(2 \times 3 = 6)$ is not in pair, we divide 7350 by 6 to get 1225, which has prime factors in pairs.

$$\therefore 1225 = 5 \times 5 \times 7 \times 7$$

$$\sqrt{1225} = 5 \times 7 = 35$$

(iii) $47068 = 2 \times 2 \times 7 \times 41 \times 41$

Since the prime factor 7 is not in pair, we divide 47068 by 7 to get 6724, which has prime factors in pairs.

$$\therefore 6724 = 2 \times 2 \times 41 \times 41$$

$$\sqrt{6724} = 2 \times 41 = 82$$

Since the prime factor 13 does not occur in pair, 1300 is not

$$\begin{array}{r|l} 5 & 65 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

a perfect square. 1300 should be multiplied by 13. So that 13 also occurs in pair and the product 16900 is a perfect square.

$$16900 = 2 \times 2 \times 5 \times 5 \times 13 \times 13$$

$$\sqrt{16900} = 2 \times 5 \times 13 = 130.$$

$$(v) 24000 = \underline{2 \times 2 \times 2 \times 2 \times 2} \times \underline{3} \times \underline{5 \times 5 \times 5}$$

Since the prime factor (3 x 5 = 15) does not occur in pair

24000 is not a perfect square.

24000 should be multiplied by 3 x 5 = 15.

So that 15 also occurs in pair and the product 360000.

$$360000 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5 \times 5 \times 5}$$

$$\begin{aligned} \sqrt{360000} &= 2 \times 2 \times 2 \times 3 \times 5 \times 5 \\ &= 600 \end{aligned}$$

$$\begin{array}{r|l} 2 & 24000 \\ \hline 2 & 12000 \\ \hline 2 & 6000 \\ \hline 2 & 3000 \\ \hline 2 & 1500 \\ \hline 2 & 750 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 5 & 245 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

70/768

Since the prime factor (2x3=6) does not occur

in pair, 7350 is not a perfect square. 7350 should be multiplied by 6. So that 2 and 3 also occurs in pair and the product 44100 is a perfect square.

$$44100 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7$$

$$\sqrt{44100} = 2 \times 3 \times 5 \times 7 = 210$$

(iii) $47068 = 2 \times 2 \times 7 \times 41 \times 41$

$$\begin{array}{r|l} 2 & 47068 \\ \hline 2 & 23534 \\ \hline 7 & 11767 \\ \hline 41 & 1681 \\ \hline 41 & 41 \\ \hline & 1 \end{array}$$

Since the prime factor 7 does not occur in pair, 47068 is not a

perfect square. 47068 should be multiplied by 7 so that 7 also occurs in pair and the product 329476 is a perfect square.

$$329476 = 2 \times 2 \times 7 \times 7 \times 41 \times 41$$

$$\sqrt{329476} = 2 \times 7 \times 41 = 574$$

$$(iv) 8836 = 2 \times 2 \times 47 \times 47$$

$$= (2 \times 47)^2$$

$$= (94)^2$$

$$\therefore \sqrt{8836} = 94$$

$$\begin{array}{r|l} 2 & 8836 \\ \hline 2 & 4418 \\ \hline 47 & 2209 \\ \hline 47 & 47 \\ \hline & 1 \end{array}$$

69/768

$$(v) 1156 = 2 \times 2 \times 17 \times 17$$

$$= (2 \times 17)^2$$

$$= (34)^2$$

$$\therefore \sqrt{1156} = 34$$

$$\begin{array}{r|l} 2 & 1156 \\ \hline 2 & 578 \\ \hline 17 & 289 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

2. For each of the following, find the smallest number by which it should be multiplied so as to get a perfect square. Also, find the square root of the perfect square.

(i) 1250

(ii) 7350

(iii) 47068

(iv) 1300

(v) 24000

Sol. (i)

$$1250 = 2 \times 5 \times 5 \times 5 \times 5$$

$$\begin{array}{r|l} 2 & 1250 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, the prime factor 2 does not occur in pair, 1250 is not a perfect square. 1250 should be multiplied by 2, so that 2 also occurs in pairs and the product 2500 is a perfect square.

$$2500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

$$\sqrt{2500} = 2 \times 5 \times 5 = 50$$



EXERCISE 3.2

1. Find the square root of :

(i) 5929

(ii) 3969

(iii) 12544

(iv) 8836

(v) 1156

Sol. (i) $5929 = \underline{7 \times 7} \times \underline{11 \times 11}$
 $= (7 \times 11)^2$
 $= (77)^2$

$$\begin{array}{r|l} 7 & 5929 \\ \hline 7 & 847 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

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$\therefore \sqrt{5929} = 77.$

(ii) $3969 = \underline{3 \times 3 \times 3 \times 3} \times \underline{7 \times 7}$
 $= (3 \times 3 \times 7)^2$
 $= (63)^2$

$$\begin{array}{r|l} 3 & 3969 \\ \hline 3 & 1323 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \sqrt{3969} = 63.$

(iii) $12544 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \times \underline{7 \times 7}$
 $= (2 \times 2 \times 2 \times 2 \times 7)^2$
 $= (112)^2$

$$\begin{array}{r|l} 2 & 12544 \\ \hline 2 & 6272 \\ \hline 2 & 3136 \\ \hline 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \sqrt{12544} = 112.$

9. Observe the following patterns and find the missing digits :

$$11^2 = 121$$

$$101^2 = 10201$$

$$1001^2 = 1002001$$

$$10001^2 = 100020001$$

$$100001^2 = 1\underline{0000}2\underline{0000}1$$

$$1000001^2 = 1\underline{000000}2\underline{000000}1$$

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10. Observe the pattern and find the missing numbers :

$$1^2 + 2^2 + 2^2 = 3^2$$

$$2^2 + 3^2 + 6^2 = 7^2$$

$$3^2 + 4^2 + 12^2 = 13^2$$

$$4^2 + \underline{5}^2 + 20^2 = 21^2$$

$$5^2 + 6^2 + \underline{30}^2 = 31^2$$

$$6^2 + \underline{7}^2 + \underline{42}^2 = \underline{43}^2$$

11. Observe the following pattern :

$$(25)^2 = (2 \times 3) \times 100 + 25 = 625$$

$$(35)^2 = (3 \times 4) \times 100 + 25 = 1225$$

$$(45)^2 = (4 \times 5) \times 100 + 25 = 2025$$

$$\vdots$$
$$(85)^2 = (8 \times 9) \times 100 + 25 = 7225$$

Using the above pattern, find the squares of the following numbers :

(i) 235

(ii) 115

(iii) 105

(iv) 65

(iii) let $2m = 20$, $m = 10$

$$m^2 - 1 = 10^2 - 1 = 100 - 1 = 99$$

$$m^2 + 1 = 10^2 + 1 = 100 + 1 = 101$$

\therefore The other two members of the Pyth

66/768

Triplet are 99 and 101.

(iv) let $2m = 26$, $m = 13$

$$m^2 - 1 = 13^2 - 1 = 169 - 1 = 168$$

$$m^2 + 1 = 13^2 + 1 = 169 + 1 = 170$$

\therefore The other two members of the Pythagorean

Triplet are 168 and 170.

8. Without adding, find the sum :

(i) $1 + 3 + 5 + 7$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23$

Sol. We know that, the sum of first n odd numbers = n^2

(i) $1 + 3 + 5 + 7 = 4^2 = 16$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = 49$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 12^2 = 144$

$\therefore (12, 35, 37)$ is a Pythagorean triplet.

(63)

$$(iv) \quad 8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

$\therefore (8, 15, 17)$ is a Pythagorean triplet.

$$(v) \quad 12^2 + 21^2 = 144 + 441 = 585 \neq 24^2$$

$\therefore (12, 21, 24)$ is not a Pythagorean triplet.

$$(vi) \quad 16^2 + 63^2 = 256 + 3969 = 4225 = 65^2$$

$\therefore (16, 63, 65)$ is a Pythagorean triplet.

7. Find the other two members of the Pythagorean triplet, one of whose members is :

(i) 10

(ii) 14

(iii) 20

(iv) 26

[HOTS]

Sol. For any natural number m the Pythagorean triplet = $2m$, $m^2 - 1$ and $m^2 + 1$

$$(i) \text{ Let } 2m = 10, m = 5$$

$$m^2 - 1 = 5^2 - 1 = 25 - 1 = 24$$

$$m^2 + 1 = 5^2 + 1 = 25 + 1 = 26$$

\therefore The other two members of the Pythagorean triplet are 24 and 26.

$$(ii) \text{ Let } 2m = 14, m = 7$$

$$m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$$

$$m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$$

\therefore The other two members of the Pythagorean triplet are 48 and 50.

(64)

5. Using the properties of squares, find the following products :

(i) 32×34

(ii) 59×61

(iii) 99×101

(iv) 1000×1002

[HOTS]

Sol. (i) $32 \times 34 = (33-1)(33+1)$
 $= (33)^2 - (1)^2 = 1089 - 1 = 1088.$

(ii) $59 \times 61 = (60-1)(60+1) = (60)^2 - (1)^2$
 $= 3600 - 1 = 3599.$ 64/768

(iii) $99 \times 101 = (100-1)(100+1) = (100)^2 - (1)^2$
 $= 10000 - 1 = 9999.$

(iv) $1000 \times 1002 = (1000-1)(1000+1)$
 $= (1000)^2 - (1)^2 = 1002000 - 1$
 $= 1002000.$

6. In the following identify the Pythagorean triplets :

(i) (3, 4, 5)

(ii) (6, 7, 8)

(iii) (12, 35, 37)

(iv) (8, 15, 17)

(v) (12, 21, 24)

(vi) (16, 63, 65)

Sol. (i) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$\therefore (3, 4, 5)$ is a Pythagorean triplet.

(ii) $6^2 + 7^2 = 36 + 49 = 85 \neq 8^2$

$\therefore (6, 7, 8)$ is not a Pythagorean triplet.

(iii) $12^2 + 35^2 = 144 + 1225 = 1369 = 37^2$

$\therefore (12, 35, 37)$ is a Pythagorean triplet.

(63)

(iv) $8^2 + 15^2 = 64 + 225 = 289 = 17^2$

(8, 15, 17) is a Pythagorean triplet

$$\begin{aligned}
 \text{(xii)} \quad 4761 &= \underline{3 \times 3} \times \underline{23 \times 23} \\
 &= (3 \times 23) \times (23 \times 3) \\
 &= (69)^2
 \end{aligned}$$

$$\begin{array}{r|l}
 3 & 4761 \\
 \hline
 3 & 1587 \\
 \hline
 23 & 529 \\
 \hline
 & 23
 \end{array}$$

$\therefore 4761$ is a perfect square.

3. Using properties of squares, find the values of the following :

(i) $15^2 - 14^2$

(ii) $22^2 - 21^2$

(iii) $51^2 - 50^2$

(iv) $100^2 - 99^2$

(v) $1001^2 - 1000^2$

(vi) $2115^2 - 2114^2$

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Sol. (i) $15^2 - 14^2 = (15+14)(15-14) = 29 \times 1 = 29$

(ii) $22^2 - 21^2 = (22+21)(22-21) = 43 \times 1 = 43$

(iii) $51^2 - 50^2 = (51+50)(51-50) = 101 \times 1 = 101$

(iv) $100^2 - 99^2 = (100+99)(100-99) = 199 \times 1 = 199$

(v) $1001^2 - 1000^2 = (1001+1000)(1001-1000) = 2001 \times 1 = 2001$

(vi) $2115^2 - 2114^2 = (2115+2114)(2115-2114) = 4229 \times 1 = 4229$

4. How many numbers lie between the squares of the following numbers? [HOTS]

(i) 10 and 11

(ii) 15 and 16

(iii) 100 and 101

Sol. We know that, there are $2n$ non perfect square numbers between the squares of the numbers n and $(n+1)$.

(i) Numbers lie between the squares 10 and 11 = $2 \times 10 = 20$.

(ii) Numbers lie between the squares 15 and 16 = $2 \times 15 = 30$.

(iii) Numbers lie between the squares 100 and 101 = $2 \times 100 = 200$.

$$\begin{aligned}
 \text{(vii)} \quad 1521 &= \underline{3 \times 3} \times \underline{13 \times 13} \\
 &= (3 \times 13) \times (3 \times 13) \\
 &= (39)^2.
 \end{aligned}$$

$$\begin{array}{r|l}
 3 & 1521 \\
 \hline
 3 & 507 \\
 \hline
 13 & 169 \\
 \hline
 13 & 13 \\
 \hline
 & 1
 \end{array}$$

$\therefore 1521$ is a perfect square.

(viii) 21952 is not a perfect square since it ends with 2.

$$\text{(ix)} \quad 28324 = \underline{2 \times 2} \times 7081$$

$$\begin{array}{r|l}
 2 & 28324 \\
 \hline
 2 & 14162 \\
 \hline
 & 7081
 \end{array}$$

Since prime factor does not occur twice, so, 28324 is not a perfect square.

$$\text{(x)} \quad 1500 = \underline{2 \times 2} \times \underline{5 \times 5} \times 5 \times 3$$

$$\begin{array}{r|l}
 2 & 1500 \\
 \hline
 2 & 750 \\
 \hline
 5 & 375 \\
 \hline
 5 & 75 \\
 \hline
 5 & 15 \\
 \hline
 & 3
 \end{array}$$

Since prime factor does not occur twice, so, 1500 is not a perfect square.

$$\begin{aligned}
 \text{(xi)} \quad 2025 &= \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5} \\
 &= (3 \times 3 \times 5) \times (3 \times 3 \times 5) \\
 &= (45)^2
 \end{aligned}$$

$$\begin{array}{r|l}
 3 & 2025 \\
 \hline
 3 & 675 \\
 \hline
 3 & 225 \\
 \hline
 3 & 75 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$\therefore 2025$ is a perfect square.

2. Check, if the following numbers are perfect squares :

(i) 9801

(v) 7938

(ix) 28324

(ii) 343

(vi) 2500

(x) 1500

(iii) 6287

(vii) 1521

(xi) 2025

(iv) 3692

(viii) 21952

(xii) 4761

Sol. (i) $9801 = \underline{3 \times 3 \times 3 \times 3 \times 11 \times 11}$

i.e. $9801 = (3 \times 3 \times 11) \times (3 \times 3 \times 11)$

i.e. $9801 = (99)^2$

$\therefore 9801$ is a perfect square.

$$\begin{array}{r|l} 3 & 9801 \\ \hline 3 & 3267 \\ \hline 3 & 1089 \\ \hline 3 & 363 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

(ii) $343 = \underline{7 \times 7 \times 7}$

Since prime factors does not occur twice. So, 343 is not a perfect square.

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(iii) 6287 is not a perfect square since it ends with 7.

(iv) 3692 is not a perfect square since it ends with 2.

(v) 7938 is not a perfect square since it ends with 8.

(vi) $2500 = \underline{2 \times 2 \times 5 \times 5 \times 5 \times 5}$
 $= (2 \times 5 \times 5) \times (2 \times 5 \times 5)$
 $= (50)^2$

$\therefore 2500$ is a perfect square.

$$\begin{array}{r|l} 2 & 2500 \\ \hline 2 & 1250 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

SQUARES AND SQUARE ROOTS



EXERCISE 3.1

1. Find the squares of the following numbers :

(i) 31

(ii) 49

(iii) 55

(iv) 121

(v) 355

(vi) 1.2

(vii) 0.016

(viii) 6.25

(ix) $\frac{12}{25}$

(x) $\frac{19}{-24}$

(xi) $\frac{-26}{15}$

(xii) $\frac{-16}{17}$

Sol. (i) $(31)^2 = 31 \times 31 = 961$

(ii) $(49)^2 = 49 \times 49 = 2401$

(iii) $(55)^2 = 55 \times 55 = 3025$ (iv) $(121)^2 = 121 \times 121 = 14641$

(v) $(355)^2 = 355 \times 355 = 126025$ (vi) $(1.2)^2 = 1.2 \times 1.2 = 1.44$

(vii) $(0.016)^2 = 0.016 \times 0.016 = 0.000256$

(viii) $(6.25)^2 = 6.25 \times 6.25 = 39.0625$

(ix) $\left(\frac{12}{25}\right)^2 = \frac{12}{25} \times \frac{12}{25} = \frac{144}{625}$

(x) $\left(\frac{19}{-24}\right)^2 = \frac{19}{-24} \times \frac{19}{-24} = \frac{361}{576}$

(xi) $\left(\frac{-26}{15}\right)^2 = \frac{-26}{15} \times \frac{-26}{15} = \frac{676}{225}$

(xii) $\left(\frac{-16}{17}\right)^2 = \frac{-16}{17} \times \frac{-16}{17} = \frac{256}{289}$